LECTURE NOTES
ON
ENGINEERING DRAWING

B. Tech II Semester (MR-20)

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(Autonomous)

(b) *With the aid of a compass* (fig. 5-35).

(i) Draw a line *AB* equal to the given length.

(ii) At *A*, draw a line *AE* perpendicular to *AB*. (Refer problem 5-3, Method III, fig. 5-5).

(iii) With centre *A* and radius *AB*, draw an arc cutting *AE* at *D*.

(iv) With centres *B* and *D* and the same radius, draw arcs intersecting at *C*.

(v) Draw lines joining *C* with *B* and *D*.

Then *ABCD* is the required square.

5-12. TO CONSTRUCT REGULAR POLYGONS

CHAROTAR COGNIFRONT This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 9 for the following problem.

Problem 5-26. *To construct a regular polygon, given the length of its side.*

Let the number of sides of the polygon be seven (i.e. heptagon).

Method I: (fig. 5-36 and fig. 5-37):

(i) Draw a line *AB* equal to the given length.

(ii) With centre *A* and radius *AB*, draw a semi-circle *BP*.

(iii) With a divider, divide the semi-circle into seven equal parts (same as the number of sides). Number the division-points as 1, 2, etc. starting from *P*.

(iv) Draw a line joining *A* with the second division-point 2.

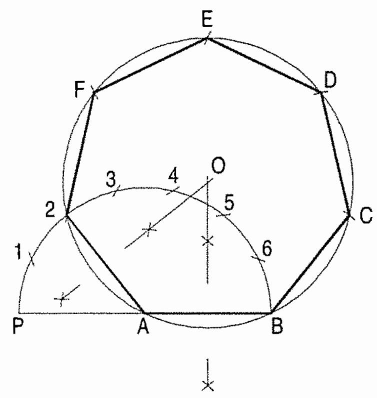

FIG. 5-36

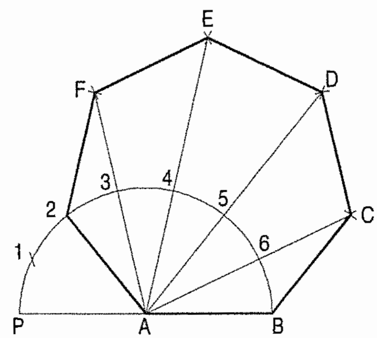

FIG. 5-37

(a) *Inscribe circle method* (fig. 5-36).

(i) Draw perpendicular bisectors of *A*2 and *AB* intersecting each other at *O*.

(ii) With centre *O* and radius *OA*, describe a circle.

(iii) With radius *AB* and starting from *B*, cut the circle at points, *C*, *D*.....2.

(iv) Draw lines *BC*, *CD* etc. thus completing the required heptagon.

(b) *Arc method* (fig. 5-37).

(i) With centre *B* and radius *AB*, draw an arc cutting the line *A*6-produced at *C*.

(ii) With centre *C* and the same radius, draw an arc cutting the line *A*5-produced at *D*.

(iii) Find points *E* and *F* in the same manner.

(iv) Draw lines *BC*, *CD* etc. and complete the heptagon.

Method II: General method for drawing any polygon (fig. 5-38):

(i) Draw a line *AB* equal to the given length.

(ii) At *B*, draw a line *BP* perpendicular and equal to *AB*.

(iii) Draw a line joining *A* with *P*.

(iv) With centre *B* and radius *AB*, draw the quadrant *AP*.

(v) Draw the perpendicular bisector of *AB* to intersect the straight line *AP* in 4 and the arc *AP* in 6.

(a) A *square* of a side equal to *AB* can be inscribed in the circle drawn with centre 4 and radius *A*4.

(b) A regular *hexagon* of a side equal to *AB* can be inscribed in the circle drawn with centre 6 and radius *A*6.

(c) The mid-point 5 of the line 4-6 is the centre of the circle of the radius *A*5 in which a regular pentagon of a side equal to *AB* can be inscribed.

(d) To locate centre 7 for the regular *heptagon* of side *AB*, step-off a division 6-7 equal to the division 5-6.

(i) With centre 7 and radius equal to *A*7, draw a circle.

(ii) Starting from *B*, cut it in seven equal divisions with radius equal to *AB*.

(iii) Draw lines *BC*, *CD* etc. and complete the heptagon.

Regular polygons of any number of sides can be drawn by this method.

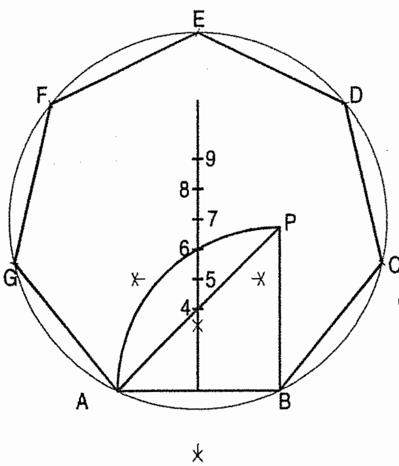

FIG. 5-38

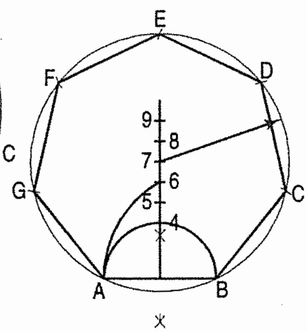

FIG. 5-39

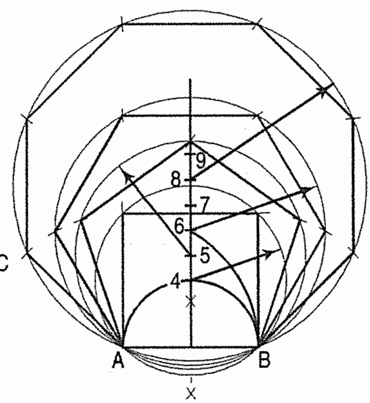

FIG. 5-40

Alternative method (fig. 5-39 and fig. 5-40):

(i) On *AB* as diameter, describe a semi-circle.

(ii) With either *A* or *B* as centre and *AB* as radius, describe an arc on the same side as the semi-circle.

(iii) Draw a perpendicular bisector of *AB* cutting the semi-circle at point 4 and the arc at point 6.

(iv) Obtain points 5, 7, 8 etc. as explained in method II.

Fig. 5-40 shows *a square, a regular pentagon, a regular hexagon and a regular octagon*, all constructed on *AB* as a common side.

5-13. SPECIAL METHODS OF DRAWING REGULAR POLYGONS

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 10 for the following problem.

Problem 5-27. *To construct a pentagon, length of a side given.*

Method I: (fig. 5-41):

(i) Draw a line *AB* equal to the given length.

(ii) With centre *A* and radius *AB*, describe a circle-1.

(iii) With centre *B* and the same radius, describe a circle-2 cuttting circle-1 at *C* and *D*.

(iv) With centre *C* and the same radius, draw an arc to cut circle-1 and circle-2 at *E* and *F* respectively.

(v) Draw a perpendicular bisector of the line *AB* to cut the arc *EF* at *G*.

(vi) Draw a line *EG* and produce it to cut circle-2 at *P*.

(vii) Draw a line *FG* and produce it to cut circle-1 at *R*.

(viii) With *P* and *R* as centres and *AB* as radius, draw arcs intersecting each other at *Q*.

(ix) Draw lines *BP*, *PQ*, *QR* and *RA*, thus completing the pentagon.

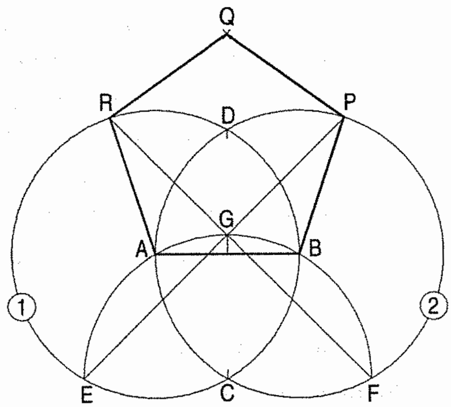

FIG. 5-41

Method II: (fig. 5-42):

(i) Draw a line *AB* equal to the given length.

(ii) Bisect *AB* in a point *C*.

(iii) Draw a line *BD* perpendicular and equal to *AB*.

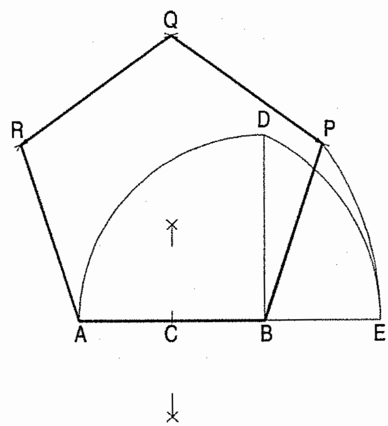

FIG. 5-42

(iv) With centre C and radius CD, draw an arc to intersect the line AB-produced at E.

(v) Then AE is the length of the diagonal of the pentagon.

(vi) Therefore, with centre A and radius AB, draw an arc intersecting the arc drawn with centre B and radius AE at R.

(vii) Again with centre A and radius AE, draw an arc intersecting the arc drawn with centre B and radius AB at P.

(viii) With centres A and B and radius AE, draw arcs intersecting each other at Q.

(ix) Draw lines BP, PQ, QR and RA, thus completing the pentagon.

Problem 5-28. *To construct a hexagon, length of a side given* (fig. 5-43 and fig. 5-44).

(a) *With T-square and 30°-60° set-square only* (fig. 5-43).

(i) Draw a line AB equal to the given length.

(ii) From A, draw lines $A1$ and $A2$ making 60° and 120° angles respectively with AB.

(iii) From B, draw lines $B3$ and $B4$ making 60° and 120° angles respectively with AB.

(iv) From O the point of intersection of $A1$ and $B3$, draw a line parallel to AB and intersecting $A2$ at F and $B4$ at C.

(v) From F, draw a line parallel to BC and intersecting $B3$ at E.

(vi) From C, draw a line parallel to AF and intersecting $A1$ at D.

(vii) Draw a line joining E and D.

Then $ABCDEF$ is the required hexagon.

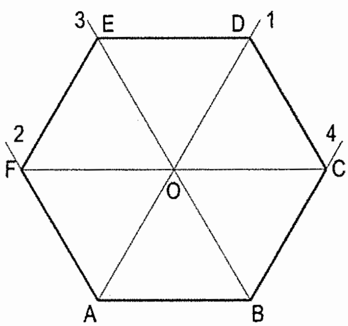

FIG. 5-43

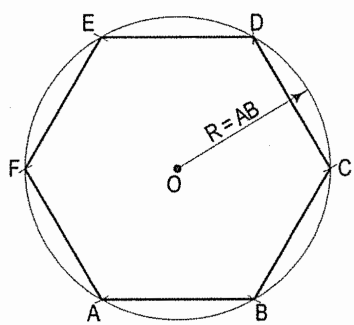

FIG. 5-44

(b) *With the aid of a compass* (fig. 5-44).

(i) With Point O as centre, draw a circle of radius equal to the given side length of the required polygon.

(ii) Draw a horizontal line passing through the centre of the circle and cutting the circle at opposite ends, say at points F and C. Mark the centre of circle as O.

(iii) Starting with either *F* or *C* as centre and side as length, go on marking the points on the circumference, *A*, *B*, *D* and *E*.

(iv) Join points *A-B-C-D-E-F*. You will get the required Hexagon (6 sided polygon).

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This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 11 for the following problem.

Problem 5-29. *To inscribe a regular octagon in a given square* (fig. 5-45).

(i) Draw the given square *ABCD*.

(ii) Draw diagonals *AC* and *BD* intersecting each other at *O*.

(iii) With centre *A* and radius *AO*, draw an arc cutting *AB* at 2 and *AD* at 7.

(iv) Similarly, with centres *B*, *C* and *D* and the same radius, draw arcs and obtain points 1, 3, 4 etc. as shown.

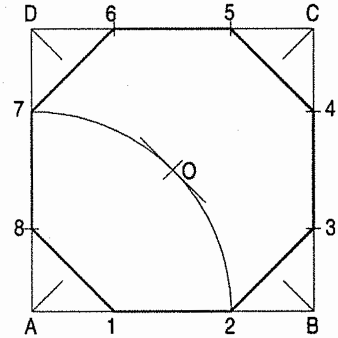


FIG. 5-45

Draw lines 2-3, 4-5, 6-7 and 8-1, thus completing the octagon.

5-14. REGULAR POLYGONS INSCRIBED IN CIRCLES

Problem 5-30. *To inscribe a regular polygon of any number of sides, say 5, in a given circle* (fig. 5-46).

(i) With centre *O*, draw the given circle.

(ii) Draw a diameter *AB* and divide it into five equal parts (same number of parts as the number of sides) and number them as shown.

(iii) With centres *A* and *B* and radius *AB*, draw arcs intersecting each other at *P*.

(iv) Draw a line *P*2 and produce it to meet the circle at *C*. Then *AC* is the length of the side of the pentagon.

(v) Starting from *C*, step-off on the circle, divisions *CD*, *DE* etc., equal to *AC*.

(vi) Draw lines *CD*, *DE* etc., thus completing the pentagon.

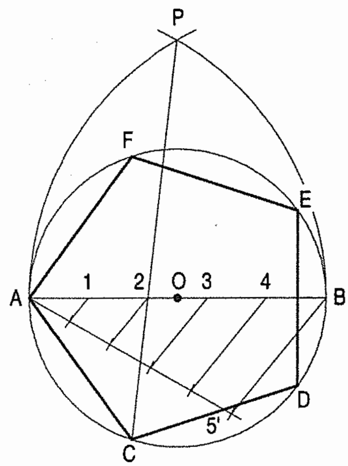


FIG. 5-46

Problem 5-31. *To inscribe a square in a given circle* (fig. 5-47).

(i) With centre *O*, draw the given circle.

(ii) Draw diameters *AB* and *CD* perpendicular to each other.

(iii) Draw lines *AC*, *CB*, *BD* and *DA*, thus completing the square.

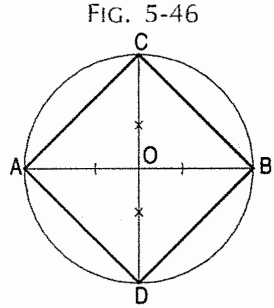


FIG. 5-47

Problem 5-32. *To inscribe a regular pentagon in a given circle* (fig. 5-48).

(i) With centre *O*, draw the given circle.

(ii) Draw diameters *AB* and *CD* perpendicular to each other.

(iii) Bisect *AO* in a point *P*. With centre *P* and radius *PC*, draw an arc cutting *OB* in *Q*.

(iv) With centre *C* and radius *CQ*, draw an arc cutting the circle in *E* and *F*.

(v) With centres *E* and *F* and the same radius, draw arcs cutting the circle in *G* and *H* respectively.

(vi) Draw lines *CE*, *EG*, *GH*, *HF* and *FC*, thus completing the required pentagon.

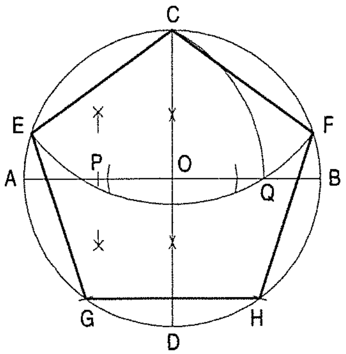
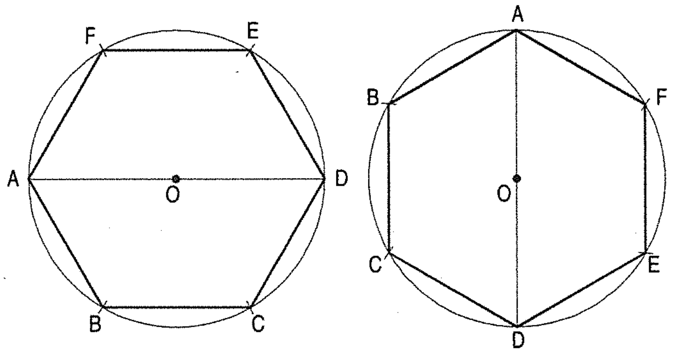


FIG. 5-48



(i) (ii)

FIG. 5-49

Problem 5-33. *To inscribe a regular hexagon in a given circle* (fig. 5-49).

Apply the same method as shown in Problem 5-28(b).

Note: (a) When two sides of the hexagon are required to be horizontal the starting point for stepping-off equal divisions should be on an end of the horizontal diameter.

(b) If they are to be vertical, the starting point should be on an end of the vertical diameter.

In either case, to avoid inaccuracy, the points should be joined with the aid of T-square and 30°-60° set-square.

Problem 5-34. *To inscribe a regular heptagon in a given circle* (fig. 5-50).

(i) With centre *O*, draw the given circle.

(ii) Draw a diameter *AB*. With centre *A* and radius *AO*, draw an arc cutting the circle at *E* and *F*.

(iii) Draw a line *EF*, cutting *AO* in *G*.

Then *EG* or *FG* is the length of the side of the heptagon.

Therefore, from any point on the circle, say *A*, step-off divisions equal to *EG*, around the circle. Join the division-points and obtain the heptagon.

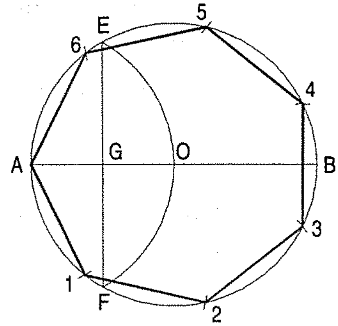


FIG. 5-50

Problem 5-35. *To inscribe a regular octagon in a given circle* (fig. 5-51).

(i) With centre *O*, draw the given circle.

(ii) Draw diameters *AB* and *CD* at right angles to each other.

(iii) Draw diameters *EF* and *GH* bisecting angles *AOC* and *COB*.

(iv) Draw lines *AE*, *EC* etc. and complete the octagon.

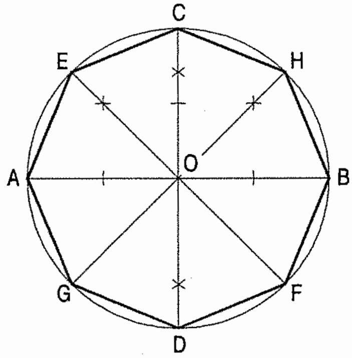


FIG. 5-51

5-15. TO DRAW REGULAR FIGURES USING T-SQUARE AND SET-SQUARES

Problem 5-36. *To describe an equilateral triangle about a given circle* (fig. 5-52).

(i) With centre *O*, draw the given circle.

(ii) Draw a vertical radius *OA*.

(iii) Draw radii *OB* and *OC* with a 30°–60° set-square, such that ∠*AOB* = ∠*AOC* = 120°.

(iv) At *A*, *B* and *C*, draw tangents to the circle, i.e. a horizontal line *EF* through *A*, and lines *FG* and *GE* through *B* and *C* respectively with a 30°–60° set-square.

Then *EFG* is the required triangle.

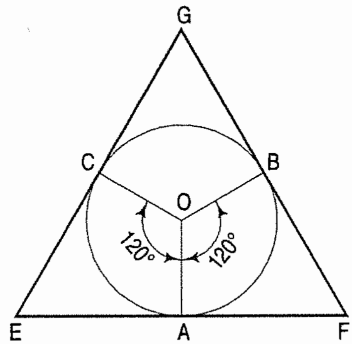


FIG. 5-52

Problem 5-37. *To draw a square about a given circle* (fig. 5-53).

(i) With centre *O*, describe the given circle.

(ii) Draw diameters *AB* and *CD* at right angles to each other as shown.

(iii) At *A* and *B*, draw vertical lines, and at *C* and *D*, draw horizontal lines intersecting at *E*, *F*, *G* and *H*.

EFGH is the required square.

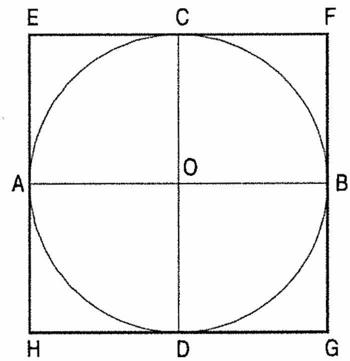


FIG. 5-53

Problem 5-38. *To describe a regular hexagon about a given circle* (fig. 5-54).

(i) With centre *O*, draw the given circle.

(ii) Draw horizontal diameter *AB*, and diameters *CD* and *EF* making 60° angle with *AB*.

(iii) Draw tangents at all the six ends, i.e. verticals at *A* and *B*, and lines with a 30°–60° set-square at the remaining points intersecting at 1, 2,.....6.

A hexagon with two sides horizontal can be drawn by drawing a vertical diameter *AB* and the other lines as shown in fig. 5-55.

Chapter 6

CURVES USED IN ENGINEERING PRACTICE

6-0. INTRODUCTION

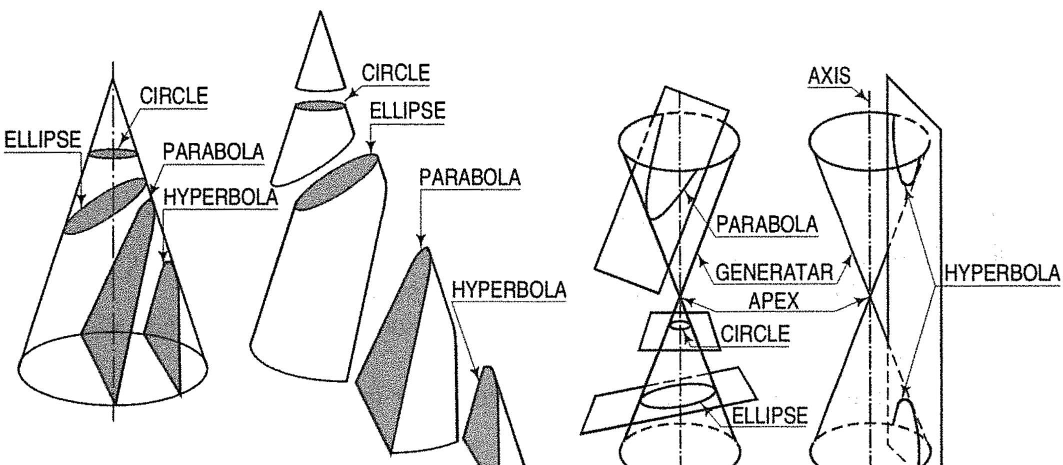
The profile of number of objects consists of various types of curves. This chapter deals with various types of curves which are commonly used in engineering practice as shown below:

1. Conic sections
2. Cycloidal curves
3. Involute
4. Evolutes
5. Spirals
6. Helix.

We shall now discuss the above in details with reference to their construction and applications.

6-1. CONIC SECTIONS

The sections obtained by the intersection of a right circular cone by a plane in different positions relative to the axis of the cone are called *conics*. Refer to fig. 6-1.



Conic sections

FIG. 6-1

(i) When the section plane is inclined to the axis and cuts all the generators on one side of the apex, the section is an *ellipse* [fig. 6-1].

(ii) When the section plane is inclined to the axis and is parallel to one of the generators, the section is a *parabola* [fig. 6-1].

(iii) A *hyperbola* is a plane curve having two separate parts or branches, formed when two cones that point towards one another are intersected by a plane that is parallel to the axes of the cones.

The conic may be defined as the locus of a point moving in a plane in such a way that the ratio of its distances from a fixed point and a fixed straight line is always constant. The fixed point is called the *focus* and the fixed line, the *directrix*.

The ratio $\dfrac{\text{distance of the point from the focus}}{\text{distance of the point from the directrix}}$ is called *eccentricity* and is denoted by e. It is always less than 1 for ellipse, equal to 1 for parabola and greater than 1 for hyperbola i.e.

(i) ellipse : $e < 1$

(ii) parabola : $e = 1$

(iii) hyperbola : $e > 1$.

The line passing through the focus and perpendicular to the directrix is called the *axis*. The point at which the conic cuts its axis is called the *vertex*.

6-1-1. ELLIPSE

Use of elliptical curves is made in arches, bridges, dams, monuments, man-holes, glands and stuffing-boxes etc. Mathematically an ellipse can be described by equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. Here '*a*' and '*b*' are half the length of major and minor axes of the ellipse and *x* and *y* co-ordinates.

(1) General method of construction of an ellipse:

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Problem 6-1. (fig. 6-2): *To construct an ellipse when the distance of the focus from the directrix is equal to 50 mm and eccentricity is* $\frac{2}{3}$.

(i) Draw any vertical line *AB* as directrix.

(ii) At any point *C* on it, draw the axis perpendicular to the *AB* (directrix).

(iii) Mark a focus *F* on the axis such that *CF* = 50 mm.

(iv) Divide *CF* into 5 equal divisions (sum of numerator and denometer of the eccentricity.).

(v) Mark the vertex *V* on the third division-point from *C*.

Thus, eccentricity, $e = \dfrac{VF}{VC} = \dfrac{2}{3}$.

(vi) A scale may now be constructed on the axis (as explained below), which will directly give the distances in the required ratio.

(vii) At *V*, draw a perpendicular *VE* equal to *VF*. Draw a line joining *C* and *E*.

Thus, in triangle *CVE*, $\dfrac{VE}{VC} = \dfrac{VF}{VC} = \dfrac{2}{3}$.

(viii) Mark any point 1 on the axis and through it, draw a perpendicular to meet *CE*-produced at 1'.

(ix) With centre *F* and radius equal to 1-1', draw arcs to intersect the perpendicular through 1 at points P_1 and P'_1.

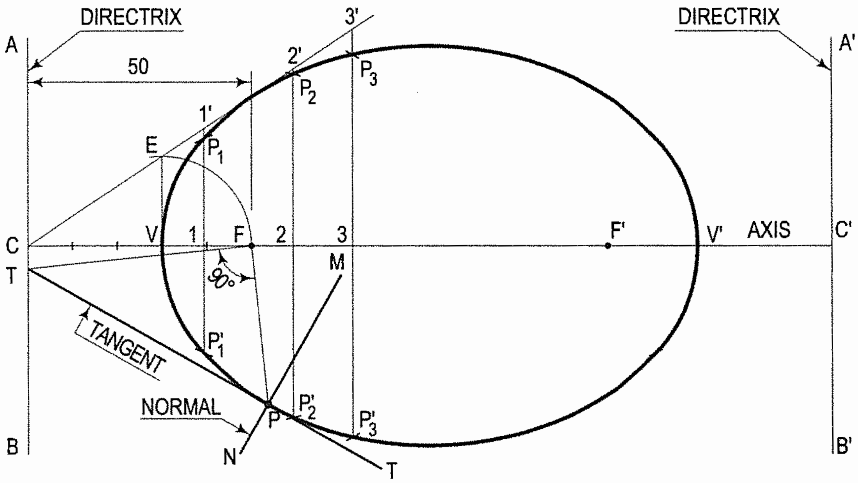
These are the points on the ellipse, because the distance of P_1 from *AB* is equal to *C*1,

$$P_1 F = 1\text{-}1'$$

and $$\frac{1-1'}{C1} = \frac{VF}{VC} = \frac{2}{3}.$$

Similarly, mark points 2, 3 etc. on the axis and obtain points P_2 and P'_2, P_3 and P'_3 etc.

(x) Draw the ellipse through these points. It is a closed curve having two foci and two directrices.



Directrix and focus

FIG. 6-2

(2) Construction of ellipse by other methods:

Ellipse is also defined as a curve traced out by a point, moving in the same plane as and in such a way that the sum of its distances from two fixed points is always the same.

(i) Each of the two fixed points is called the *focus*.

(ii) The line passing through the two foci and terminated by the curve, is called the *major axis*.

(iii) The line bisecting the major axis at right angles and terminated by the curve, is called the *minor axis*.

Conjugate axes: *Those axes are called conjugate axes when they are parallel to the tangents drawn at their extremities.*

In fig. 6-3, *AB* is the major axis, *CD* the minor axis and F_1 and F_2 are the foci. The foci are equidistant from the centre *O*.

The points A, P, C etc. are on the curve and hence, according to the definition,

$$(AF_1 + AF_2) = (PF_1 + PF_2) = (CF_1 + CF_2) \text{ etc.}$$

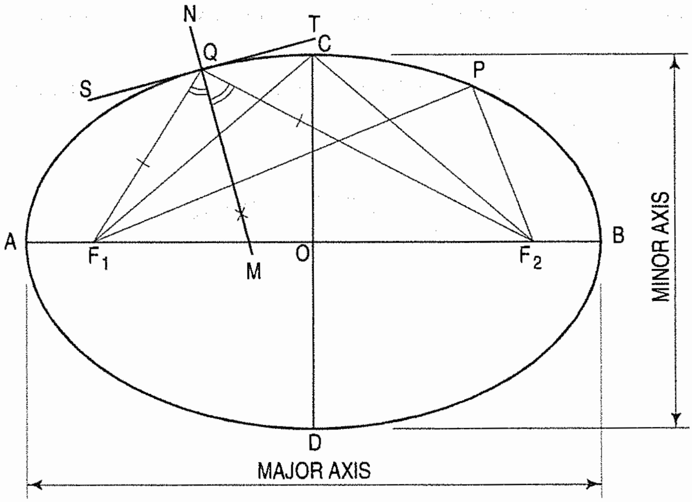
But $(AF_1 + AF_2) = AB$. $\therefore (PF_1 + PF_2) = AB$, the major axis.

Therefore, *the sum of the distances of any point on the curve from the two foci is equal to the major axis.*

Again, $(CF_1 + CF_2) = AB$.

But $CF_1 = CF_2$ $\therefore CF_1 = CF_2 = \frac{1}{2} AB$.

Hence, *the distance of the ends of the minor axis from the foci is equal to half the major axis.*



Conjugate axes

FIG. 6-3

Problem 6-2. *To construct an ellipse, given the major and minor axes.*

The ellipse is drawn by, first determining a number of points through which it is known to pass and then, drawing a smooth curve through them, either freehand or with a french curve. Larger the number of points, more accurate the curve will be.

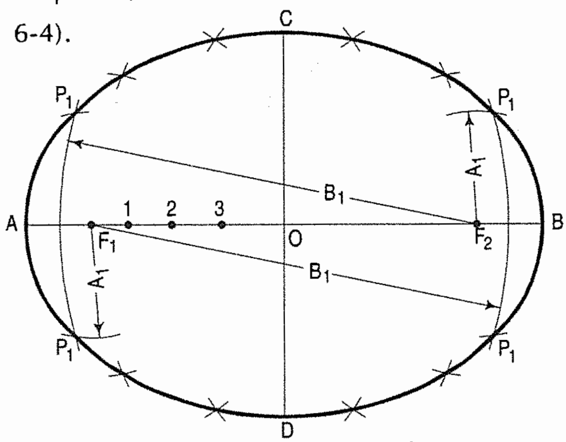
Method I: *Arcs of circles method* (fig. 6-4).

(i) Draw a line AB equal to the major axis and a line CD equal to the minor axis, bisecting each other at right angles at O.

(ii) With centre C and radius equal to half AB (i.e. AO) draw arcs cutting AB at F_1 and F_2, the foci of the ellipse.

(iii) Mark a number of points 1, 2, 3 etc. on AB.

(iv) With centres F_1 and F_2 and radius equal to $A1$, draw arcs on both sides of AB.



Arc of circle method

FIG. 6-4

(v) With same centres and radius equal to $B1$, draw arcs intersecting the previous arcs at four points marked P_1.

(vi) Similarly, with radii $A2$ and $B2$, $A3$ and $B3$ etc. obtain more points.

(vii) Draw a smooth curve through these points. This curve is the required ellipse.

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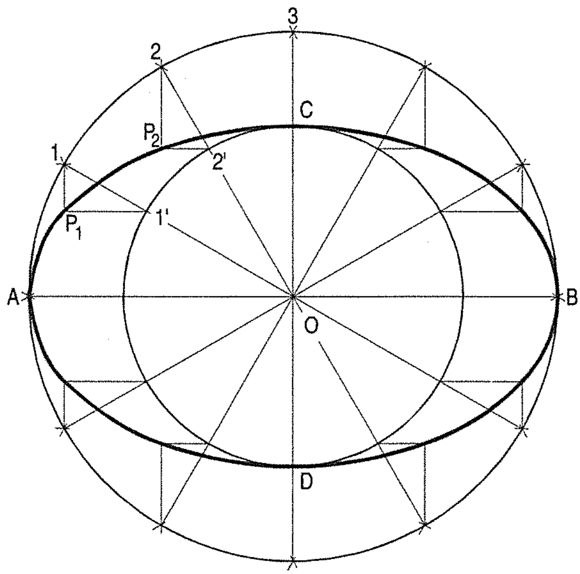
Method II: *Concentric circles method* (fig. 6-5).



Concentric circle method

FIG. 6-5

(i) Draw the major axis AB and the minor axis CD intersecting each other at O.

(ii) With centre O and diameters AB and CD respectively, draw two circles.

(iii) Divide the major-axis-circle into a number of equal divisions, say 12 and mark points 1, 2 etc. as shown.

(iv) Draw lines joining these points with the centre O and cutting the minor-axis-circle at points 1', 2' etc.

(v) Through point 1 on the major-axis-circle, draw a line parallel to CD, the minor axis.

(vi) Through point 1' on the minor-axis-circle, draw a line parallel to AB, the major axis. The point P_1, where these two lines intersect is on the required ellipse.

(vii) Repeat the construction through all the points. Draw the ellipse through A, P_1, P_2... etc.

Method III: *Loop of the thread method* (fig. 6-6).

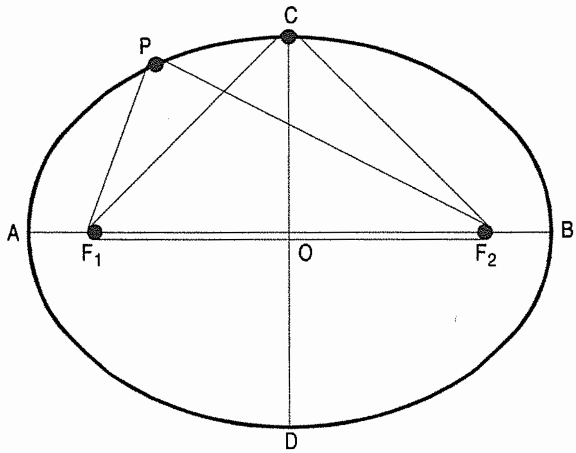
This is practical application of the first method.

(i) Draw the two axes AB and CD intersecting at O. Locate the foci F_1 and F_2.

(ii) Insert a pin at each focus-point and tie a piece of thread in the form of a loop around the pins, in such a way that the pencil point when placed in the loop (keeping the thread tight), is just on the end C of the minor axis.

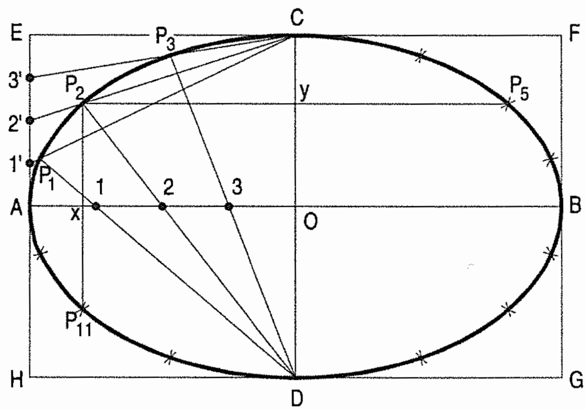
(iii) Move the pencil around the foci, maintaining an even tension in the thread throughout and obtain the ellipse.

It is evident that $PF_1 + PF_2 = CF_1 + CF_2$ etc.



Loop of the thread method
FIG. 6-6

Method IV: Oblong method (fig. 6-7).



Oblong method
FIG. 6-7

(i) Draw the two axes *AB* and *CD* intersecting each other at *O*.

(ii) Construct the oblong *EFGH* having its sides equal to the two axes.

(iii) Divide the semi-major-axis *AO* into a number of equal parts, say 4, and *AE* into the same number of equal parts, numbering them from *A* as shown.

(iv) Draw lines joining 1', 2' and 3' with *C*.

(v) From *D*, draw lines through 1, 2 and 3 intersecting C'_1, C'_2 and C'_3 at points P_1, P_2 and P_3 respectively.

(vi) Draw the curve through *A*, P_1.....*C*. It will be one quarter of the ellipse.

(vii) Complete the curve by the same construction in each of the three remaining quadrants.

As the curve is symmetrical about the two axes, points in the remaining quadrants may be located by drawing perpendiculars and horizontals from P_1, P_2 etc. and making each of them of equal length on both the sides of the two axes.

For example, $P_2x = x P_{11}$ and $P_2y = yP_5$.

An ellipse can be inscribed within a parallelogram by using the above method as shown in fig. 6-8.

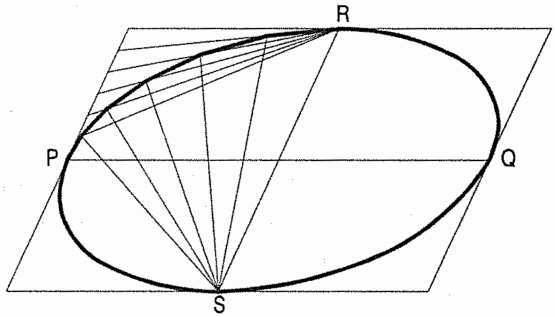
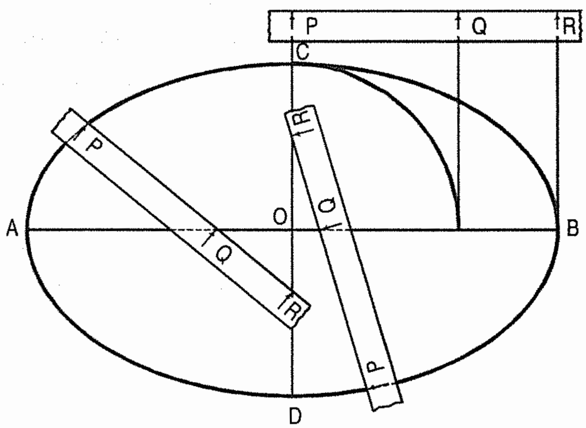


FIG. 6-8

Lines *PQ* and *RS*, joining the mid-points of the opposite sides of the parallelogram are called conjugate axes.

Method V: *Trammel method* (fig. 6-9).



Trammel method

FIG. 6-9

(i) Draw the two axes *AB* and *CD* intersecting each other at *O*. Along the edge of a strip of paper which may be used as a trammel, mark *PQ* equal to half the minor axis and *PR* equal to half the major axis.

(ii) Place the trammel so that *R* is on the minor axis *CD* and *Q* on the major axis *AB*. Then *P* will be on the required ellipse. By moving the trammel to new positions, always keeping *R* on *CD* and *Q* on *AB*, obtain other points. Draw the ellipse through these points.

Problem 6-3. (fig. 6-10): *ABC is a triangle such that AB = 100 mm, AC = 80 mm and BC = 60 mm. Draw an ellipse passing through points A, B and C.*

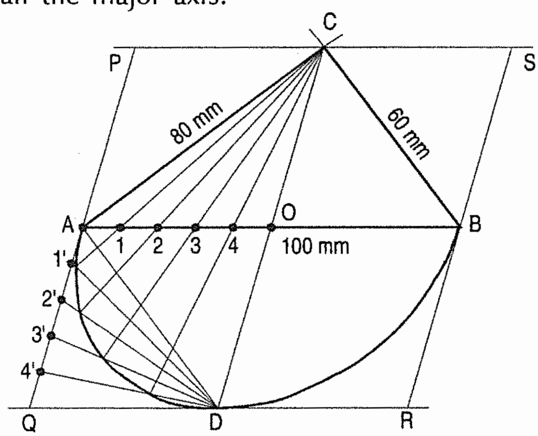


Fig. 6-10

(i) Draw horizontal line AB = 100 mm. Take A as centre draw an arc of 80 mm. Similarly B as centre and the radius equal to 60 mm, draw the arc such that it intersects previously drawn arc at the point C. Join ABC to complete triangle.

(ii) Mark the mid point of AB such that $OA = OB$ = 50 mm. Join OC and extend CO such that $CO = OD$.

(iii) Draw parallel lines from C and D to the line AB. Similarly draw parallel lines from A and B to the line CD and complete the rhombus ($PQRS$).

(iv) Divide AO into convenient number of equal parts $A1 = 12 = 23 = 34 = 4O$ and AQ to same number of equal parts $A1' = 1'2' = 2'3' = 3'4' = 4'Q$. Join A, 1', 2', 3', 4' with D. Join $C1$ and extend it to intersect line $D1'$. Similarly join $C2$, $C3$, $C4$ and extend it to intersect $D2'$, $D3'$, $D4'$ respective. Draw smooth curve passing through all intersection.

(v) Complete the ellipse by above method for the remaining part.

(3) **Normal and tangent to an ellipse:** The normal to an ellipse at any point on it bisects the angle made by lines joining that point with the foci.

The tangent to the ellipse at any point is perpendicular to the normal at that point.

Problem 6-4. (fig. 6-3): *To draw a normal and a tangent to the ellipse at a point Q on it.*

Join Q with the foci F_1 and F_2.

(i) Draw a line NM bisecting $\angle F_1 QF_2$. NM is the normal to the ellipse.

(ii) Draw a line ST through Q and perpendicular to NM. ST is the tangent to the ellipse at the point Q.

Problem 6-5. (fig. 6-11): *To draw a curve parallel to an ellipse and at distance R from it.*

This may be drawn by *two* methods:

(a) A large number of arcs of radius equal to the required distance R, with centres on the ellipse, may be described. The curve drawn touching these arcs will be parallel to the ellipse.

(b) It may also be obtained by drawing a number of normals to the ellipse, making them equal to the required distance R and then drawing a smooth curve through their ends.

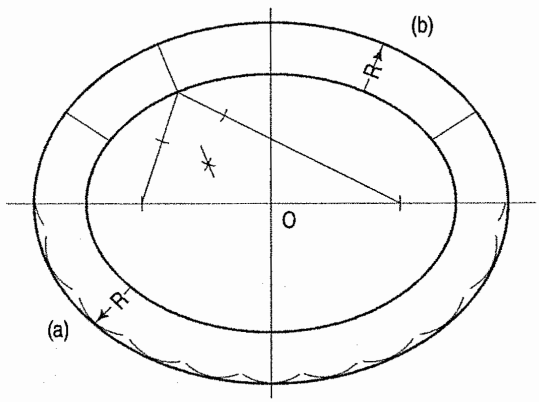


FIG. 6-11

Problem 6-6. (fig. 6-12): *To find the major axis and minor axis of an ellipse whose conjugate axes and angle between them are given.*

Conjugate axes PQ and RS, and the angle α between them are given.

(i) Draw the two axes intersecting each other at O.

(ii) Complete the parallelogram and inscribe the ellipse in it as described in problem 6-2, method (iv).

(iii) With O as centre and OR as radius, draw the semi-circle cutting the ellipse at a point E.

(iv) Draw the line RE.

(v) Through O draw a line parallel to RE and cutting the ellipse at points C and D. CD is the minor axis.

(vi) Through O, draw a line perpendicular to CD and cutting the ellipse at points A and B. AB is the major axis.

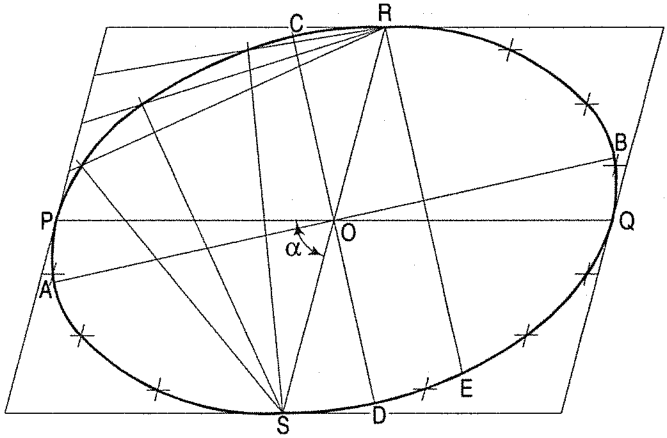


FIG. 6-12

Problem 6-7. (fig. 6-13): *To find the centre, major axis and minor axis of a given ellipse.*

(i) Draw any two chords 1-2 and 3-4 parallel to each other.

(ii) Find their mid-points P and Q, and draw a line passing through them, cutting the ellipse at points R and S. Bisect the line RS in the point O which is the centre of the ellipse.

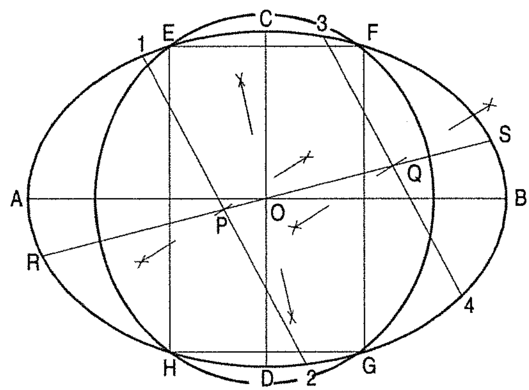


FIG. 6-13

With O as centre and any convenient radius, draw a circle cutting the ellipse in points E, F, G and H. Complete the rectangle $EFGH$. Through O, draw a line parallel to EF cutting the ellipse in points A and B. Again through O, draw a line parallel to FG cutting the ellipse at points C and D. AB and CD are respectively the major axis and the minor axis.

6-1-2. PARABOLA

CHAROTAR COGNIFRONT

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Chapter 4

SCALES

4-1. INTRODUCTION

Drawings of small objects can be prepared of the same size as the objects they represent. A 150 mm long pencil may be shown by a drawing of 150 mm length. Drawings drawn of the same size as the objects, are called full-size drawings. The ordinary full-size scales are used for such drawings.

A scale is defined as the ratio of the linear dimensions of element of the object as represented in a drawing to the actual dimensions of the same element of the object itself.

4-2. SCALES

The scales generally used for general engineering drawings are shown in table 4-1 [SP : 46].

TABLE 4-1

(i)	Reducing scales	1 : 2	1 : 5	1 : 10
		1 : 20	1 : 50	1 : 100
		1 : 200	1 : 500	1 : 1000
		1 : 2000	1 : 5000	1 : 10000
(ii)	Enlarging scales	50 : 1	20 : 1	10 : 1
		5 : 1	2 : 1	
(iii)	Full size scales			1 : 1

All these scales are usually 300 mm long and sub-divided throughout their lengths. The scale is indicated on the drawing at a suitable place near the title. The complete designation of a scale consists of word scale followed by the ratio, i.e. scale 1 : 1 or scale, full size.

It may not be always possible to prepare full-size drawings. They are, therefore, drawn proportionately smaller or larger. When drawings are drawn smaller than the actual size of the objects (as in case of buildings, bridges, large machines etc.) the scale used is said to be a *reducing scale* (1 : 5). Drawings of small machine parts, mathematical instruments, watches etc. are made larger than their real size. These are said to be drawn on an *enlarging scale* (5 : 1).

The scales can be expressed in the following *three* ways:

(1) Engineer's scale: In this case, the relation between the dimension on the drawing and the actual dimension of the object is mentioned numerically in the style as 10 mm = 5 m etc.

(2) Graphical scale: The scale is drawn on the drawing itself. As the drawing becomes old, the engineer's scale may shrink and may not give accurate results.

However, such is not the case with graphical scale because if the drawing shrinks, the scale will also shrink. Hence, the graphical scale is commonly used in survey maps.

(3) Representative fraction: The ratio of the length of the object represented on drawing to the actual length of the object represented is called the Representative Fraction (i.e. R.F.).

$$\text{R.F.} = \frac{\text{Length of the drawing}}{\text{Actual length of object}}$$

When a 1 cm long line in a drawing represents 1 metre length of the object, the R.F. is equal to $\frac{1\text{ cm}}{1\text{ m}} = \frac{1\text{ cm}}{1 \times 100\text{ cm}} = \frac{1}{100}$ and the scale of the drawing will be 1 : 100 or $\frac{1}{100}$ full size. *The R.F. of a drawing is greater than unity when it is drawn on an enlarging scale*. For example, when a 2 mm long edge of an object is shown in a drawing by a line 1 cm long, the R.F. is $\frac{1\text{ cm}}{2\text{ mm}} = \frac{10\text{ mm}}{2\text{ mm}} = 5$. Such a drawing is said to be drawn on scale 5 : 1 or *five times full-size*.

4-3. SCALES ON DRAWINGS

When an unusual scale is used, it is constructed on the drawing sheet. To construct a scale the following information is essential:

(1) The R.F. of the scale.

(2) The units which it must represent, for example, millimetres and centimetres, or feet and inches etc.

(3) The maximum length which it must show.

The length of the scale is determined by the formula:

Length of the scale = R.F. × maximum length required to be measured.

It may not be always possible to draw as long a scale as to measure the longest length in the drawing. The scale is therefore drawn 15 cm to 30 cm long, longer lengths being measured by marking them off in parts.

4-4. TYPES OF SCALES

The scales used in practice are classified as under:

(1) Plain scales

(2) Diagonal scales

(3) Comparative scales

(4) Vernier scales

(5) Scale of chords.

(1) Plain scales: A plain scale consists of a line divided into suitable number of equal parts or units, the first of which is sub-divided into smaller parts. Plain scales represent either two units or a unit and its sub-division.

In every scale,

(i) The zero should be placed at the end of the first main division, i.e. between the unit and its sub-divisions.

(ii) From the zero mark, the units should be numbered to the right and its sub-divisions to the left.

(iii) The names of the units and the sub-divisions should be stated clearly below or at the respective ends.

(iv) The name of the scale (e.g. scale, 1 : 10) or its R.F. should be mentioned below the scale.

Problem 4-1. (fig. 4-1): *Construct a scale of 1 : 4 to show centimetres and long enough to measure upto 5 decimetres.*

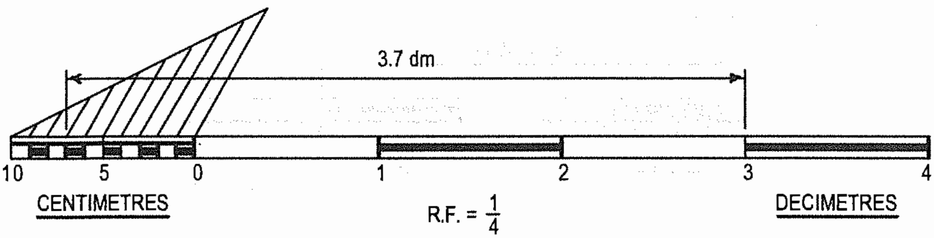


FIG. 4-1

(i) Determine R.F. of the scale. Here it is $\frac{1}{4}$.

(ii) Determine length of the scale.

Length of the scale = R.F. × maximum length = $\frac{1}{4}$ × 5 dm = 12.5 cm.

(iii) Draw a line 12.5 cm long and divide it into 5 equal divisions, each representing 1 dm.

(iv) Mark 0 at the end of the first division and 1, 2, 3 and 4 at the end of each subsequent division to its right.

(v) Divide the first division into 10 equal sub-divisions, each representing 1 cm.

(vi) Mark cms to the left of 0 as shown in the figure.

To distinguish the divisions clearly, *show the scale as a rectangle of small width (about 3 mm) instead of only a line*. Draw the division-lines showing decimetres throughout the width of the scale. Draw the lines for the sub-divisions slightly shorter as shown. Draw thick and dark horizontal lines in the middle of all alternate divisions and sub-divisions. This helps in taking measurements. Below the scale, print DECIMETRES on the right-hand side, CENTIMETRES on the left-hand side, and the R.F. in the middle.

To set-off any distance, say 3.7 dm, place one leg of the divider on 3 dm mark and the other on 7 cm mark. The distance between the ends of the two legs will represent 3.7 dm.

Problem 4-2. (fig. 4-2): *Draw a scale of 1 : 60 to show metres and decimetres and long enough to measure upto 6 metres.*

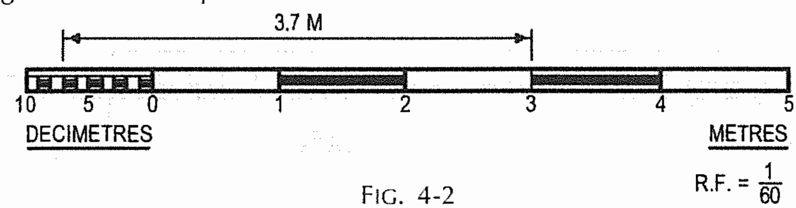


FIG. 4-2

(i) Determine R.F. of the scale, here R.F. $= \frac{1}{60}$.

(ii) Determine length of the scale.

Length of the scale $= \frac{1}{60} \times 6$ m $= \frac{1}{10}$ metre $= 10$ cm.

(iii) Draw a line 10 cm long and divide it into 6 equal parts.

(iv) Divide the first part into 10 equal divisions and complete the scale as shown. The length 3.7 metres is shown on the scale.

Problem 4-3. (fig. 4-3): *Construct a scale of 1.5 inches = 1 foot to show inches and long enough to measure upto 4 feet.*

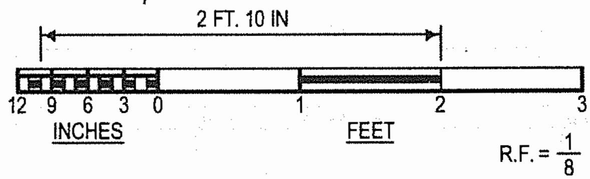


FIG. 4-3

(i) Determine R.F. of the scale. R.F. $= \frac{1.5 \text{ inches}}{1 \times 12 \text{ inches}} = \frac{1}{8}$.

(ii) Draw a line, $1.5 \times 4 = 6$ inches long.

(iii) Divide it into four equal parts, each part representing one foot.

(iv) Divide the first division into 12 equal parts, each representing 1". Complete the scale as explained in problem 4-1. The distance 2'-10" is shown measured in the figure.

Problem 4-4. (fig. 4-4): *Construct a scale of R.F.* $= \frac{1}{60}$ *to read yards and feet, and long enough to measure upto 5 yards.*

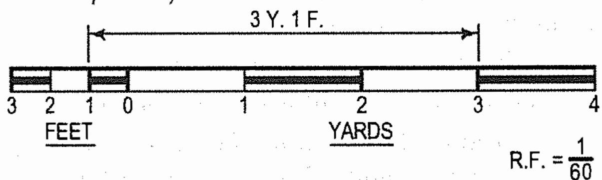


FIG. 4-4

(i) Length of the scale = R.F. × max. length $= \frac{1}{60} \times 5$ yd

$= \frac{1}{12}$ yd $= 3$ inches.

(ii) Draw a line 3 inches long and divide it into 5 equal parts.

(iii) Divide the first part into 3 equal divisions.

(iv) Mark the scale as shown in the figure.

Problem 4-5. (fig. 4-5): *Construct a scale of R.F.* $= \frac{1}{84480}$ *to show miles and furlongs and long enough to measure upto 6 miles.*

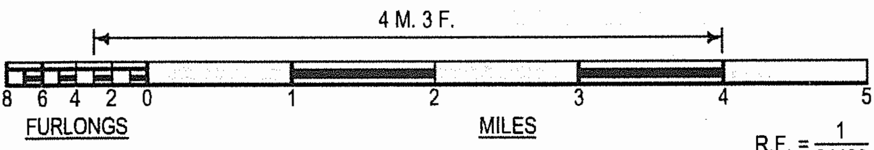


FIG. 4-5

(i) Length of the scale $= \frac{1}{84480} \times 6 = \frac{1}{14080}$ miles $= 4\frac{1}{2}$".

(ii) Draw a line $4\frac{1}{2}$" long and divide it into 6 equal parts. Divide the first part into 8 equal divisions and complete the scale as shown.

The distance 4 miles and 3 furlongs is shown measured in the figure.

(2) Diagonal scales: A diagonal scale is used when very minute distances such as 0.1 mm etc. are to be accurately measured or when measurements are required in three units; for example, dm, cm and mm, or yard, foot and inch.

Small divisions of short lines are obtained by the principle of diagonal division, as explained below.

Principle of diagonal scale: To obtain divisions of a given short line *AB* in multiples of $\frac{1}{10}$ its length, e.g. 0.1 *AB*, 0.2 *AB*, 0.3 *AB* etc. (fig. 4-6).

(i) At one end, say *B*, draw a line perpendicular to *AB* and along it, step-off ten equal divisions of any length, starting from *B* and ending at *C*.

(ii) Number the division-points, 9, 8, 7,.....1 as shown.

(iii) Join *A* with *C*.

(iv) Through the points 1, 2 etc. draw lines parallel to *AB* and cutting *AC* at 1', 2' etc. It is evident that triangles 1'1*C* , 2'2*C* ... *ABC* are similar.

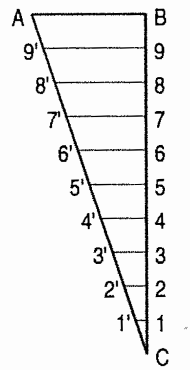


FIG. 4-6

Since *C*5 = 0.5*BC*, the line 5'5 = 0.5*AB*.

Similarly, 1'1 = 0.1*AB*, 2'2 = 0.2*AB* etc.

Thus, each horizontal line below *AB* becomes progressively shorter in length by $\frac{1}{10}$ *AB* giving lengths in multiples of 0.1*AB*.

Problem 4-6. (fig. 4-7): *Construct a diagonal scale of 3 : 200 i.e. 1 : 66 $\frac{2}{3}$ showing metres, decimetres and centimetres and to measure upto 6 metres.*

Length of the scale $= \frac{3}{200} \times 6$ m = 9 cm.

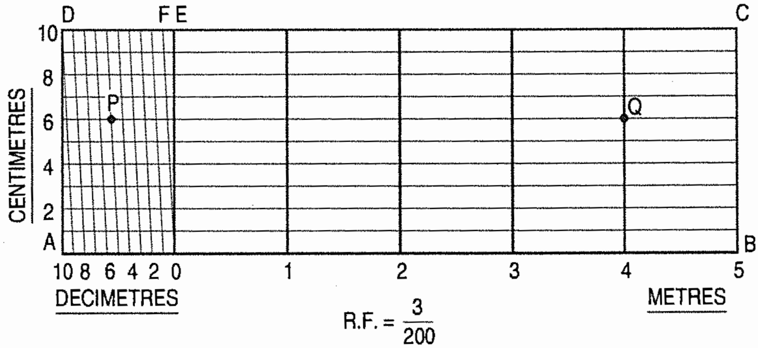


FIG. 4-7

(i) Draw a line *AB* 9 cm long and divide it into 6 equal parts. Each part will show a metre.

(ii) Divide the first part *A*0 into 10 equal divisions, each showing a decimetre or 0.1 m.

(iii) At *A* erect a perpendicular and step-off along it, 10 equal divisions of any length, ending at *D*. Complete the rectangle *ABCD*.

(iv) Erect perpendiculars at metre-divisions 0, 1, 2, 3 and 4.

(v) Draw horizontal lines through the division-points on *AD*.

(vi) Join *D* with the end of the first division along *A*0, viz. the point 9.

(vii) Through the remaining points i.e. 8, 7, 6 etc. draw lines parallel to *D*9.

In △ *OFE*, *FE* represents 1 dm or 0.1 m. Each horizontal line below *FE* progressively diminishes in length by 0.1*FE*. Thus, the next line below *FE* is equal to 0.9*FE* and represents 0.9 A 1 dm = 0.9 dm or 0.09 m or 9 cm.

Any length between 1 cm or 0.01 m and 6 m can be measured from this scale. To show a distance of 4.56 metres, i.e. 4 m, 5 dm and 6 cm, place one leg of the divider at *Q* where the vertical through 4 m meets the horizontal through 6 cm and the other leg at *P* where the diagonal through 5 dm meets the same horizontal.

Problem 4-7. (fig. 4-8): *Construct a diagonal scale of R.F. = $\frac{1}{4000}$ to show metres and long enough to measure upto 500 metres.*

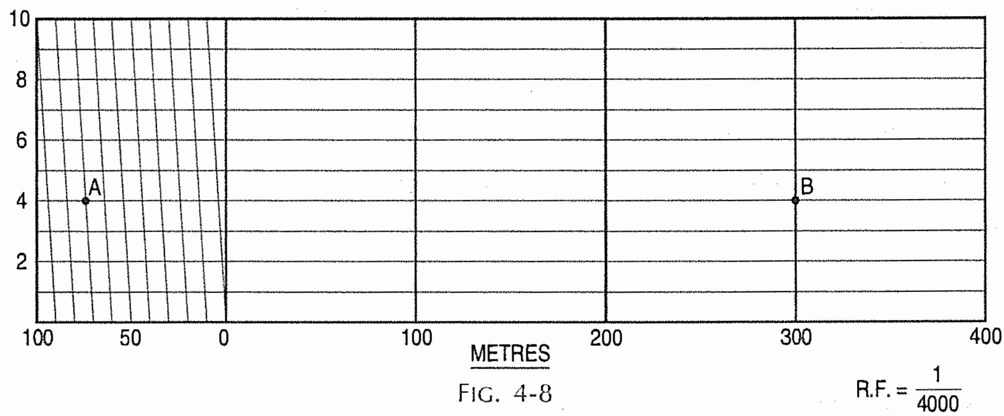


FIG. 4-8

Length of the scale = $\frac{1}{4000}$ × 500 m = $\frac{1}{8}$ metre = 12.5 cm.

(i) Draw a line 12.5 cm long and divide it into 5 equal parts. Each part will show 100 metres.

(ii) Divide the first part into ten equal divisions. Each division will show 10 metres.

(iii) At the left-hand end, erect a perpendicular and on it, step-off 10 equal divisions of any length.

(iv) Draw the rectangle and complete the scale as explained in problem 4-6.

The distance between points *A* and *B* shows 374 metres.

Problem 4-8. (fig. 4-9): *Draw a diagonal scale of 1 : 2.5, showing centimetres and millimetres and long enough to measure upto 20 centimetres.*

Length of the scale = $\frac{1}{2.5} \times 20$ cm = 8 cm.

(i) Draw a line 8 cm long and divide it into 4 equal parts. Each part will represent a length of 5 cm.

(ii) Divide the first part into 5 equal divisions. Each division will show 1 cm.

(iii) At the left-hand end of the line, draw a vertical line and on it step-off 10 equal divisions of any length.

Complete the scale as explained in problem 4-6. The distance between points *C* and *D* shows 13.4 cm.

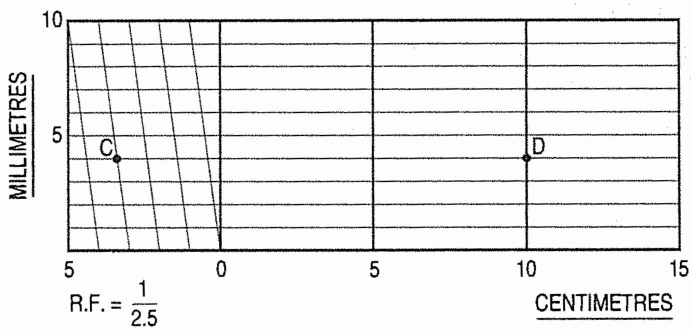


FIG. 4-9

Problem 4-9. (fig. 4-10): *Construct a diagonal scale of R.F. = $\frac{1}{32}$ showing yards, feet and inches and to measure upto 4 yards.*

Length of the scale = $\frac{1}{32} \times 4$ yd = 18 yd = $4\frac{1}{2}$".

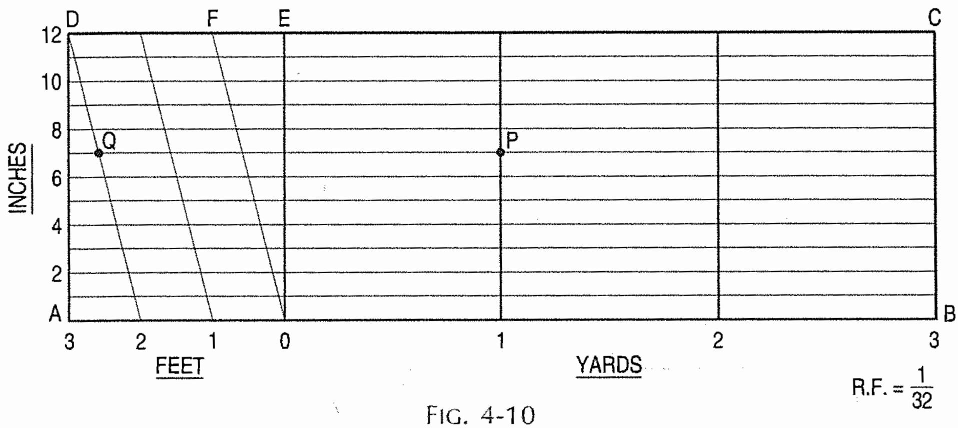


FIG. 4-10

(i) Draw a line *AB* $4\frac{1}{2}$" long.

(ii) Divide it into 4 equal parts to show yards. Divide the first part *A*0 into 3 equal divisions showing feet.

(iii) At *A*, erect a perpendicular and step-off along it, 12 equal divisions of any length, ending at *D*. Complete the scale as explained in problem 4-6.

To show a distance of 1 yard, 2 feet and 7 inches, place one leg of the divider at *P*, where the horizontal through 7" meets the vertical from 1 yard and the other leg at Q where the diagonal through 2' meets the same horizontal.

Problem 4-10. (fig. 4-11): *Draw a scale of full-size, showing $\frac{1}{100}$ inch and to measure upto 5 inches.*

(i) Draw a line *AB* 5" long and divide it into five equal parts. Each part will show one inch.

(ii) Sub-divide the first part into 10 equal divisions. Each division will measure $\frac{1}{10}$ inch.

(iii) At *A*, draw a perpendicular to *AB* and on it, step-off ten equal divisions of any length, ending at *D*.

(iv) Draw the rectangle *ABCD* and complete the scale as explained in problem 4-6. The line *QP* shows 2.68 inches.

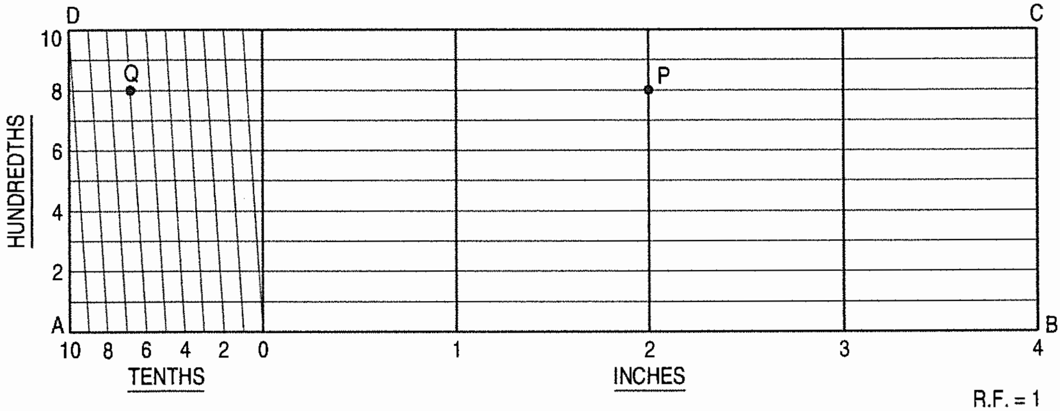


FIG. 4-11

Problem 4-11. (fig. 4-12): *The area of a field is 50,000 sq m. The length and the breadth of the field, on the map is 10 cm and 8 cm respectively. Construct a diagonal scale which can read upto one metre. Mark the length of 235 metre on the scale. What is the R.F. of the scale?*

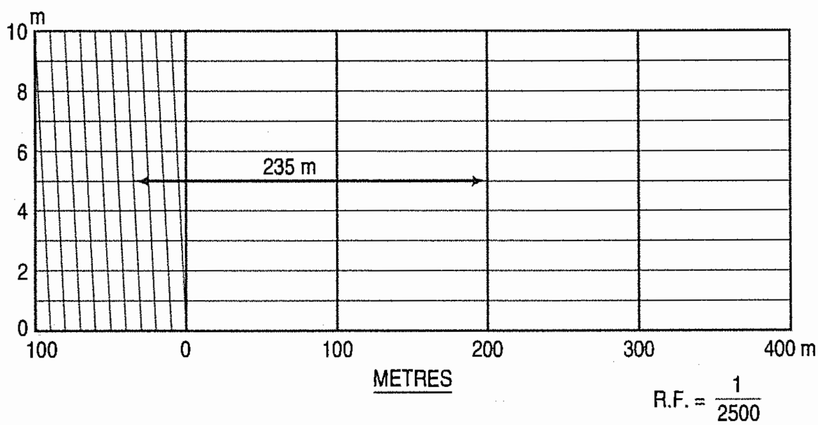


FIG. 4-12

The area of the field = 50,000 sq m.

The area of the field on the map = 10 cm × 8 cm = 80 cm^2.

$\therefore$ $$1 \text{ sq cm} = \frac{50000}{80} = 625 \text{ sq m.}$$

$\therefore$ $$1 \text{ cm} = 25 \text{ m.}$$

$$\text{Now representative fraction} = \frac{1 \text{ cm}}{25 \text{ m}} = \frac{1}{2500}.$$

$$\text{Length of the scale} = \frac{1}{2500} \times \frac{500 \times 100}{1} = \frac{50000}{2500} = 20 \text{ cm.}$$

Take 20 cm length and divide it into 5 equal parts. Complete the scale as shown in fig. 4-12.

(3) Comparative scales: Scales having same representative fraction but graduated to read different units are called *comparative scales*. A drawing drawn with a scale reading inch units can be read in metric units by means of a metric comparative scale, constructed with the same representative fraction. Comparative scales may be plain scales or diagonal scales and may be constructed separately or one above the other.

Problem 4-12. [fig. 4-13(i) and fig. 4-13(ii)]: *A drawing is drawn in inch units to a scale $\frac{3}{8}$ full size. Draw the scale showing $\frac{1}{8}$ inch divisions and to measure upto 15 inches. Construct a comparative scale showing centimetres and millimetres, and to read upto 40 centimetres.*

(i) *Inch scale:*

Length of the scale

$$= \frac{3}{8} \times 15 = \frac{45}{8}$$

$$= 5\frac{5}{8} \text{ inches.}$$

Construct the diagonal scale as shown in fig. 4-13(i).

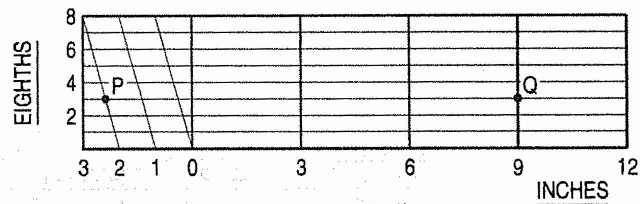


FIG. 4-13(i)

(ii) *Comparative scale:*

Length of the scale

$$= \frac{3}{8} \times 40$$

$$= 15 \text{ cm.}$$

Construct the diagonal scale as shown in fig. 4-13(ii).

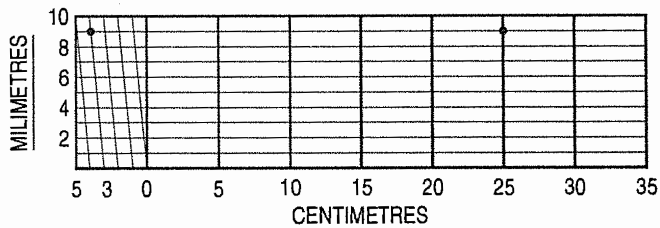


FIG. 4-13(ii)

The line *PQ* on the inch scale shows a length equal to $11\frac{3}{8}''$. Its equivalent, when measured on the comparative scale is 28.9 cm.

Problem 4-13. (fig. 4-14): *Draw comparative scales of R.F. = $\frac{1}{485000}$ to read upto 80 kilometres and 80 versts. 1 verst = 1.067 km.*

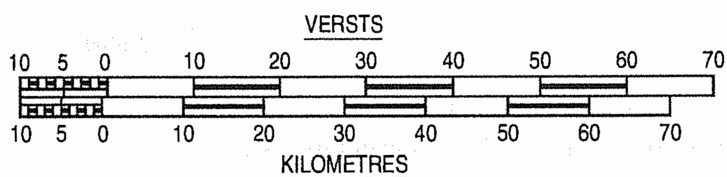


FIG. 4-14

$$\text{Length of kilometre scale} = \frac{1}{485000} \times 80 \times 1000 \times 100 = 16.5 \text{ cm.}$$

$$\text{Length of verst scale} = \frac{1}{485000} \times 80 \times 1.067 \times 1000 \times 100 = 17.6 \text{ cm.}$$

Draw the two scales one above the other as shown in the figure.

Problem 4-14. (fig. 4-15): *On a road map, a scale of miles is shown. On measuring from this scale, a distance of 25 miles is shown by a line 10 cm long. Construct this scale to read miles and to measure upto 40 miles. Construct a comparative scale, attached to this scale, to read kilometres upto 60 kilometres. 1 mile = 1.609 km.*

(i) *Scale of miles:*

$$\text{Length of the scale} = \frac{10 \times 40}{25} = 16 \text{ cm.}$$

Draw a line 16 cm long and construct a plain scale to show miles.

(ii) *Scale of kilometres:*

$$\text{R.F.} = \frac{10}{25 \times 1.609 \times 1000 \times 100} = \frac{1}{402250}$$

$$\text{Length of the scale} = \frac{1}{402250} \times 60 \times 1000 \times 100 = 14.9 \text{ cm.}$$

Construct the plain scale 14.9 cm long, above the scale of miles and attached to it, to read kilometres.

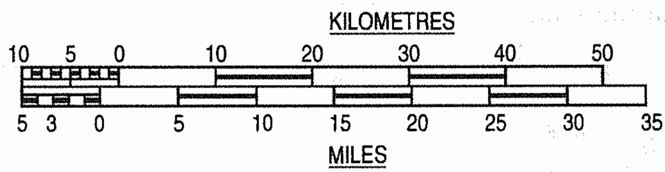


FIG. 4-15

Problem 4-15. (fig. 4-16): *The distance between Bombay and Poona is 180 km. A passenger train covers this distance in 6 hours. Construct a plain scale to measure time upto a single minute. The R.F. of the scale is* $\frac{1}{200000}$. *Find the distance covered by the train in 36 minutes.*

$$\text{Speed of the train} = \frac{180}{6} = 30 \text{ km/hour.}$$

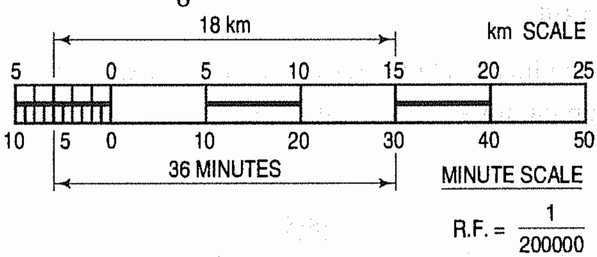


FIG. 4-16

(i) Distance scale (kilometres scale):

Length of the scale = R.F. × maximum distance

$$= \frac{1}{200000} \times 30 \times 1000 \times 100 = 15 \text{ cm.}$$

(ii) Time scale (minute scale):

Speed of the train = 30 km/hour.

i.e. 30 km is covered in 60 minutes.

As length of the scale of 15 cm represents 30 km, 60 minutes which is the time required to cover 30 km, can be represented on the same length of the scale.

(iii) Draw a line 15 cm long and divide it into 6 equal parts. Each part represents 5 km for the distance scale and 10 minutes for the time scale.

(iv) Divide the first part of the distance scale and the time scale into 5 and 10 equal parts respectively. Complete the scales as shown. The distance covered in 36 minutes is shown on the scale.

Problem 4-16. (fig. 4-17): *On a Russian map, a scale of versts is shown. On measuring it with a metric scale, 150 versts are found to measure 15 cm. Construct comparative scales for the two units to measure upto 200 versts and 200 km respectively. 1 verst = 1.067 km.*

(i) *Scale of verst:*

$$\text{Length of the scale} = \frac{15 \times 200}{150} = 20 \text{ cm.}$$

Draw a line 20 cm long and construct a plain scale to show versts.

(ii) *Scale of kilometres:*

$$\text{R.F.} = \frac{15}{150 \times 1.609 \times 1000 \times 10} = \frac{1}{160900}$$

$$\text{Length of the scale} = \frac{1}{160900} \times 200 \times 1000 \times 10 = 12.4 \text{ cm.}$$

Construct the plain scale 12.4 cm long, above the scale of versts and attached to it, to read kilometres (fig. 4-17).

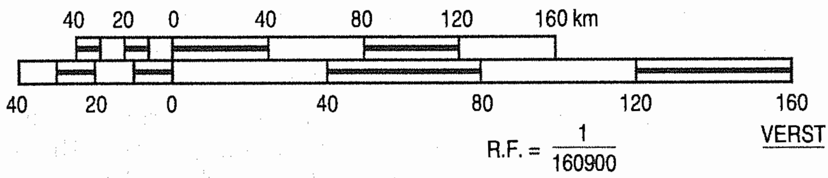


FIG. 4-17

(4) **Vernier scales:** Vernier scales, like diagonal scales, are used to read to a very small unit with great accuracy. A vernier scale consists of two parts — a primary scale and a vernier. The primary scale is a plain scale fully divided into minor divisions.

As it would be difficult to sub-divide the minor divisions in the ordinary way, it is done with the help of the vernier. The graduations on the vernier are derived from those on the primary scale.

(a) ***Principle of vernier:*** Fig. 4-18 shows a part of a plain scale in which the length *A*0 represents 10 cm. If we divide *A*0 into ten equal parts, each part will represent 1 cm. It would not be easy to divide each of these parts into ten equal divisions to get measurements in millimetres.

Now, if we take a length $B0$ equal to 10 + 1 = 11 such equal parts, thus representing 11 cm, and divide it into ten equal divisions, each of these divisions will represent $\frac{11}{10}$ = 1.1 cm or 11 mm.

The difference between one part of $A0$ and one division of $B0$ will be equal 1.1 – 1.0 = 0.1 cm or 1 mm.

Similarly, the difference between two parts of each will be 0.2 cm or 2 mm.

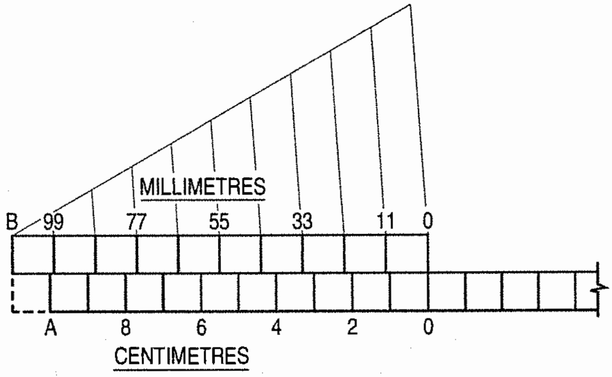


FIG. 4-18

The *upper scale* $B0$ is the vernier. The combination of the plain scale and the vernier is the vernier scale.

In general, if a line representing n units is divided into n equal parts, each part will show $\frac{n}{n}$ = 1 unit. But, if a line equal to n + 1 of these units is taken and then divided into n equal parts, each of these parts will be equal to $\frac{n+1}{n} = 1 + \frac{1}{n}$ units.

The difference between one such part and one former part will be equal to $\frac{n+1}{n} - \frac{n}{n} = \frac{1}{n}$ unit.

Similarly, the difference between two parts from each will be $\frac{2}{n}$ unit.

(b) ***Least count of a vernier:*** It is the difference of 1 primary scale division and 1 vernier scale division. It is denoted by $\angle C$.

$\angle C$ = 1 primary scale division – 1 vernier scale division.

The vernier scales are classified as under:

(i) *Forward vernier:* In this case, the length of one division of the vernier scale is smaller than the length of one division of the primary scale. The vernier divisions are marked in the same direction as that of the main scale.

(ii) *Backward vernier:* The length of each division of vernier scale is greater than the length of each division of the primary scale. The numbering is done in the opposite direction as that of the primary scale.

Problem 4-17. (fig. 4-19): *Draw a vernier scale of R.F.* = $\frac{1}{25}$ *to read centimetres upto 4 metres and on it, show lengths representing 2.39 m and 0.91 m.*

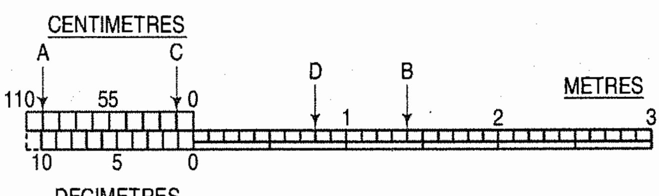


FIG. 4-19

Length of the scale = $\frac{1}{25} \times 4 \times 100$ = 16 cm.

(i) Draw a line 16 cm long and divide it into 4 equal parts to show metres. Divide each of these parts into 10 equal parts to show decimetres.

(ii) To construct a vernier, take 11 parts of dm length and divide it into 10 equal parts. Each of these parts will show a length of 1.1 dm or 11 cm.

To measure a length representing 2.39 m, place one leg of the divider at *A* on 99 cm mark and the other leg at *B* on 1.4 m mark. The length *AB* will show 2.39 metres (0.99 + 1.4 = 2.39).

Similarly, the length, *CD* shows 0.91 metre (0.8 + 0.11 = 0.91).

The necessity of dividing the plain scale into minor divisions throughout its length is quite evident from the above measurements.

Problem 4-18. (fig. 4-20): *Construct a full-size vernier scale of inches and show on it lengths 3.67", 1.54" and 0.48".*

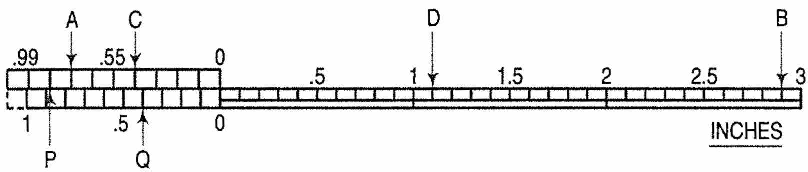


FIG. 4-20

(i) Draw a plain full-size scale 4" long and divide it fully to show 0.1" lengths.

(ii) Construct a vernier of length equal to 10 + 1 = 11 parts and divide it into 10 equal parts. Each of these parts will $\dfrac{11 \times 0.1}{10} = 0.11''$.

The line *AB* shows a length of 3.67" (0.77" + 2.9" = 3.67"). Similarly, lines *CD* and *PQ* show lengths of 1.54" (0.44" + 1.1" = 1.54") and 0.48" (0.88" – 0.4" = 0.48") respectively.

Problem 4-19. (fig. 4-21): *Construct a vernier scale of R.F.* = $\dfrac{1}{80}$ *to read inches and to measure upto 15 yards.*

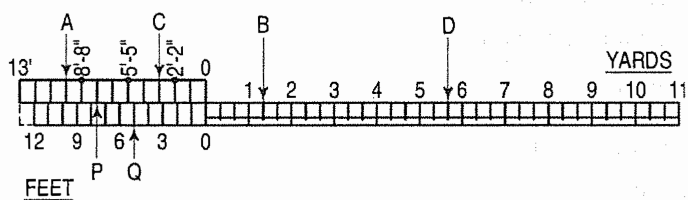


FIG. 4-21

Length of the scale = $\dfrac{1}{80} \times 15$ yd = $\dfrac{3}{16}$ yd = $6\dfrac{3}{4}''$.

(i) Draw the plain scale $6\dfrac{3}{4}''$ long and divide it fully to show yards and feet.

(ii) To construct the vernier, take a length of 12 + 1 = 13 feet-divisions and divide it into 12 equal parts. Each part will represent $\dfrac{13}{12}$ ft or 1'-1".

Lines *AB*, *CD* and *PQ* show respectively lengths representing 4 yd 1 ft 9 in (9' – 9" + 4'), 6 yd 2 ft 3 in (3' – 3" + 17') and 0 yd 2 ft 7 in (7' – 7" – 5").

Chapter 8

ORTHOGRAPHIC PROJECTION

8-0. INTRODUCTION

Practical solid geometry or *descriptive geometry* deals with the representation of points, lines, planes and solids on a flat surface (such as a sheet of paper), in such a manner that their relative positions and true forms can be accurately determined.

8-1. PRINCIPLE OF PROJECTION

If straight lines are drawn from various points on the contour of an object to meet a plane, the object is said to be projected on that plane. The figure formed by joining, in correct sequence, the points at which these lines meet the plane, is called the *projection* of the object. The lines from the object to the plane are called *projectors*.

8-2. METHODS OF PROJECTION

In engineering drawing following *four* methods of projection are commonly used, they are:

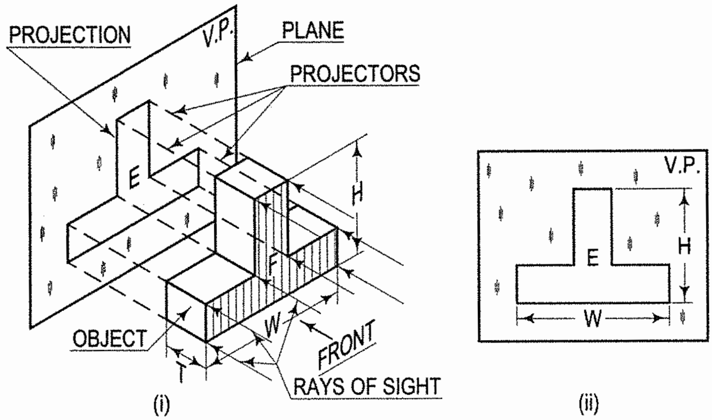
(1) Orthographic projection
(2) Isometric projection
(3) Oblique projection
(4) Perspective projection.

In the above methods (2), (3) and (4) represent the object by a pictorial view as eyes see it. In these methods of projection a three dimensional object is represented on a projection plane by one view only. While in the orthographic projection an object is represented by two or three views on the mutual perpendicular projection planes. Each projection view represents two dimensions of an object. For the complete description of the three dimensional object at least *two* or *three* views are required.

8-3. ORTHOGRAPHIC PROJECTION

When the projectors are parallel to each other and also perpendicular to the plane, the projection is called *orthographic projection*.

Step 1: Imagine that a person looks at the block [fig. 8-1(i)] from a theoretically infinite distance, so that the rays of sight from his eyes are parallel to one another and perpendicular to the front surface *F*. The view of this block will be the shaded figure, showing the front surface of the object in its true shape and proportion.



Step 2: If these rays of sight are extended further to meet perpendicularly a vertical plane (marked V.P.) set up behind the block.

Step 3: The points at which they meet the plane are joined in proper sequence, the resulting figure (marked *E*) will also be exactly similar to the front surface and this is known as *an elevation or front-view*. This figure is the projection of the block. *The lines from the block to the plane are the projectors.* As the projectors are perpendicular to the plane on which the projection is obtained, it is the orthographic projection. The projection is shown separately in fig. 8-1(ii). It shows only two dimensions of the block viz. the height *H* and the width *W*. It does not show the thickness. Thus, we find that only one projection is insufficient for complete description of the block.

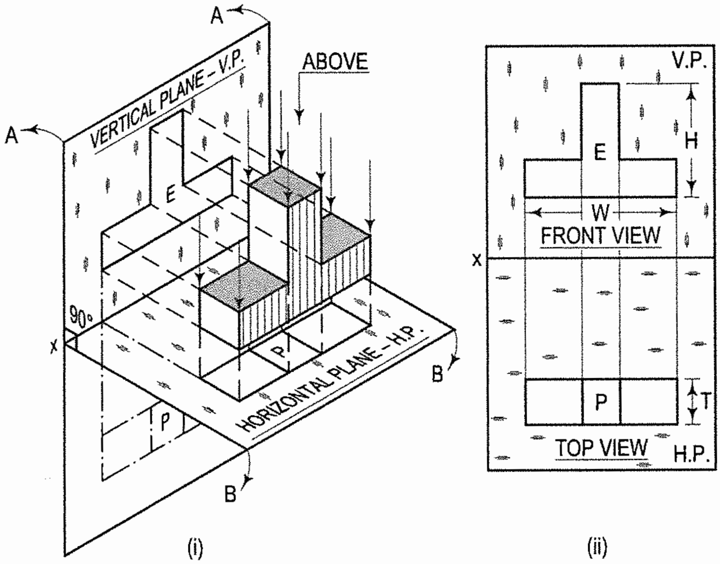


FIG. 8-2

Let us further assume that another plane marked H.P. (horizontal plane) [fig. 8-2(i)] is hinged at right angles to the first plane, so that the block is in front of the V.P. and above the H.P. The projection on the H.P. (figure *P*) shows the top surfaces of the block. If a person looks at the block from above, he will obtain the same view as the figure *P* and is known as a *plan* or *top-view*. It shows the width *W* and the thickness *T* of the block. It however does not show the height of the block.

One of the planes is now rotated or turned around on the hinges so that it lies in extension of the other plane. This can be done in *two* ways:

(i) by turning the V.P. in direction of arrows *A*

(ii) by turning the H.P. in direction of arrows *B*.

The H.P. when turned and brought in line with the V.P. is shown by dashed lines. The two projections can now be drawn on a flat sheet of paper, in correct relationship with each other, as shown in fig. 8-2(ii).

When studied together, they supply all information regarding the shape and the size of the block. Any solid may thus be represented by means of orthographic projections or orthographic views.

8-4. PLANES OF PROJECTION

The two planes employed for the purpose of orthographic projections are called *reference planes or principal planes of projection*. They intersect each other at right angles. The *vertical plane* of projection (in front of the observer) is usually denoted by the letters V.P. It is often called the *frontal plane* and denoted by the letters F.P.

The other plane is the *horizontal plane* of projection known as the H.P. The line in which they intersect is termed the *reference line* and is denoted by the letters *xy*. The projection on the V.P. is called the *front view* or the *elevation* of the object. The projection on the H.P. is called the *top view* or the *plan*.

8-5. FOUR QUADRANTS

When the planes of projection are extended beyond the line of intersection, they form four quadrants or dihedral angles which may be numbered as in fig. 8-3. The object may be situated in any one of the quadrants, its position relative to the planes being described as "*above or below the H.P.*" and "*in front of or behind the V.P.*"

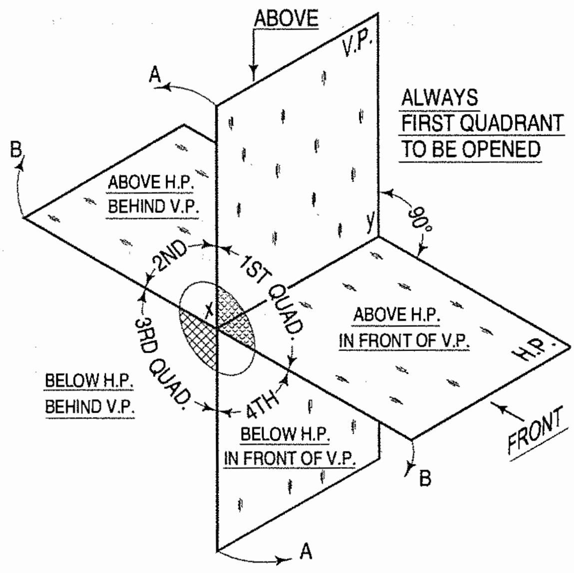


FIG. 8-3

The planes are assumed to be transparent. The projections are obtained by drawing perpendiculars from the object to the planes, i.e. by looking from the front and from above. They are then shown on a flat surface by rotating one of the planes as already explained. *It should be remembered that the first and the third quadrants are always opened out while rotating the planes.* The positions of the views with respect to the reference line will change according to the quadrant in which the object may be situated. This has been explained in detail in the next chapter.

8-6. FIRST-ANGLE PROJECTION

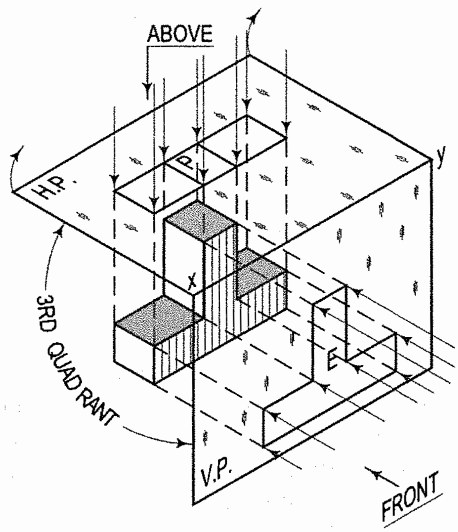
We have assumed the object to be situated in front of the V.P. and above the H.P. i.e. in the first quadrant and then projected it on these planes. This method of projection is known as *first-angle projection method*. The object lies between the observer and the plane of projection. In this method, when the views are drawn in their relative positions, the top view comes below the front view. In other words, the view seen from above is placed on the other side of (i.e. below) the front view. Each projection shows the view of that surface (of the object) which is remote from the plane on which it is projected and which is nearest to the observer.

TABLE 8-1
DIFFERENCE BETWEEN FIRST-ANGLE PROJECTION METHOD AND THIRD-ANGLE PROJECTION METHOD

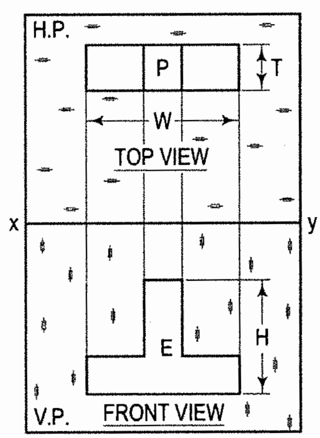
No.	First-angle projection method	Third-angle projection method
1.	The object is kept in the *first quadrant.*	The object is assumed to be kept in the *third quadrant.*
2.	The object lies between the observer and the plane of projection.	The plane of projection lies between the observer and the object.
3.	The plane of projection is assumed to be non-transparent.	The plane of projection is assumed to be *transparent.*
4.	In this method, when the views are drawn in their relative positions, the *plan* comes *below* the *elevation,* the view of the object as observed from the *left-side* is drawn to the *right of elevation.*	In this method, when the views are drawn in their relative positions, the *plan,* comes *above* the elevation, *left hand side view* is drawn to the *left hand side of the elevation.*
5.	This method of projection is now recommended by the "Bureau of Indian Standards' from 1991.	This method of projection is used in U.S.A. and also in other countries.

8-7. THIRD-ANGLE PROJECTION

In this method of projection, the object is assumed to be situated in the third quadrant [fig. 8-4(i)]. The planes of projection are assumed to be transparent. They lie between the object and the observer. When the observer views the object from the front, the rays of sight intersect the V.P. The figure formed by joining the points of intersection in correct sequence is the front view of the object. The top view is obtained in a similar manner by looking from above. When the two planes are brought in line with each other, the views will be seen as shown in fig. 8-4(ii). The top view in this case comes above the front view.



(i)



(ii)

FIG. 8-4

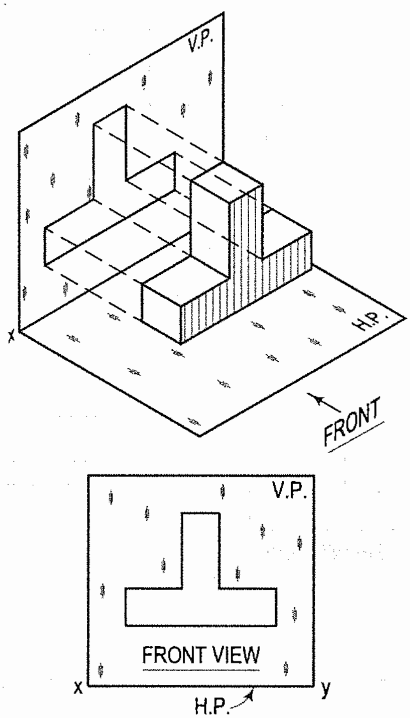
In other words, the view seen from above the object is placed on the same side of (i.e. above) the front view.

Each projection shows the view of that surface (of the object) which is nearest to the plane on which it is projected.

On comparison, it is quite evident that the views obtained by the two methods of projection are completely identical in shape, size and all other details. The difference lies in their relative positions only.

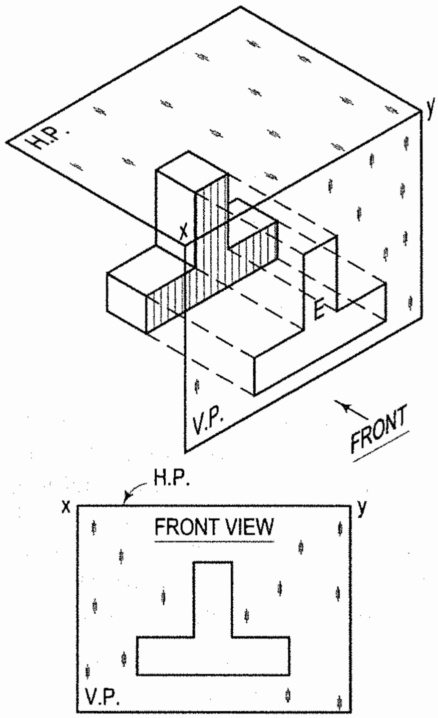
8-8. REFERENCE LINE

Studying the projections independently, it can be seen that while considering the front view (fig. 8-5 and fig. 8-6), which is the view as seen from the front, the H.P. coincides with the line *xy*. In other words, *xy* represents the H.P.



FIRST-ANGLE PROJECTION

FIG. 8-5

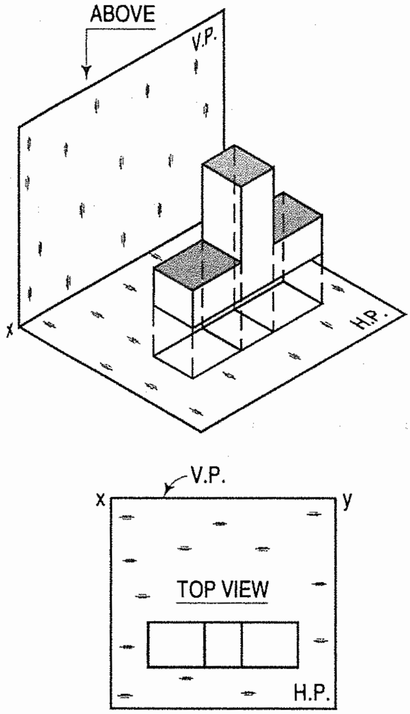


THIRD-ANGLE PROJECTION

FIG. 8-6

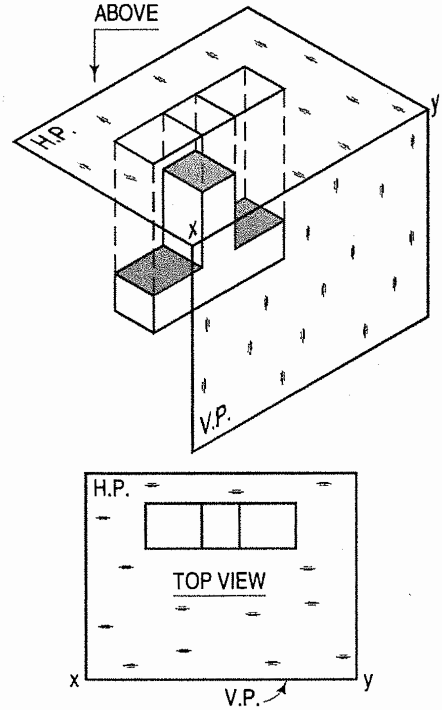
Similarly, while considering the top view (fig. 8-7 and fig 8-8), which is the view obtained by looking from above, the same line *xy* represents the V.P. Hence, when the two projections are drawn in correct relationship with each other (fig. 8-9), *xy* represents both the H.P. and the V.P. This line *xy* is called the *reference line*. The squares or rectangles for individual planes are thus unnecessary and are therefore discarded.

Further, in first-angle projection method, the H.P. is always assumed to be so placed as to coincide with the ground on or above which the object is situated. Hence, in this method, the line *xy* is also the line for the ground.



FIRST-ANGLE PROJECTION

FIG. 8-7



THIRD-ANGLE PROJECTION

FIG. 8-8

In third-angle projection method, the H.P. is assumed to be placed above the object. The object may be situated on or above the ground. Hence, in this method, the line *xy* does not represent the ground. The line for the ground, denoted by letters GL, may be drawn parallel to *xy* and below the front view [fig. 8-9(ii)].

In brief, when an object is situated on the ground, in first-angle projection method, the bottom of its front view will coincide with xy; in third-angle projection method, it will coincide with GL, while xy will be above the front view and parallel to Ground line.

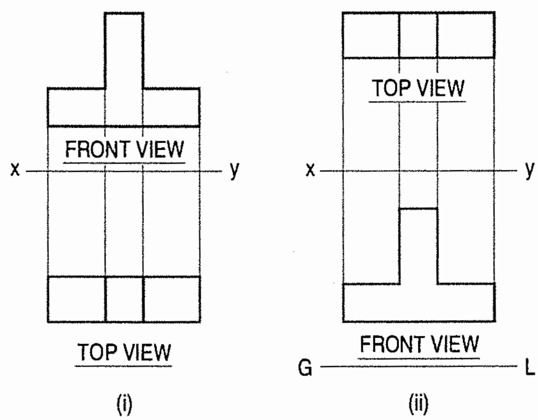


FIG. 8-9

Symbols for methods of projection: For every drawing it is absolutely essential to indicate the method of projection adopted. This is done by means of a symbolic figure drawn within the title block on the drawing sheet.

The symbolic figure for the first-angle projection method is shown in fig. 8-10, while that for the third-angle projection method is shown in fig. 8-11 which are self expanatory. These symbolic figures are actually the projections of a frustum of cone of convenient dimensions according to the size of drawing.

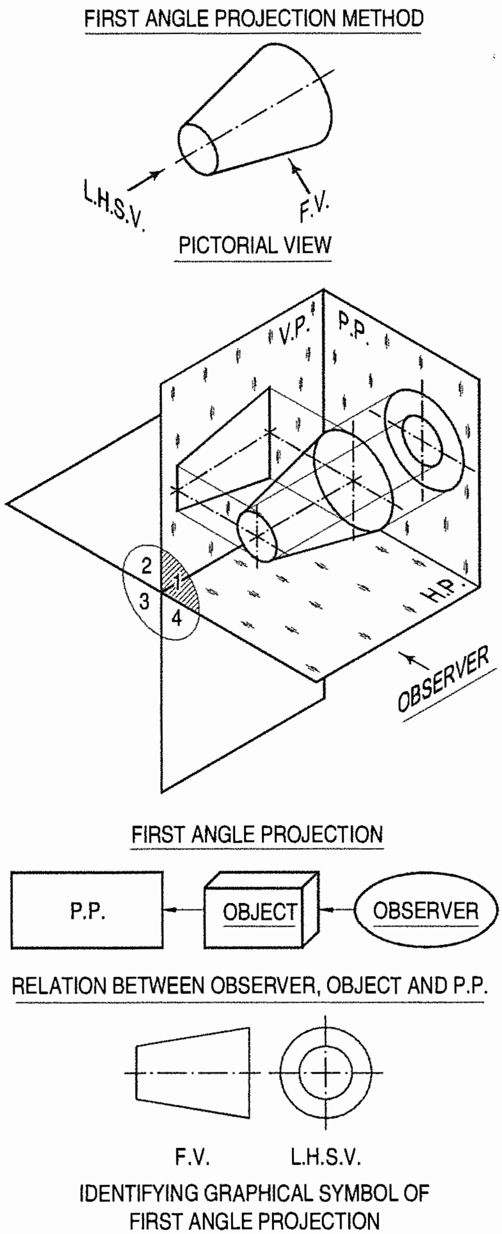


FIG. 8-10

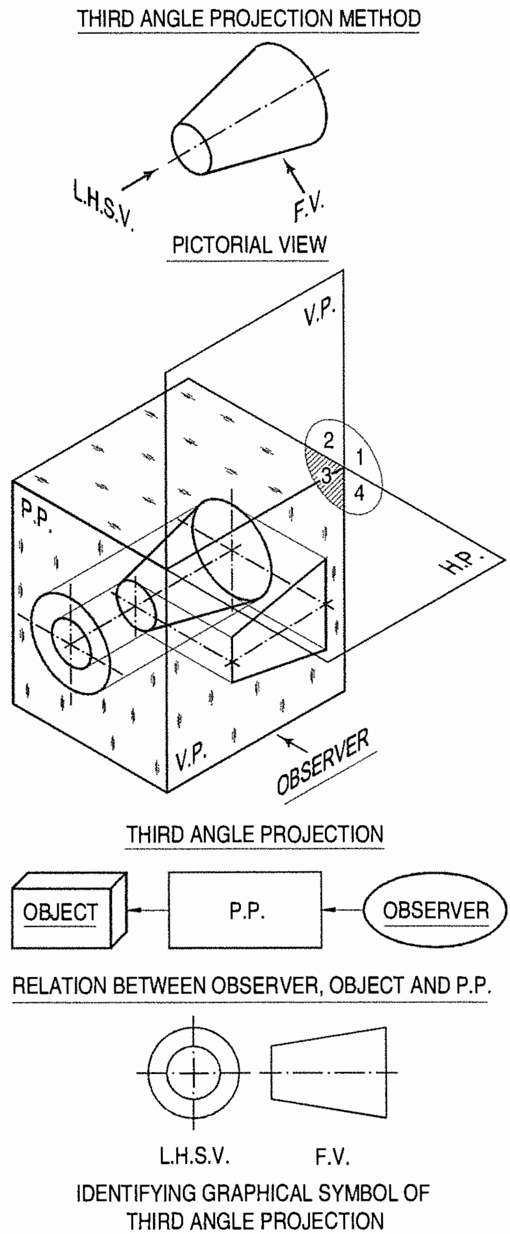


FIG. 8-11

Six views of an Object: There are *three* important elements of this projection system, namely

(a) an object

(b) plane of projection

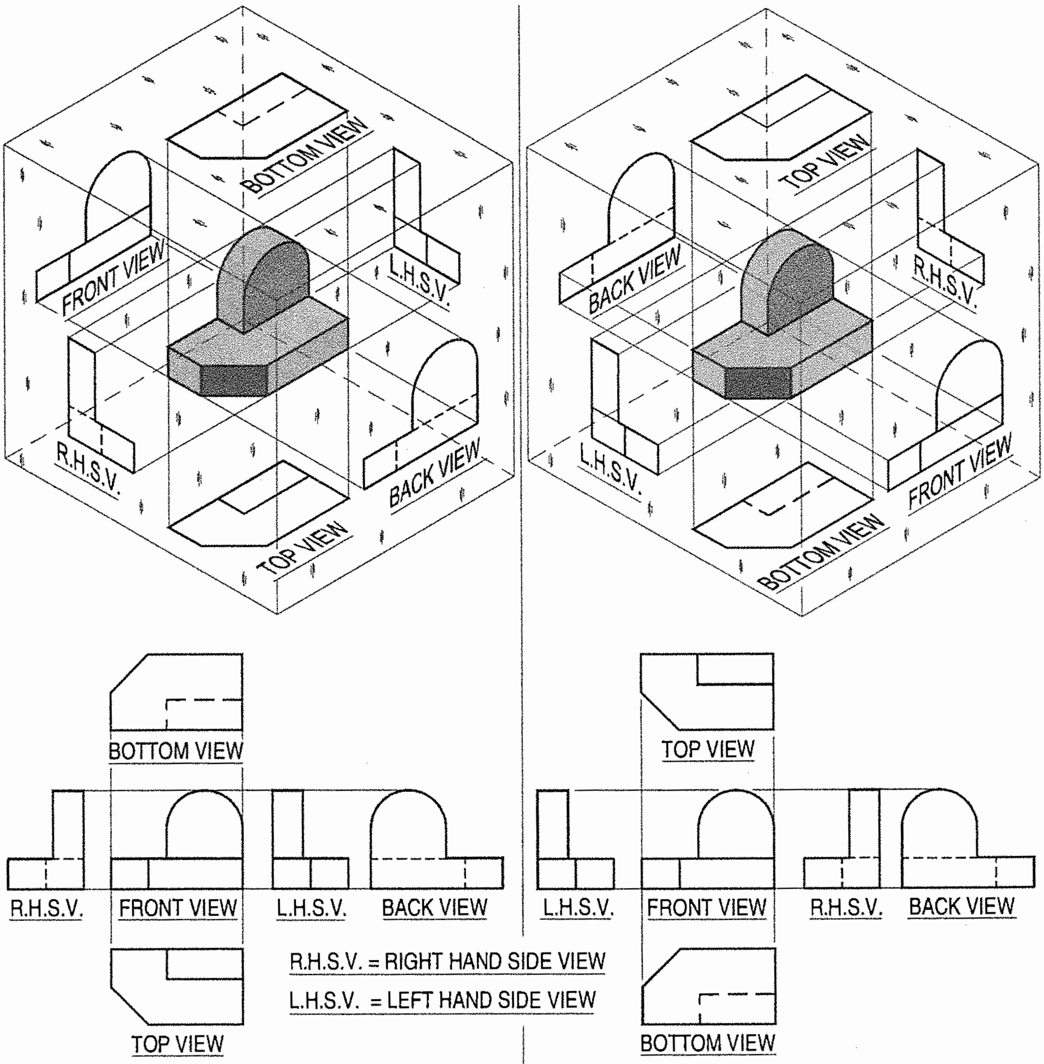
(c) an observer.

Very often, two views are not sufficient to describe an object completely. The planes of projection being imaginary, following six views are obtained:

(1) Front view
(2) Top view
(3) Left hand side view
(4) Right hand side view
(5) Back view
(6) Bottom view

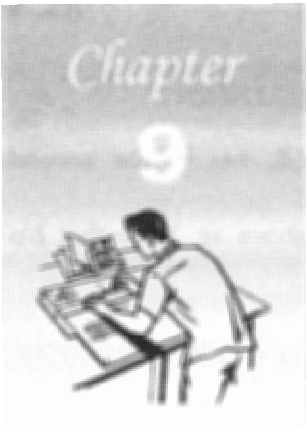
These projections are shown projected on the respective planes, placed by the methods of first-angle projection and third-angle projection as shown in fig. 8-12 and fig. 8-13 respectively.

Ordinarily, two views — the front view and top view are shown. Two other views i.e. L.H.S.V. or R.H.S.V. may be required to describe an object completely. Only in exceptional cases, when an object is of a very complex nature, five or six views may be found necessary.



FIRST ANGLE PROJECTION
FIG. 8-12

THIRD ANGLE PROJECTION
FIG. 8-13



PROJECTIONS OF POINTS

9-0. INTRODUCTION

A point may be situated, in space, in any one of the four quadrants formed by the two principal planes of projection or may lie in any one or both of them. Its projections are obtained by extending projectors perpendicular to the planes.

One of the planes is then rotated so that the first and third quadrants are opened out. The projections are shown on a flat surface in their respective positions either above or below or in *xy*.

CHAROTAR COGNIFRONT

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 21 for the projections of points.

9-1. A POINT IS SITUATED IN THE FIRST QUADRANT

The pictorial view [fig. 9-1(i)] shows a point *A* situated above the H.P. and in front of the V.P., i.e. in the first quadrant. *a'* is its front view and *a* the top view. After rotation of the plane, these projections will be seen as shown in fig. 9-1(ii).

The front view *a'* is above *xy* and the top view *a* below it. The line joining *a'* and *a* (which also is called a projector), intersects *xy* at right angles at a point *o*. It is quite evident from the pictorial view that *a'o* = *Aa*, i.e. the distance of the front view from *xy* = the distance of *A* from the H.P. viz. *h*. Similarly, *ao* = *Aa'*, i.e. the distance of the top view from *xy* = the distance of *A* from the V.P. viz. *d*.

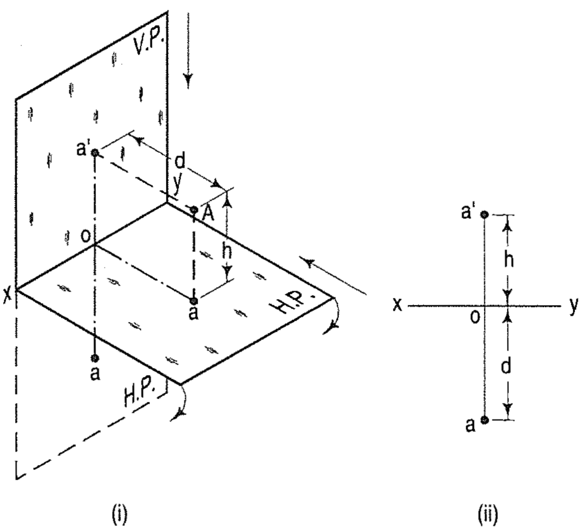


FIG. 9-1

9-2. A POINT IS SITUATED IN THE SECOND QUADRANT

A point *B* (fig. 9-2) is above the H.P. and behind the V.P., i.e. in the second quadrant. *b*' is the front view and *b* the top view.

When the planes are rotated, both the views are seen above *xy*. Note that *b'o* = *Bb* and *bo* = *Bb*'.

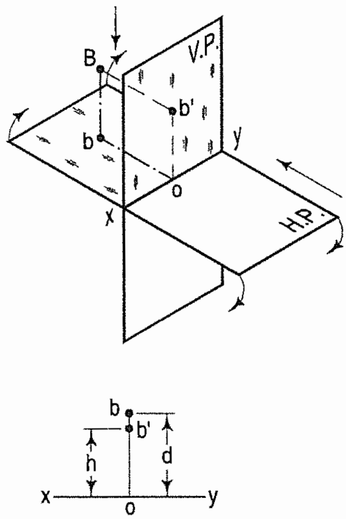


FIG. 9-2

9-3. A POINT IS SITUATED IN THE THIRD QUADRANT

A point *C* (fig. 9-3) is below the H.P. and behind the V.P., i.e. in the third quadrant. Its front view *c*' is below *xy* and the top view *c* above *xy*. Also *c'o* = *Cc* and *co* = *Cc*'.

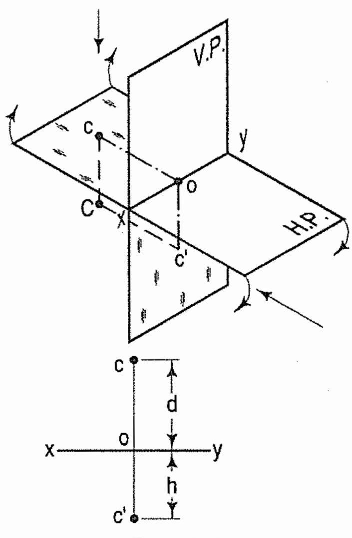


FIG. 9-3

9-4. A POINT IS SITUATED IN THE FOURTH QUADRANT

A point *E* (fig. 9-4) is below the H.P. and in front of the V.P., i.e. in the fourth quadrant. Both its projections are below *xy*, and *e'o* = *Ee* and *eo* = *Ee'*.

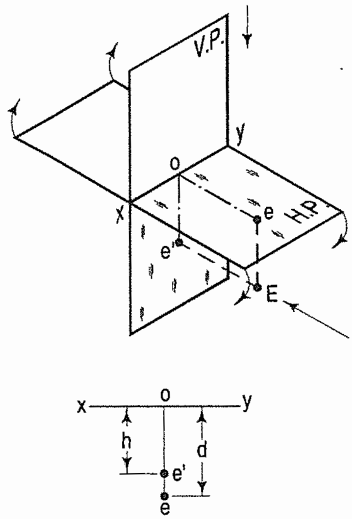


FIG. 9-4

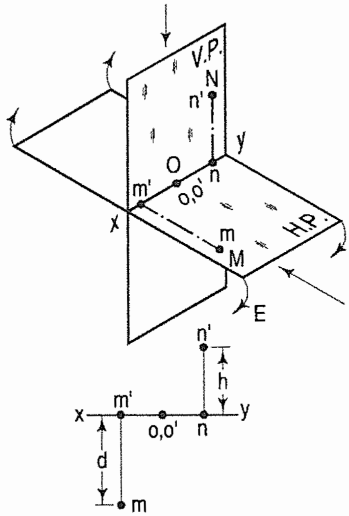


FIG. 9-5

Referring to fig. 9-5, we see that,

(i) A point *M* is in the H.P. and in front of the V.P. Its front view *m'* is in *xy* and the top view *m* below it.

(ii) A point *N* is in the V.P. and above the H.P. Its top view *n* is in *xy* and the front view *n'* above it.

(iii) A point *O* is in both the H.P. and the V.P. Its projection *o* and *o'* coincide with each other in *xy*.

9-5. GENERAL CONCLUSIONS

(i) The line joining the top view and the front view of a point is always perpendicular to *xy*. It is called a *projector*.

(ii) When a point is above the H.P., its front view is above *xy*; when it is below the H.P., the front view is below *xy*. The distance of a point from the H.P. is shown by the length of the projector from its front view to *xy*, e.g. *a'o*, *b'o* etc.

(iii) When a point is in front of the V.P., its top view is below *xy*; when it is behind the V.P., the top view is above *xy*. The distance of a point from the V.P. is shown by the length of the projector from its top view to *xy*, e.g. *ao*, *bo* etc.

(iv) When a point is in a reference plane, its projection on the other reference plane is in *xy*.

Problem 9-1. (fig. 9-1): *A point A is 25 mm above the H.P. and 30 mm in front of the V.P. Draw its projections.*

(i) Draw the reference line *xy* [fig. 9-1(ii)].

(ii) Through any point *o* in it, draw a perpendicular.

As the point is above the H.P. and in front of the V.P. its front view will be above *xy* and the top view below *xy*.

(iii) On the perpendicular, mark a point *a*' above *xy*, such that *a*'*o* = 25 mm. Similarly, mark a point *a* below *xy*, so that *ao* = 30 mm. *a*' and *a* are the required projections.

Problem 9-2. (fig. 9-6): *A point A is 20 mm below the H.P. and 30 mm behind the V.P. Draw its projections.*

As the point is below the H.P. and behind the V.P., its front view will be below *xy* and the top view above *xy*.

Draw the projections as explained in problem 9-1 and as shown in fig. 9-6.

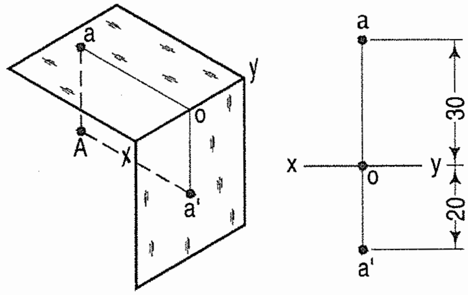


FIG. 9-6

Problem 9-3. (fig. 9-7): *A point P is in the first quadrant. Its shortest distance from the intersection point of H.P., V.P. and Auxiliary vertical plane, perpendicular to the H.P. and V.P. is 70 mm and it is equidistant from principal planes (H.P. and V.P.). Draw the projections of the point and determine its distance from the H.P. and V.P.*

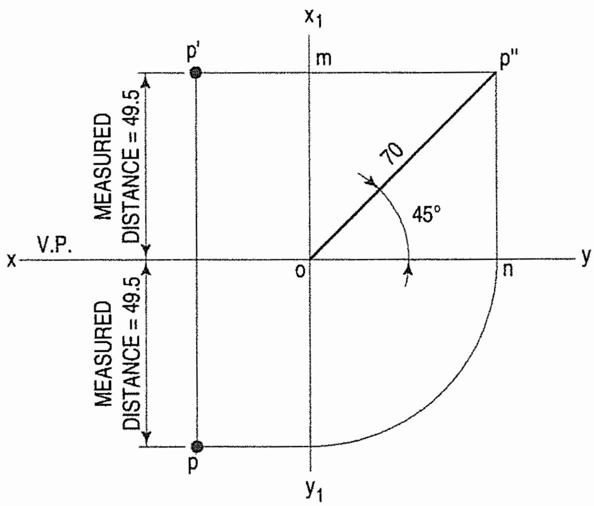


FIG. 9-7

Note: O represents intersection of H.P., V.P. and A.V.P.

(i) Draw *xy* and x_1 y_1 perpendicular reference lines.

(ii) *O* represents intersection of H.P., V.P. and A.V.P.

(iii) Draw from *O* a line inclined at 45° of 70 mm length.

(iv) Project from *P*" on *xy* line and x_1 y_1. The projections are *n* and *m* respectively as shown in figure. From *O* draw arc intersecting x_1 y_1.

(v) Draw a parallel line at convinent distance from x_1 y_1. Extend *P*"*m* to intersect a parallel line at *p*' and *p* as shown.

(vi) Measure distance from *xy* line, which is nearly 49.4974 mm say 49.5 mm.

Projections on auxiliary plane: Sometime projections of object on the principal (H.P. and V.P.) are insufficient. In such situation, another projection plane perpendicular to the principal planes is taken. This plane is known as auxiliary plane. The projection on the auxiliary plane is known as side view or side elevation. Refer fig. 9-8.

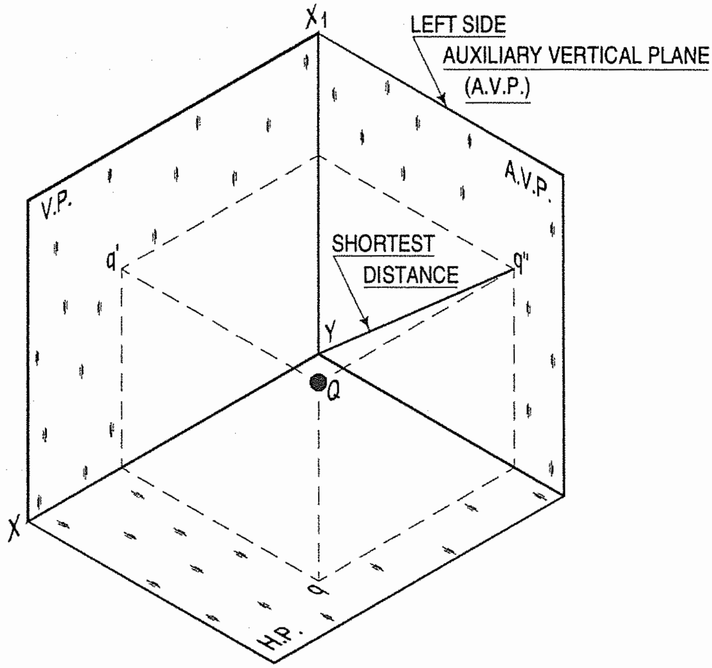


FIG. 9-8

The A.V.P. can be also taken right side also. For more details on projection on auxiliary plane, refer chapter 11.

EXERCISES 9

1. Draw the projections of the following points on the same ground line, keeping the projectors 25 mm apart.

   A, in the H.P. and 20 mm behind the V.P.

   B, 40 mm above the H.P. and 25 mm in front of the V.P.

   C, in the V.P. and 40 mm above the H.P.

   D, 25 mm below the H.P. and 25 mm behind the V.P.

   E, 15 mm above the H.P. and 50 mm behind the V.P.

   F, 40 mm below the H.P. and 25 mm in front of the V.P.

   G, in both the H.P. and the V.P.

2. A point *P* is 50 mm from both the reference planes. Draw its projections in all possible positions.

3. State the quadrants in which the following points are situated:

   (a) A point *P*; its top view is 40 mm above *xy*; the front view, 20 mm below the top view.

   (b) A point *Q*, its projections coincide with each other 40 mm below *xy*.

4. A point P is 15 mm above the H.P. and 20 mm in front of the V.P. Another point Q is 25 mm behind the V.P. and 40 mm below the H.P. Draw projections of P and Q keeping the distance between their projectors equal to 90 mm. Draw straight lines joining (i) their top views and (ii) their front views.

5. Projections of various points are given in fig. 9-9. State the position of each point with respect to the planes of projection, giving the distances in centimetres.

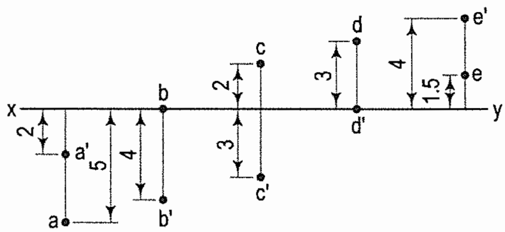


FIG. 9-9

6. Two points A and B are in the H.P. The point A is 30 mm in front of the V.P., while B is behind the V.P. The distance between their projectors is 75 mm and the line joining their top views makes an angle of 45° with xy. Find the distance of the point B from the V.P.

7. A point P is 20 mm below H.P. and lies in the third quadrant. Its shortest distance from xy is 40 mm. Draw its projections.

8. A point A is situated in the first quadrant. Its shortest distance from the intersection point of H.P., V.P. and auxiliary plane is 60 mm and it is equidistant from the principal planes. Draw the projections of the point and determine its distance from the principal planes.

9. A point 30 mm above xy line is the plan-view of two points P and Q. The elevation of P is 45 mm above the H.P. while that of the point Q is 35 mm below the H.P. Draw the projections of the points and state their position with reference to the principal planes and the quadrant in which they lie.

10. A point Q is situated in first quadrant. It is 40 mm above H.P. and 30 mm in front of V.P. Draw its projections and find its shortest distance from the intersection of H.P., V.P. and auxiliary plane.

Chapter 10

PROJECTIONS OF STRAIGHT LINES

10-0. INTRODUCTION

A straight line is the shortest distance between two points. Hence, the projections of a straight line may be drawn by joining the respective projections of its ends which are points.

The position of a straight line may also be described with respect to the two reference planes. It may be:

1. Parallel to one or both the planes.
2. Contained by one or both the planes.
3. Perpendicular to one of the planes.
4. Inclined to one plane and parallel to the other.
5. Inclined to both the planes.
6. Projections of lines inclined to both the planes.
7. Line contained by a plane perpendicular to both the reference planes.
8. True length of a straight line and its inclinations with the reference planes.
9. Traces of a line.
10. Methods of determining traces of a line.
11. Traces of a line, the projections of which are perpendicular to xy.
12. Positions of traces of a line.

10-1. LINE PARALLEL TO ONE OR BOTH THE PLANES (Fig. 10-1)

(a) Line *AB* is parallel to the H.P.

a and *b* are the top views of the ends *A* and *B* respectively. It can be clearly seen that the figure *ABba* is a rectangle. Hence, the top view *ab* is equal to *AB*.

a'b' is the front view of *AB* and is parallel to *xy*.

(b) Line *CD* is parallel to the V.P.

The line *c'd'* is the front view and is equal to *CD*; the top view *cd* is parallel to *xy*.

(c) Line *EF* is parallel to the H.P. and the V.P.

ef is the top view and *e'f'* is the front view; both are equal to *EF* and parallel to *xy*.

Hence, when a line is parallel to a plane, its projection on that plane is equal to its true length; while its projection on the other plane is parallel to the reference line.

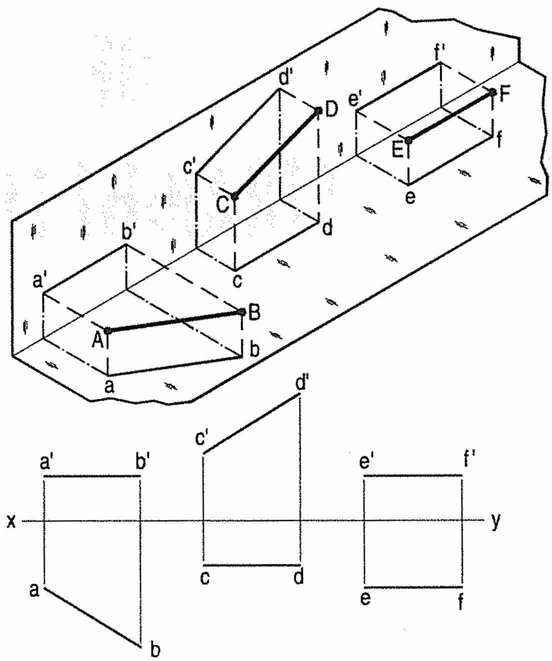


FIG. 10-1

10-2. LINE CONTAINED BY ONE OR BOTH THE PLANES (FIG. 10-2)

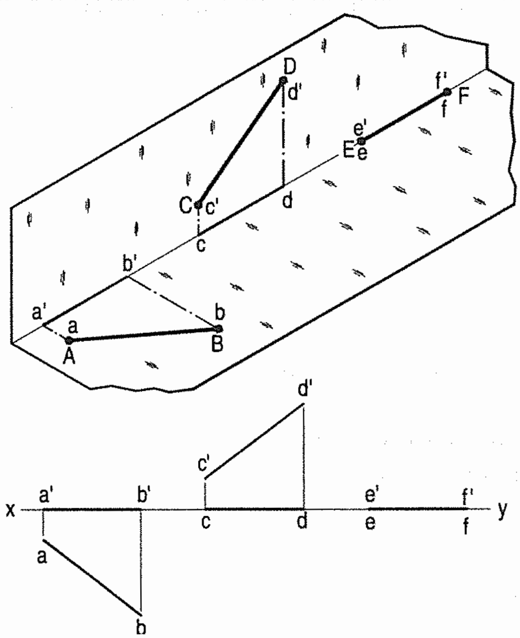


FIG. 10-2

Line *AB* is in the H.P. Its top view *ab* is equal to *AB*; its front view *a' b'* is in *xy*.

Line *CD* is in the V.P. Its front view *c'd'* is equal to *CD*; its top view *cd* is in *xy*.

Line *EF* is in both the planes. Its front view *e' f'* and the top view *ef* coincide with each other in *xy*.

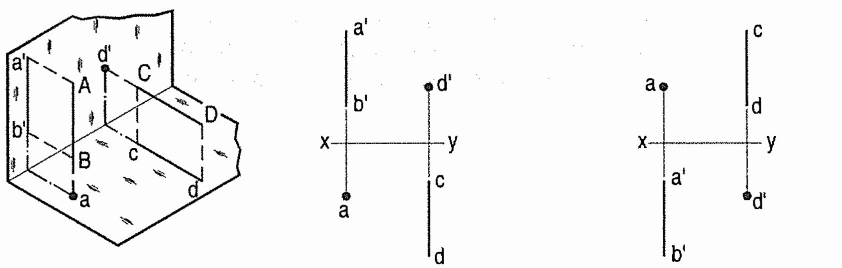
Hence, when a line is contained by a plane, its projection on that plane is equal to its true length; while its projection on the other plane is in the reference line.

10-3. LINE PERPENDICULAR TO ONE OF THE PLANES (FIG. 10-3)

CHAROTAR COGNIFRONT

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 22 for the line perpendicular to one of the planes.

When a line is perpendicular to one reference plane, it will be parallel to the other.



(FIRST-ANGLE PROJECTION) (THIRD-ANGLE PROJECTION)

FIG. 10-3

(a) Line *AB* is perpendicular to the H.P. The top views of its ends coincide in the point *a*. Hence, the top view of the line *AB* is the point *a*. Its front view *a' b'* is equal to *AB* and perpendicular to *xy*.

(b) Line *CD* is perpendicular to the V.P. The point *d'* is its front view and the line *cd* is the top view. *cd* is equal to *CD* and perpendicular to *xy*.

Hence, when a line is perpendicular to a plane its projection on that plane is a point; while its projection on the other plane is a line equal to its true length and perpendicular to the reference line.

In *first-angle projection method,* when top views of two or more points coincide, the point which is comparatively *farther* away from *xy* in the front view will be visible; and when their front views coincide, that which is *farther* away from *xy* in the top view will be visible.

In *third-angle projection method,* it is just the reverse. When top views of two or more points coincide the point which is comparatively *nearer xy* in the front view will be visible; and when their front views coincide, the point which is *nearer xy* in the top view will be visible.

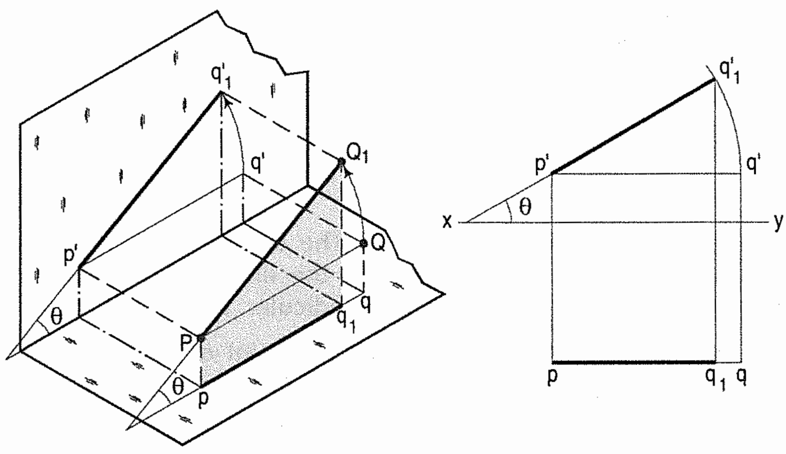
10-4. LINE INCLINED TO ONE PLANE AND PARALLEL TO THE OTHER

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 23 for the line inclined to one plane and parallel to the other.

The inclination of a line to a plane is the angle which the line makes with its projection on that plane.

(a) Line PQ_1 [fig. 10-4(i)] is inclined at an angle θ to the H.P. and is parallel to the V.P. The inclination is shown by the angle θ which PQ_1 makes with its own projection on the H.P., viz. the top view pq_1.

The projections [fig. 10-4(ii)] may be drawn by first assuming the line to be parallel to both the H.P. and the V.P. Its front view $p'q'$ and the top view pq will both be parallel to xy and equal to the true length. When the line is turned about the end P to the position PQ_1 so that it makes the angle θ with the H.P. while remaining parallel to the V.P., in the front view the point q' will move along an arc drawn with p' as centre and $p'q'$ as radius to a point q'_1 so that $p'q'_1$ makes the angle θ with xy. In the top view, q will move towards p along pq to a point q_1 on the projector through q'_1. $p'q'_1$ and pq_1 are the front view and the top view respectively of the line PQ_1.



(i) (ii)

FIG. 10-4

(b) Line RS_1 [fig. 10-5(i)] is inclined at an angle ø to the V.P. and is parallel to the H.P. The inclination is shown by the angle ø which RS_1 makes with its projection on the V.P., viz. the front view $r's'_1$. Assuming the line to be parallel to both the H.P. and the V.P., its projections $r's'$ and rs are drawn parallel to xy and equal to its true length [fig. 10-5(ii)].

When the line is turned about its end R to the position RS_1 so that it makes the angle ø with the V.P. while remaining parallel to the H.P., in the top view the point s will move along an arc drawn with r as centre and rs as radius to a point s_1 so that rs_1 makes the angle ø with xy. In the front view, the point s' will move towards r' along the line $r's'$ to a point s'_1 on the projector through s_1. rs_1 and $r's'_1$ are the projections of the line RS_1.

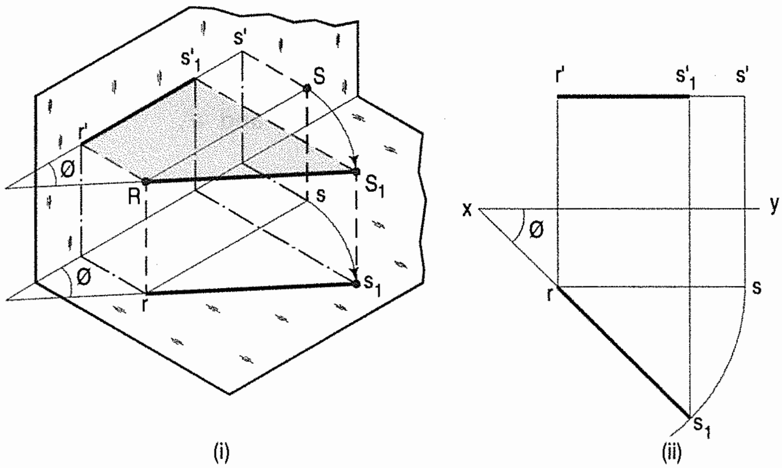


FIG. 10-5

Therefore, when the line is inclined to the H.P. and parallel to the V.P., its top view is shorter than its true length, but parallel to xy; its front view is equal to its true length and is inclined to xy at its true inclination with the H.P. And when the line is inclined to the V.P. and parallel to the H.P., its front view is shorter than its true length but parallel to xy; its top view is equal to its true length and is inclined to xy at its true inclination with the V.P.

Hence, when a line is inclined to one plane and parallel to the other, its projection on the plane to which it is inclined, is a line shorter than its true length but parallel to the reference line. Its projection on the plane to which it is parallel, is a line equal to its true length and inclined to the reference line at its true inclination.

In other words, the inclination of a line with the H.P. is seen in the front view and that with the V.P. is seen in the top view.

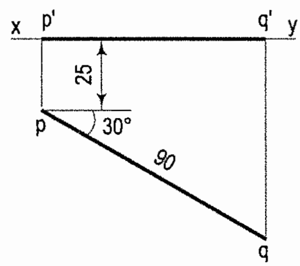


FIG. 10-6

Problem 10-1. (fig. 10-6): *A line PQ, 90 mm long, is in the H.P. and makes an angle of 30° with the V.P. Its end P is 25 mm in front of the V.P. Draw its projections.*

As the line is in the H.P., its top view will show the true length and the true inclination with the V.P. Its front view will be in xy.

(i) Mark a point p, the top view 25 mm below xy. Draw a line pq equal to 90 mm and inclined at 30° to xy.

(ii) Project p to p' and q to q' on xy.

pq and $p'q'$ are the required top view and front view respectively.

Problem 10-2. (fig. 10-7): *The length of the top view of a line parallel to the V.P. and inclined at 45° to the H.P. is 50 mm. One end of the line is 12 mm above the H.P. and 25 mm in front of the V.P. Draw the projections of the line and determine its true length.*

As the line is parallel to the V.P., its top view will be parallel to *xy* and the front view will show its true length and the true inclination with the H.P.

(i) Mark *a*, the top view, 25 mm below *xy* and *a'*, the front view, 12 mm above *xy*.

(ii) Draw the top view *ab* 50 mm long and parallel to *xy* and draw a projector through *b*.

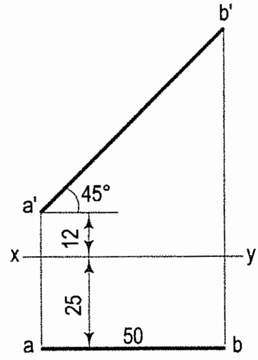


FIG. 10-7

(iii) From *a'* draw a line making 45° angle with *xy* and cutting the projector through *b* at *b'*. Then *a'b'* is the front view and also the true length of the line.

Problem 10-3. (fig. 10-8): *The front view of a 75 mm long line measures 55 mm. The line is parallel to the H.P. and one of its ends is in the V.P. and 25 mm above the H.P. Draw the projections of the line and determine its inclination with the V.P.*

As the line is parallel to the H.P., its front view will be parallel to *xy*.

(i) Mark *a*, the top view of one end in *xy*, and *a'*, its front view, 25 mm above *xy*.

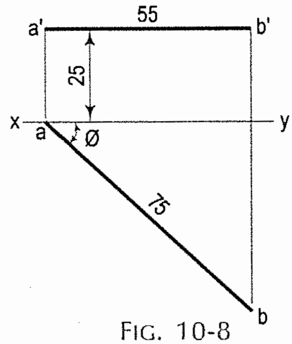


FIG. 10-8

(ii) Draw the front view *a'b'*, 55 mm long and parallel to *xy*. With *a* as centre and radius equal to 75 mm, draw an arc cutting the projector through *b'* at *b*.

Join *a* with *b*. *ab* is the top view of the line. Its inclination with *xy*, viz. ø is the inclination of the line with the V.P.

EXERCISES 10(a)

1. Draw the projections of a 75 mm long straight line, in the following positions:

(a) (i) Parallel to both the H.P. and the V.P. and 25 mm from each.

(ii) Parallel to and 30 mm above the H.P. and in the V.P.

(iii) Parallel to and 40 mm in front of the V.P. and in the H.P.

(b) (i) Perpendicular to the H.P., 20 mm in front of the V.P. and its one end 15 mm above the H.P.

(ii) Perpendicular to the V.P., 25 mm above the H.P. and its one end in the V.P.

(iii) Perpendicular to the H.P., in the V.P. and its one end in the H.P.

(c) (i) Inclined at 45° to the V.P., in the H.P. and its one end in the V.P.

(ii) Inclined at 30° to the H.P. and its one end 20 mm above it; parallel to and 30 mm in front of the V.P.

(iii) Inclined at 60° to the V.P. and its one end 15 mm in front of it; parallel to and 25 mm above the H.P.

2. A 100 mm long line is parallel to and 40 mm above the H.P. Its two ends are 25 mm and 50 mm in front of the V.P. respectively. Draw its projections and find its inclination with the V.P.
3. A 90 mm long line is parallel to and 25 mm in front of the V.P. Its one end is in the H.P. while the other is 50 mm above the H.P. Draw its projections and find its inclination with the H.P.
4. The top view of a 75 mm long line measures 55 mm. The line is in the V.P., its one end being 25 mm above the H.P. Draw its projections.
5. The front view of a line, inclined at 30° to the V.P is 65 mm long. Draw the projections of the line, when it is parallel to and 40 mm above the H.P., its one end being 30 mm in front of the V.P.
6. A vertical line *AB*, 75 mm long, has its end *A* in the H.P. and 25 mm in front of the V.P. A line *AC*, 100 mm long, is in the H.P. and parallel to the V.P. Draw the projections of the line joining *B* and *C*, and determine its inclination with the H.P.
7. Two pegs fixed on a wall are 4.5 metres apart. The distance between the pegs measured parallel to the floor is 3.6 metres. If one peg is 1.5 metres above the floor, find the height of the second peg and the inclination of the line joining the two pegs, with the floor.
8. Draw the projections of the lines in Exercises 1 to 6, assuming them to be in the third quadrant, taking the given positions to be below the H.P. instead of above the H.P., and behind the V.P., instead of in front of the V.P.

10-5. LINE INCLINED TO BOTH THE PLANES

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 24 for the line inclined to both the planes.

(a) A line *AB* (fig. 10-9) is inclined at θ to the H.P. and is parallel to the V.P. The end *A* is in the H.P. *AB* is shown as the hypotenuse of a right-angled triangle, making the angle θ with the base.

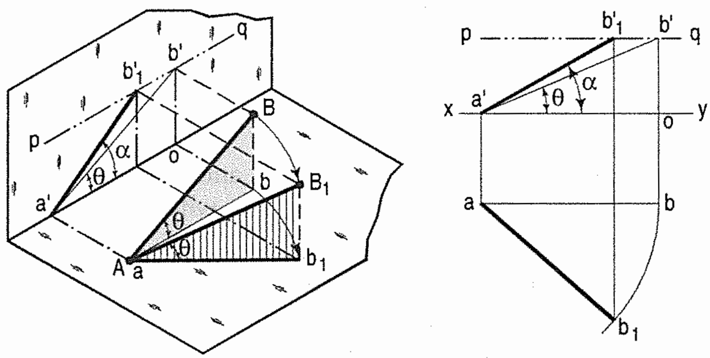


FIG. 10-9

The top view *ab* is shorter than *AB* and parallel to *xy*. The front view *a'b'* is equal to *AB* and makes the angle θ with *xy*.

Keeping the end *A* fixed and the angle θ with the H.P. constant, if the end *B* is moved to any position, say B_1, the line becomes inclined to the V.P. also.

In the top view, *b* will move along an arc, drawn with *a* as centre and *ab* as radius, to a position b_1. The new top view ab_1 is equal to *ab* but shorter than *AB*.

In the front view, *b'* will move to a point b'_1 keeping its distance from *xy* constant and equal to *b'o*; i.e. it will move along the line *pq*, drawn through *b'* and parallel to *xy*. This line *pq* is the locus or path of the end *B* in the front view. b'_1 will lie on the projector through b_1. The new front view $a'b'_1$ is shorter than *a'b'* (i.e. *AB*) and makes an angle α with *xy*. α is greater than θ.

Thus, it can be seen that as long as the inclination θ of *AB* with the H.P. is constant, even when it is inclined to the V.P.

(i) its length in the top view, viz. *ab* remains constant; and

(ii) the distance between the paths of its ends in the front view, viz. *b'o* remains constant.

(b) The same line *AB* (fig. 10-10) is inclined at ø to the V.P. and is parallel to the H.P. Its end *A* is in the V.P. *AB* is shown as the hypotenuse of a right-angled triangle making the angle ø with the base.

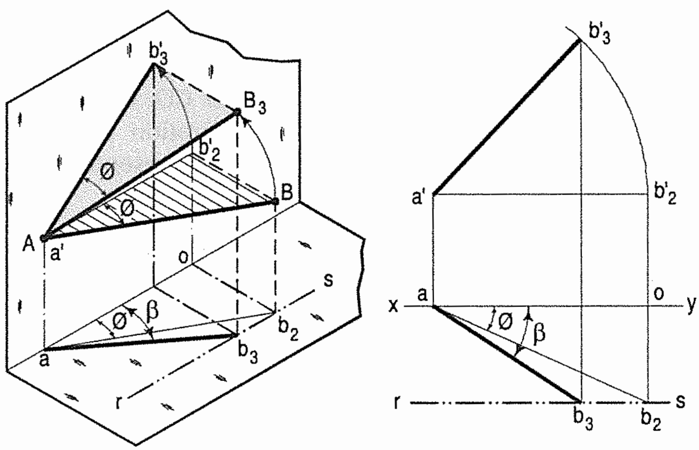


FIG. 10-10

The front view $a'b'_2$ is shorter than *AB* and parallel to *xy*. The top view ab_2 is equal to *AB* and makes an angle ø with *xy*.

Keeping the end *A* fixed and the angle ø with the V.P. constant, if *B* is moved to any position, say B_3, the line will become inclined to the H.P. also.

In the front view, b'_2, will move along the arc, drawn with a' as centre and $a'b'_2$ as radius, to a position b'_3. The new front view $a'b'_3$ is equal to $a'b'_2$ but is shorter than *AB*.

In the top view, b_2 will move to a point b_3 along the line *rs*, drawn through b_2 and parallel to *xy*, thus keeping its distance from the path of *a*, viz. b_2o constant. *rs* is the locus or path of the end *B* in the top view. The point b_3 lies on the projector through b'_3. The new top view ab_3 is shorter than ab_2 (i.e. *AB*) and makes an angle β with *xy*. β is greater than ø.

Here also we find that, as long as the inclination of *AB* with the V.P. does not change, even when it becomes inclined to the H.P.

(i) its length in the front view, viz. $a'b'_2$ remains constant; and

(ii) the distance between the paths of its ends in the top view, viz. b_2o remains constant.

Hence, when a line is inclined to both the planes, its projections are shorter than the true length and inclined to xy at angles greater than the true inclinations. These angles viz. α and β are called apparent angles of inclination.

10-6. PROJECTIONS OF LINES INCLINED TO BOTH THE PLANES

From Art. 10-5(a) above, we find that as long as the inclination of *AB* with the H.P. is constant

(i) its length in the top view, viz. *ab* remains constant, and

(ii) in the front view, the distance between the loci of its ends, viz. *b'o* remains constant.

In other words if

(i) its length in the top view is equal to *ab*, and

(ii) the distance between the paths of its ends in the front view is equal to *b'o*, the inclination of *AB* with the H.P. will be equal to θ.

Similarly, from Art. 10-5(b) above, we find that as long as the inclination of *AB* with the V.P. is constant

(i) its length in the front view, viz. $a'b'_2$ remains constant, and

(ii) in the top view, the distance between the loci of its ends, viz. b_2o remains constant.

The reverse of this is also true, viz.

(i) if its length in the front view is equal to $a'b'_2$, and

(ii) the distance between the paths of its ends in the top view is equal to b_2o, the inclination of *AB* with the V.P. will be equal to ø.

Combining the above two findings, we conclude that when *AB* is inclined at θ to the H.P. and at ø to the V.P.

(i) its lengths in the top view and the front view will be equal to ab_2 and $a'b'_2$ respectively, and

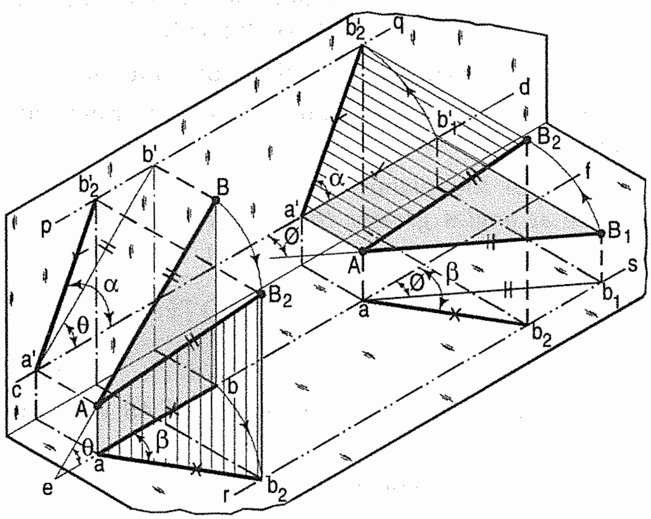
(ii) the distances between the paths of its ends in the front view and the top view will be equal to b'_2o and b_2o respectively.

The two lengths when arranged with their ends in their respective paths and in projections with each other will be the projections of the line *AB*, as illustrated in problem 10-4.

Problem 10-4. *Given the line AB, its inclinations θ with the H.P. and ø with the V.P. and the position of one end A. To draw its projections.*

Mark the front view *a*' and the top view *a* according to the given position of *A* (fig. 10-12).

Let us first determine the lengths of *AB* in the top view and the front view and the paths of its ends in the front view and the top view.

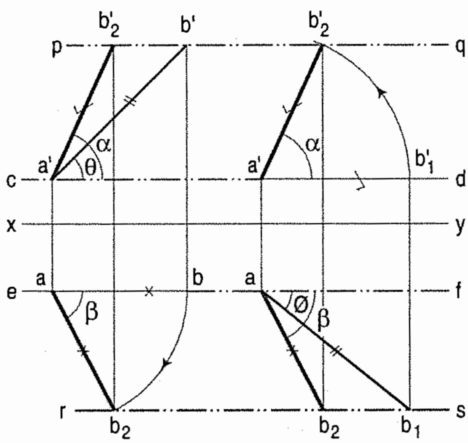


(i) (ii)

FIG. 10-11

(1) Assume *AB* to be parallel to the V.P. and inclined at θ to the H.P. *AB* is shown in the pictorial view as a side of the trapezoid *ABba* [fig. 10-11(i)]. Draw the front view *a'b'* equal to *AB* [fig. 10-12(i)] and inclined at θ to *xy*. Project the top view *ab* parallel to *xy*. Through *a*' and *b*', draw lines *cd* and *pq* respectively parallel to *xy*. *ab* is the length of *AB* in the top view and, *cd* and *pq* are the paths of *A* and *B* respectively in the front view.

(2) Again, assume AB_1 (equal to *AB*) to be parallel to the H.P. and inclined at ø to the V.P. In the pictorial view [fig. 10-11(ii)], AB_1 is shown as a side of the trapezoid $AB_1b'_1a'$. Draw the top view ab_1 equal to *AB* [fig. 10-12(ii)] and inclined at ø to *xy*. Project the front view $a'b'_1$ parallel to *xy*. Through *a* and b_1, draw lines *ef* and *rs* respectively parallel to *xy*. $a'b'_1$ is the length of *AB* in the front view and, *ef* and *rs* are the paths of *A* and *B* respectively in the top view.



(i) (ii)

FIG. 10-12

We may now arrange

(i) *ab* (the length in the top view) between its paths *ef* and *rs*, and

(ii) $a'b'_1$ (the length in the front view) between the paths *cd* and *pq*, keeping them in projection with each other, in one of the following *two* ways:

(a) In case (1) [fig. 10-11(i)], if the side *Bb* is turned about *Aa*, so that *b* comes on the path *rs*, the line *AB* will become inclined at ø to the V.P. Therefore, with *a* as centre [fig. 10-12(i)] and radius equal to *ab*, draw an arc cutting *rs* at a point b_2. Project b_2 to b'_2 on the path *pq*.

Draw lines joining a with b_2, and a' with b'_2. ab_2 and $a'b'_2$ are the required projections. Check that $a'b'_2 = a'b'_1$.

(b) Similarly, in case (2) [fig. 10-11(ii)], if the side $B_1b'_1$ is turned about Aa' till b_1' is on the path pq, the line AB_1 will become inclined at θ to the H.P.

Hence, with a' as centre [fig. 10-12(ii)] and radius equal to $a'b'_1$, draw an arc cutting pq at a point b'_2. Project b'_2 to b_2 in the top view on the path rs.

Draw lines joining a with b_2, and a' with b'_2. ab_2 and $a'b'_2$ are the required projections. Check that $ab_2 = ab$.

Fig. 10-13 shows (in pictorial and orthographic views) the projections obtained with both the above steps combined in one figure and as described below.

First, determine

(i) the length ab in the top view and the path pq in the front view and

(ii) the length $a'b'_1$ in the front view and the path rs in the top view.

Then, with a as centre and radius equal to ab, draw an arc cutting rs at a point b_2. With a' as centre and radius equal to $a'b'_1$, draw an arc cutting pq at a point b'_2.

Draw lines joining a with b_2 and a' with b'_2. ab_2 and $a'b'_2$ are the required projections. Check that b_2 and b'_2 lie on the same projector.

It is quite evident from the figure that the apparent angles of inclination α and β are greater than the true inclinations θ and ø respectively.

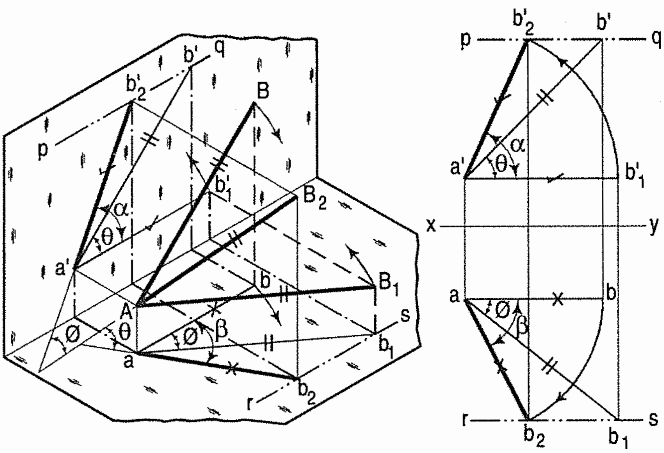


FIG. 10-13

10-7. LINE CONTAINED BY A PLANE PERPENDICULAR TO BOTH THE REFERENCE PLANES

As the two reference planes are at right angles to each other, the sum total of the inclinations of a line with the two planes, viz. θ and ø can never be more than 90°. When θ + ø = 90°, the line will be contained by a third plane called the profile plane, perpendicular to both the H.P. and the V.P.

A line *EF* (fig. 10-14), is inclined at θ to the H.P. and at ø [equal to (90° – θ)] to the V.P. The line is thus contained by the profile plane marked P.P.

The front view *e'f'* and the top view *ef* are both perpendicular to *xy* and shorter than *EF*.

Therefore, when a line is inclined to both the reference planes and contained by a plane perpendicular to them, i.e. when the sum of its inclinations with the H.P. and the V.P. is 90°, its projections are perpendicular to *xy* and shorter than the true length.

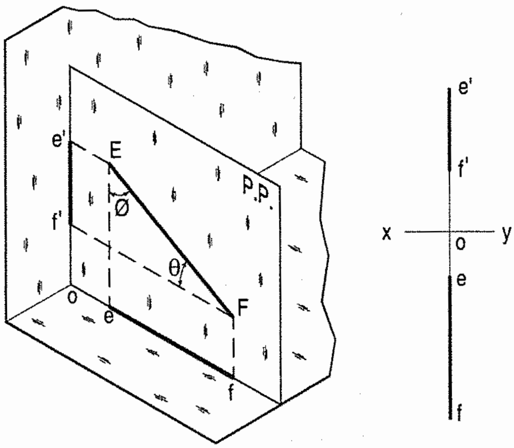


FIG. 10-14

10-8. TRUE LENGTH OF A STRAIGHT LINE AND ITS INCLINATIONS WITH THE REFERENCE PLANES

When projections of a line are given, its true length and inclinations with the planes are determined by the application of the following rule:

When a line is parallel to a plane, its projection on that plane will show its true length and the true inclination with the other plane.

The line may be made parallel to a plane, and its true length obtained by any one of the following *three* methods:

Method I:

Making each view parallel to the reference line and projecting the other view from it. This is the exact reversal of the processes adopted in Art. 10-5 for obtaining the projections.

Method II:

Rotating the line about its projections till it lies in the H.P. or in the V.P.

Method III:

Projecting the views on auxiliary planes parallel to each view.

(This method will be dealt with in chapter 11).

The following problem shows the application of the first two methods and problem 10-29 and problem 10-31 show application of third method.

Problem 10-5. *The top view ab and the front view a'b' of a line AB are given. To determine its true length and the inclinations with the H.P. and the V.P.*

Method I:

Fig. 10-15(i) shows *AB* the line, *a'b'* its front view and *ab* its top view. If the trapezoid *ABba* is turned about *Aa* as axis, so that *AB* becomes parallel to the V.P., in the top view, *b* will move along an arc drawn with centre *a* and radius equal to *ab*, to b_1, so that ab_1 is parallel to *xy*. In the front view, *b'* will move along its locus *pq*, to a point b'_1 on the projector through b_1.

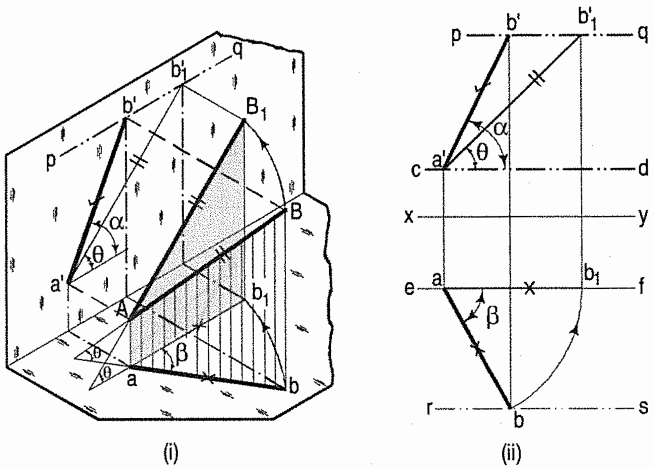


FIG. 10-15

(i) Therefore, with centre a and radius equal to ab [fig. 10-15(ii)], draw an arc to cut ef at b_1.

(ii) Draw a projector through b_1 to cut pq (the path of b') at b'_1.

(iii) Draw the line $a'b'_1$ which is the true length of AB. The angle θ, which it makes with xy is the inclination of AB with the H.P.

Again, in fig. 10-16(i) AB is shown as a side of a trapezoid $ABb'a'$. If the trapezoid is turned about Aa' as axis so that AB is parallel to the H.P., the new top view will show its true length and true inclination with the V.P.

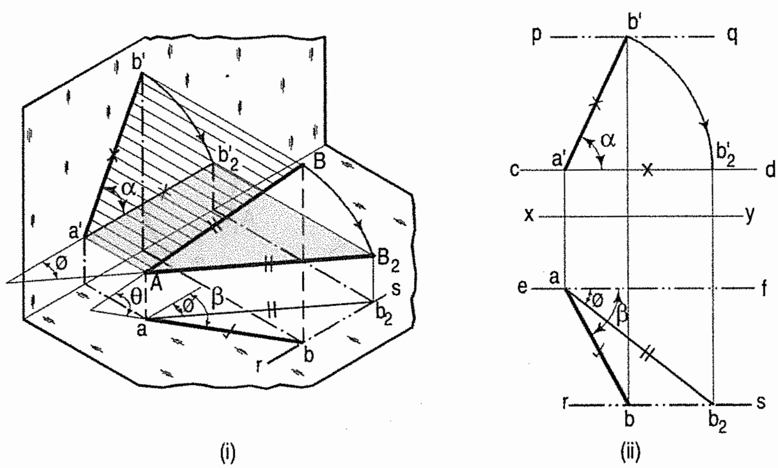


FIG. 10-16

(i) With a' as centre and radius equal to $a'b'$ [fig. 10-16(ii)], draw an arc to cut cd at b'_2.

(ii) Draw a projector through b'_2 to cut rs (the path of b) at b_2.

(iii) Draw the line ab_2, which is the true length of AB. The angle ø which it makes with xy is the inclination of AB with the V.P.

Fig. 10-17(i) shows the above two steps combined in one figure.

The same results will be obtained by keeping the end *B* fixed and turning the end *A* [fig. 10-17(ii)], as explained below.

(i) With centre *b* and radius equal to *ba*, draw an arc cutting *rs* at a_1 (thus making *ba* parallel to *xy*).

(ii) Project a_1 to a'_1 on *cd* (the path of *a*') a'_1b' is the true length and θ is the true inclination of *AB* with the H.P.

(iii) Similarly, with centre *b*' and radius equal to *b'a'*, draw an arc cutting *pq* at a_2'.

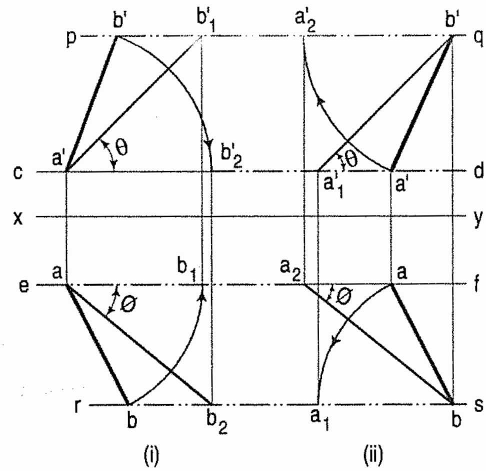


FIG. 10-17

(iv) Project a_2' to a_2 on *ef* (the path of *a*). a_2b is the true length and ø is the true inclination of *AB* with the V.P.

Method II:

Referring to the pictorial view in fig. 10-18(i) we find that *AB* is the line, *ab* its top view and *a'b'* its front view.

In the trapezoid *ABb'a'* (i) *a'A* and *b'B* are both perpendicular to *a'b'* and are respectively equal to ao_1 and bo_2 (the distances of *a* and *b* from *xy* in the top view), and (ii) the angle between *AB* and *a'b'* is the angle of inclination ø of *AB* with the V.P.

Assume that this trapezoid is rotated about *a'b'*, till it lies in the V.P.

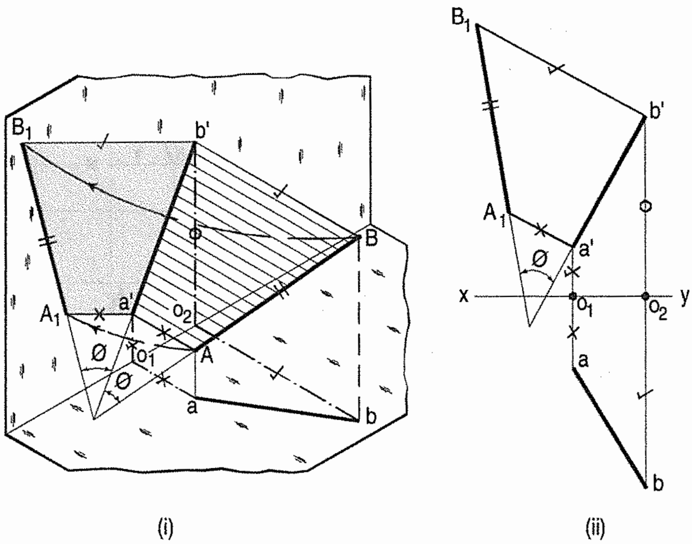


FIG. 10-18

In the orthographic view [fig. 10-18(ii)], this trapezoid is obtained by drawing perpendiculars to *a'b'*, viz. $a'A_1$ (equal to ao_1) and $b'B_1$ (equal to bo_2) and then joining A_1 with B_1. The line A_1B_1 is the true length of *AB* and its inclination ø with *a' b'* is the inclination of *AB* with the V.P.

Similarly, in trapezoid *ABba* in fig. 10-19(i), *AB* is the line and *ab* its top view. *Aa* and *Bb* are both perpendicular to *ab* and are respectively equal to $a'o_1$ and $b'o_2$ (the distances of *a'* and *b'* from *xy* in the front view). The angle θ between *AB* and *ab* is the inclination of *AB* with the H.P.

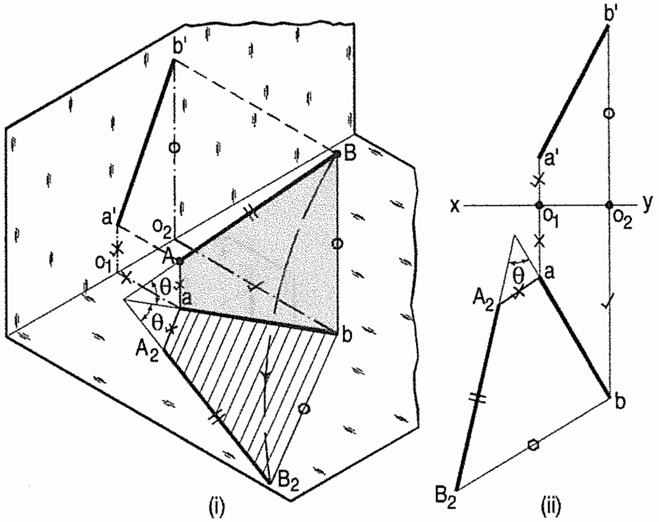


FIG. 10-19

This figure may now be assumed to be rotated about *ab* as axis, so that it lies in the H.P.

In the orthographic view [fig. 10-19(ii)], this trapezoid is obtained by erecting perpendiculars to *ab*, viz. aA_2 equal to $a'o_1$ and bB_2 equal to $b'o_2$ and joining A_2 with B_2. The line A_2B_2 is the true length of *AB* and its inclination θ with *ab* is the inclination of *AB* with the H.P.

Note: The perpendiculars on *ab* or *a'b'* can also be drawn on its other side assuming the trapezoid to be rotated in the opposite direction.

10-9. TRACES OF A LINE

When a line is inclined to a plane, it will meet that plane, produced if necessary. The point in which the line or line-produced meets the plane is called its *trace*.

The point of intersection of the line with the H.P. is called the *horizontal trace*, usually denoted as H.T. and that with the V.P. is called the *vertical trace* or V.T.

Refer to fig. 10-20.

(i) A line *AB* is parallel to the H.P. and the V.P. It has no trace.

(ii) A line *CD* is inclined to the H.P. and parallel to the V.P. It has only the H.T. but no V.T.

(iii) A line *EF* is inclined to the V.P. and parallel to the H.P. It has only the V.T. but no H.T.

Thus, when a line is parallel to a plane it has no trace upon that plane.

Refer to fig. 10-21.

(i) A line *PQ* is perpendicular to the H.P. Its H.T. coincides with its top view which is a point. It has no V.T.

(ii) A line *RS* is perpendicular to the V.P. Its V.T. coincides with its front view which is a point. It has no H.T.

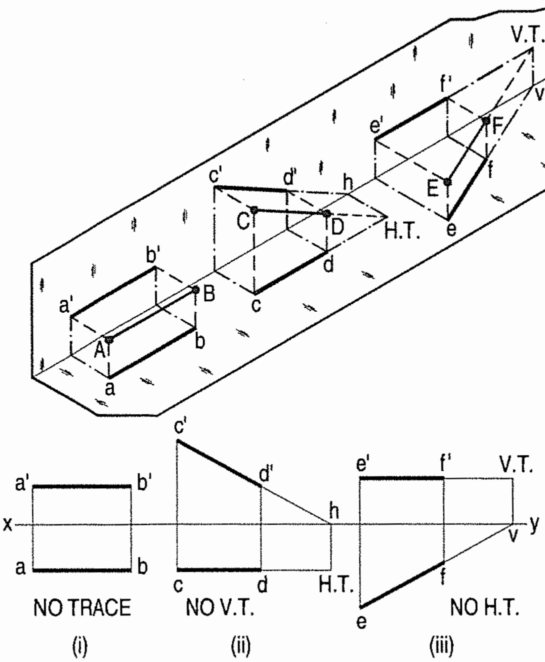


FIG. 10-20

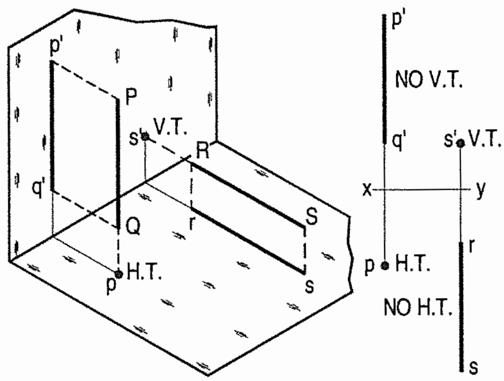


FIG. 10-21

Hence, when a line is perpendicular to a plane, its trace on that plane coincides with its projection on that plane. It has no trace on the other plane.

Refer to fig. 10-22.

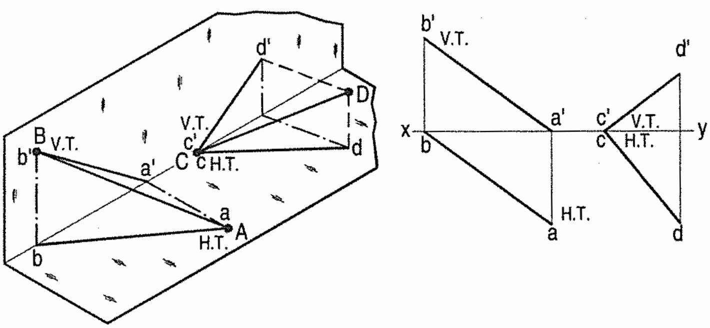


FIG. 10-22

(i) A line *AB* has its end *A* in the H.P. and the end *B* in the V.P. Its H.T. coincides with *a* the top view of *A* and the V.T. coincides with *b*' the front view of *B*.

(ii) A line *CD* has its end *C* in both the H.P. and the V.P. Its H.T. and V.T. coincide with *c* and *c*' (the projections of *C*) in *xy*.

Hence, when a line has an end in a plane, its trace upon that plane coincides with the projection of that end on that plane.

10-10. METHODS OF DETERMINING TRACES OF A LINE

Method I:

Fig. 10-23(i) shows a line *AB* inclined to both the reference planes. Its end *A* is in the H.P. and *B* is in the V.P.

a'b' and *ab* are the front view and the top view respectively [fig. 10-23(ii)].

The H.T. of the line is on the projector through *a'* and coincides with *a*. The V.T. is on the projector through *b* and coincides with *b'*.

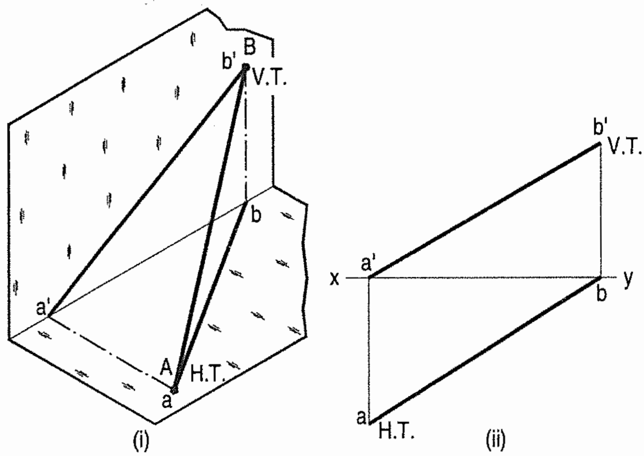


FIG. 10-23

Let us now assume that *AB* is shortened from both its ends, its inclination with the planes remaining constant. The H.T. and V.T. of the new line *CD* are still the same as can be seen clearly in fig. 10-24(i).

c'd' and *cd* are the projections of *CD* [fig. 10-24(ii)]. Its traces may be determined as described below.

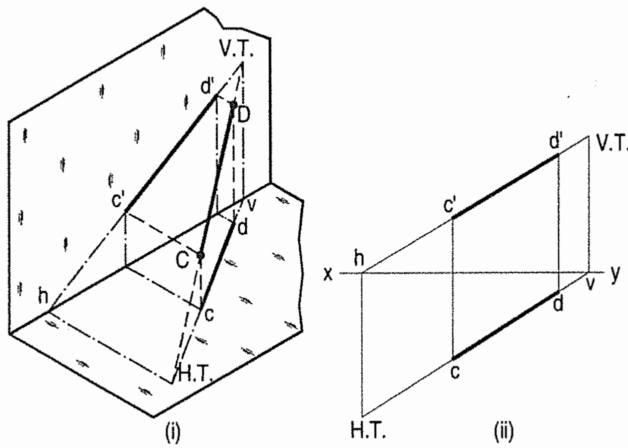


FIG. 10-24

(i) Produce the front view *c'd'* to meet *xy* at a point *h*.

(ii) Through *h*, draw a projector to meet the top view *cd*-produced, at the H.T. of the line.

(iii) Similarly, produce the top view *cd* to meet *xy* at a point *v*.

(iv) Through *v*, draw a projector to meet the front view *c'd'*-produced, at the V.T. of the line.

Method II:

c'd' and *cd* are the projections of the line *CD* [fig. 10-25(ii)]. Determine the true length C_1D_1 from the front view *c'd'* by trapezoid method. The point of intersection between *c'd'*-produced and C_1D_1-produced is the V.T. of the line.

Similarly, determine the true length C_2D_2 from the top view *cd*. Produce them to intersect at the H.T. of the line.

The above is quite evident from the pictorial view shown in fig. 10-25(i).

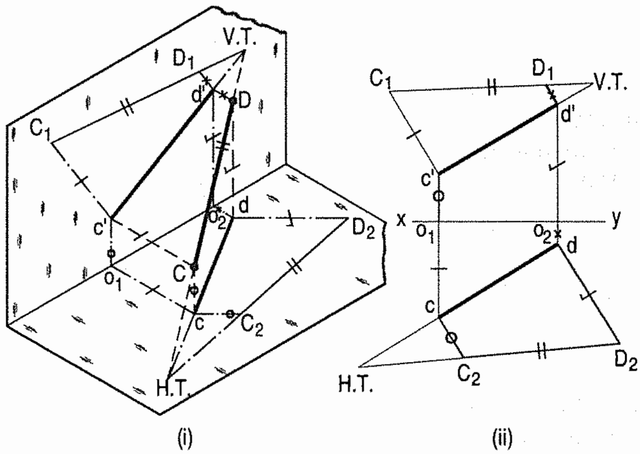


FIG. 10-25

10-11. TRACES OF A LINE, THE PROJECTIONS OF WHICH ARE PERPENDICULAR TO *xy*

When the projections of a line are perpendicular to *xy*, i.e. when the sum of its inclinations with the two principal planes of projection is 90°, it is not possible to find the traces by the first method. Method II must, therefore, be adopted as shown in fig. 10-26.

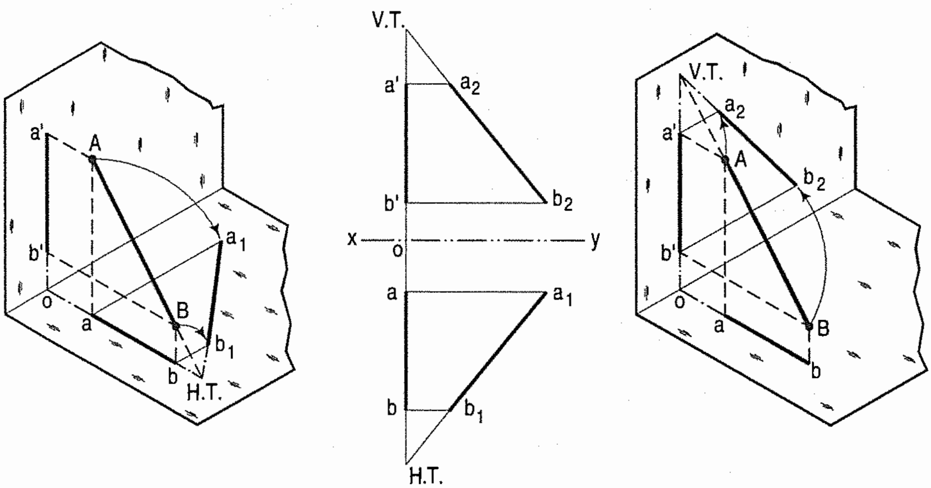


FIG. 10-26

10-12. POSITIONS OF TRACES OF A LINE

Although the line may be situated in the third quadrant, its both traces may be above or below *xy*, as shown in problem 10-6 and in fig. 10-27 and fig. 10-28. When a line intersects a plane, its traces on that plane will be contained by its projection on that plane as shown in problem 10-7.

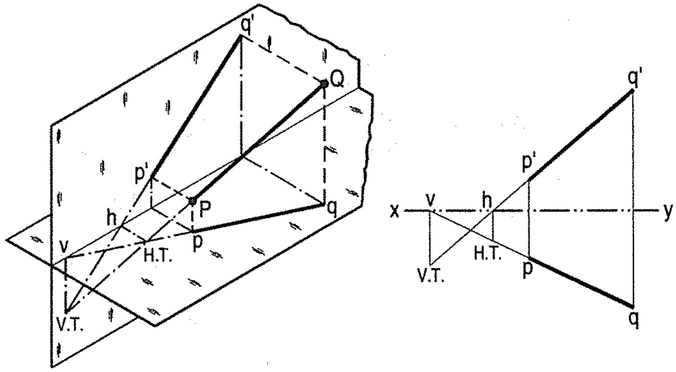


FIG. 10-27

Problem 10-6. *Projections of a line PQ are given. Determine the positions of its traces.*

Let *pq* and *p'q'* be the projections of *PQ* (fig. 10-27 and fig. 10-28).

(i) Produce the top view *pq* to meet *xy* at *v*. Draw a projector through *v* to meet the front view *p'q'*-produced at the V.T.

(ii) Through *h*, the point of intersection between *p'q'*-produced and *xy*, draw a projector to meet the top view *pq*-produced at the H.T.

Note that in fig. 10-27, the traces are below *xy* while in fig. 10-28 they are above it.

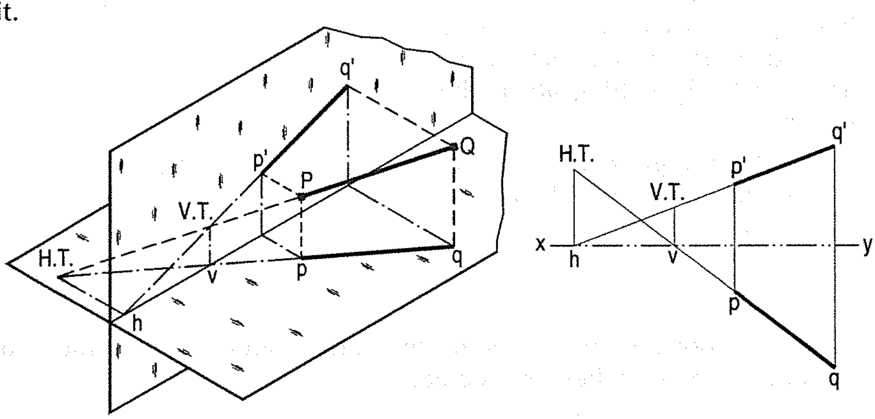


FIG. 10-28

Problem 10-7. *A point A is 50 mm below the H.P. and 12 mm behind the V.P. A point B is 10 mm above the H.P. and 25 mm in front of the V.P. The distance between the projectors of A and B is 40 mm. Determine the traces of the line joining A and B.*

Draw the projections *ab* and *a'b'* of the line *AB*.

Method I: (fig. 10-29):

(i) Through *v*, the point of intersection between *ab* and *xy*, draw a projector to meet *a'b'* at the V.T. of the line.

(ii) Similarly, through *h*, the point of intersection between *a'b'* and *xy*, draw a projector to cut *ab* at the H.T. of the line.

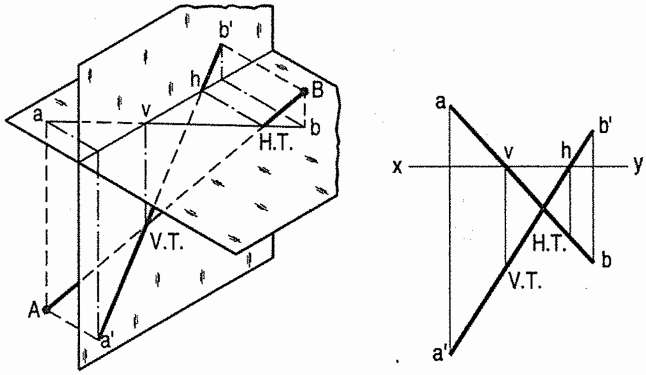


FIG. 10-29

Method II: (fig. 10-30):

At the ends a' and b', draw perpendiculars to $a'b'$, viz. $a'A_1$ equal to ao_1 and $b'B_1$ equal to bo_2 on its opposite sides (as a and b are on opposite sides of xy).

Draw the line A_1B_1 intersecting $a'b'$ at the V.T. of the line.

Similarly, at the ends a and b, draw perpendiculars to ab, viz. aA_2 equal to $a'o_1$ and bB_2 equal to $b'o_2$, on its opposite sides (as a' and b' are on opposite sides of xy). Join A_2 with B_2 cutting ab at the H.T. of the line.

Note that $A_1B_1 = A_2B_2 = AB$ and that θ and ø are the inclinations of AB with the H.P. and the V.P. respectively.

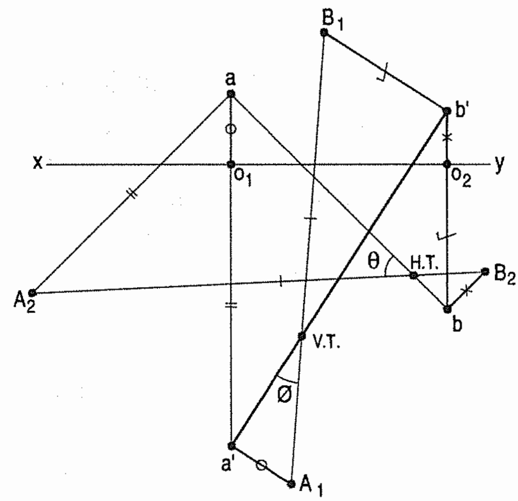


FIG. 10-30

10-13. ADDITIONAL ILLUSTRATIVE PROBLEMS

In the following problems, the ends of the lines should be assumed to be in the first quadrant, unless otherwise stated.

Problem 10-8. (fig. 10-31): *A line AB, 50 mm long, has its end A in both the H.P. and the V.P. It is inclined at 30° to the H.P. and at 45° to the V.P. Draw its projections.*

As the end A is in both the planes, its top view and the front view will coincide in xy.

(i) Assuming AB to be parallel to the V.P. and inclined at θ (equal to 30°) to the H.P., draw its front view ab' (equal to AB) and project the top view ab.

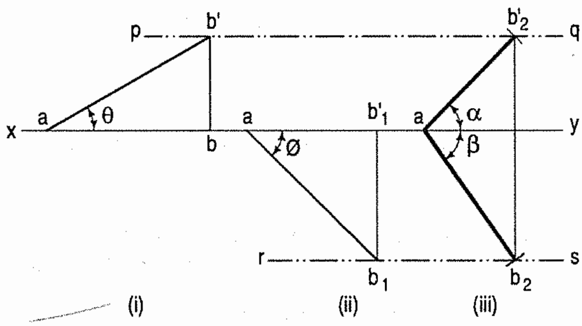


FIG. 10-31

(ii) Again assuming *AB* to be parallel to the H.P. and inclined at ø (equal to 45°) to the V.P., draw its top view ab_1 (equal to *AB*). Project the front view ab'_1.

ab and ab'_1 are the lengths of *AB* in the top view and the front view respectively, and *pq* and *rs* are the loci of the end *B* in the front view and the top view respectively.

(iii) With *a* as centre and radius equal to ab'_1, draw an arc cutting *pq* in b'_2. With the same centre and radius equal to *ab*, draw an arc cutting *rs* in b_2.

Draw lines joining *a* with b'_2 and b_2. ab'_2 and ab_2 are the required projections.

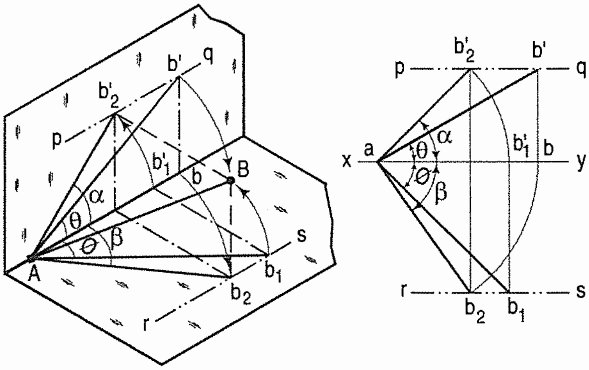


FIG. 10-32

Fig. 10-32 shows in pictorial and orthographic views, the solution obtained with all the above steps combined in one figure only.

Problem 10-9. (fig. 10-33): *A line PQ 75 mm long, has its end P in the V.P. and the end Q in the H.P. The line is inclined at 30° to the H.P. and at 60° to the V.P. Draw its projections.*

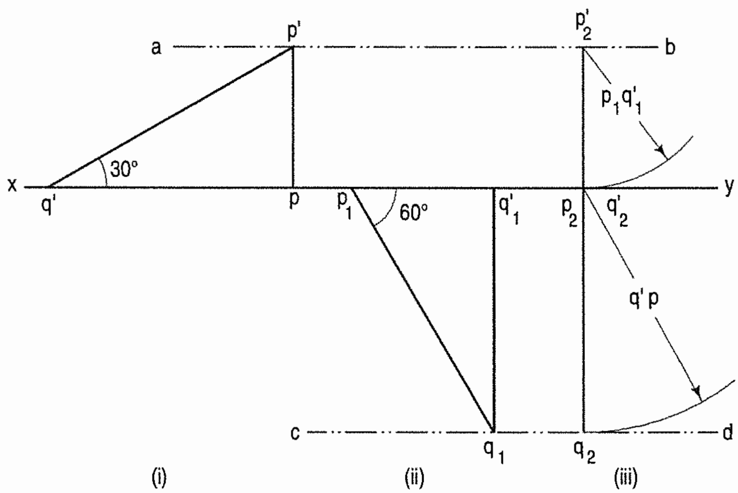


FIG. 10-33

The top view of *P* and the front view of *Q* will be in *xy*. As shown in the previous problem, determine

(i) the length of PQ in the top view, viz. $q'p$ and the path ab of the end P in the front view;

(ii) the length $p_1q'_1$ in the front view and the path cd of the end Q in the top view.

(iii) Mark any point p_2 (the top view of P) in xy and project its front view p'_2 in ab.

(iv) With p'_2 as centre and radius equal to $p_1q'_1$, draw an arc cutting xy in q'_2. It coincides with p_2.

(v) With p_2 as centre and radius equal to $q'p$, draw an arc cutting cd in q_2. p_2q_2 and $p'_2q'_2$ are the required projections. They lie in a line perpendicular to xy because the sum of the two inclinations is equal to 90°.

Problem 10-10. (fig. 10-34): *A line PQ 100 mm long, is inclined at 30° to the H.P. and at 45° to the V.P. Its mid-point is in the V.P. and 20 mm above the H.P. Draw its projections, if its end P is in the third quadrant and Q in the first quadrant.*

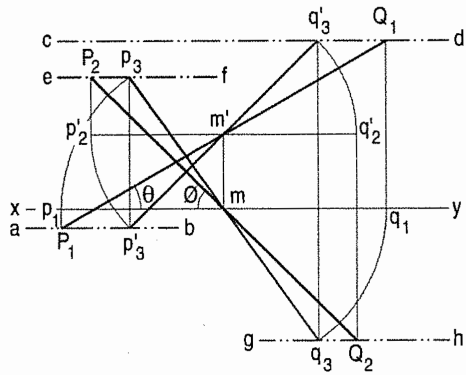


FIG. 10-34

The front view and the top view of P will be below and above xy respectively, while those of Q will be above and below xy respectively.

(i) Mark m, the top view of the mid-point in xy and project its front view m', 20 mm above xy.

(ii) Through m', draw a line making an angle θ (equal to 30°) with xy and with the same point as centre and radius equal to $\frac{1}{2}$ PQ, cut it at P_1 below xy and at Q_1 above xy. Project P_1Q_1 to p_1 q_1 on xy. p_1q_1 is the length of PQ in the top view. ab and cd are the paths of P and Q respectively in the front view.

(iii) Similarly, through m, draw a line making angle ø (equal to 45°) with xy and cut it with the same radius at P_2 above xy and at Q_2 below it.

(iv) Project P_2Q_2 to $p'_2q'_2$ on the horizontal line through m'. $p'_2q'_2$ is the length of PQ in the front view and ef and gh are the paths of P and Q respectively in the top view.

(v) With m as centre and radius equal to mp_1 or mq_1, draw arcs cutting ef at p_3 and gh at q_3. With m' as centre and radius equal to $m'p'_2$ or $m'q'_2$, draw arcs cutting ab at p'_3 and cd at q'_3. p_3q_3 and $p'_3q'_3$ are the required projections.

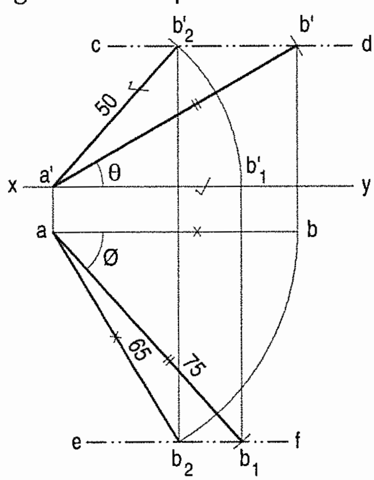


FIG. 10-35

Problem 10-11. (fig. 10-35): *The top view of a 75 mm long line AB measures 65 mm, while the length of its front view is 50 mm. Its one end A is in the H.P. and 12 mm in front of the V.P. Draw the projections of AB and determine its inclinations with the H.P. and the V.P.*

(i) Mark the front view a' and the top view a of the given end A.

(ii) Assuming AB to be parallel to the V.P., draw a line ab equal to 65 mm and parallel to xy. With a' as centre and radius equal to 75 mm, draw an arc cutting the projector through b at b'. The line cd through b' and parallel to xy, is the locus of B in the front view and θ is the inclination of AB with the H.P.

(iii) Similarly, draw a line $a'b'_1$ in xy and equal to 50 mm. With a as centre and radius equal to AB, draw an arc cutting the projector through b'_1 at b_1. ef is the locus of B in the top view and ϕ is the inclination of AB with the V.P.

(iv) With a' as centre and radius equal to $a'b'_1$, draw an arc cutting cd in b'_2. With a as centre and radius equal to ab, draw an arc cutting ef in b_2. $a'b'_2$ and ab_2 are the required projections.

Problem 10-12. (fig. 10-36): *A line AB, 65 mm long, has its end A 20 mm above the H.P. and 25 mm in front of the V.P. The end B is 40 mm above the H.P. and 65 mm in front of the V.P. Draw the projections of AB and show its inclinations with the H.P. and the V.P.*

(i) As per given positions, draw the loci cd and gh of the end A, and ef and jk of the end B in the front view and the top view respectively.

(ii) Mark any point a (the top view of A) in gh and project it to a' on cd. With a' as centre and radius equal to 65 mm, draw an arc cutting ef in b'. Join a' with b'. θ, the inclination of $a'b'$ with xy, is the inclination of AB with the H.P. Project b' to b on gh. ab is the length of AB in the top view.

(iii) With a as centre and radius equal to 65 mm, draw an arc cutting jk in b_1. Join a with b_1. ϕ, the inclination of ab_1 with xy, is the inclination of AB with the V.P. Project b_1 to b'_1 on cd. $a'b'_1$ is the length of AB in the front view.

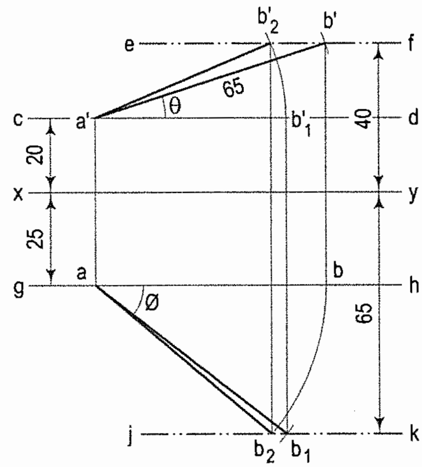


FIG. 10-36

Arrange ab and $a'b'_1$ between their respective paths as shown. $a'b'_2$ and ab_2 are the required projections of AB.

Problem 10-13. (fig. 10-37 and fig. 10-38): *The projectors of the ends of a line AB are 50 mm apart. The end A is 20 mm above the H.P. and 30 mm in front of the V.P. The end B is 10 mm below the H.P. and 40 mm behind the V.P. Determine the true length and traces of AB, and its inclinations with the two planes.*

Draw two projectors 50 mm apart. On one projector, mark the top view a and the front view a' of the end A. On the other, mark the top view b and the front view b' of the end B, as per given distances. ab and $a'b'$ are the projections of AB.

Determine the true length, traces and inclinations by any one of the following two methods:

Method I:

By making the line parallel to a plane (fig. 10-37):

(i) Keeping *a* fixed, turn *ab* to a position ab_1, thus making it parallel to *xy*. Project b_1 to b'_1 on the locus of *b'*. $a'b'_1$ is the true length of *AB* and θ is its true inclination with the H.P.

(ii) Similarly, turn *a'b'* to the position a'_1b' and project a'_1 to a_1 on the path of *a* (because the end *a* has been moved). a_1b is the true length of *AB* and ø is its inclination with the V.P.

Traces:

(i) Through *v* the point of intersection of the top view *ab* with *xy*, draw a projector to cut *a'b'* at the V.T.

(ii) Through *h* the point of intersection of the front view *a'b'* with *xy*, draw a projector to cut *ab* at the H.T. of the line.

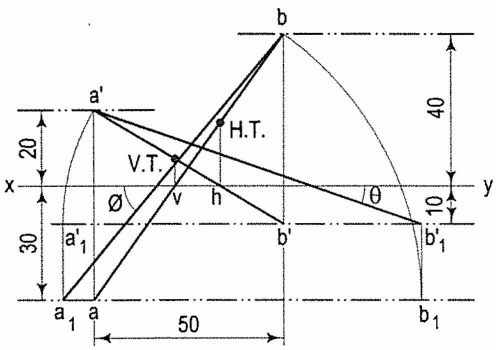


FIG. 10-37

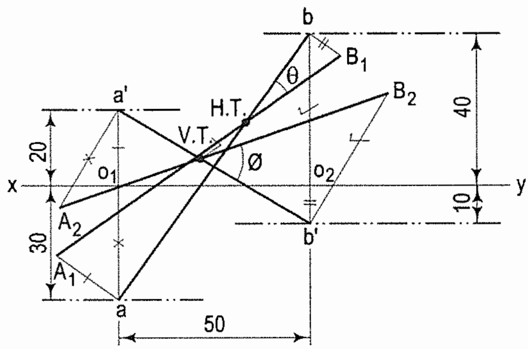


FIG. 10-38

Method II:

By rotating the line about its projections till it lies in H.P. or V.P. (fig. 10-38):

(i) At the ends *a* and *b* of the top view *ab*, draw perpendiculars to *ab*, viz. aA_1 equal to $a'o_1$ and bB_1 equal to $b'o_2$, on opposite sides of it (because *a'* and *b'* are on opposite sides of *xy*). A_1B_1 is the true length of *AB*. θ (its inclination with *ab*) is the inclination of *AB* with the H.P. and the point at which A_1B_1 intersects *ab* is the H.T. of *AB*.

(ii) Similarly, at the ends *a'* and *b'* of the front view *a'b'*, draw perpendiculars to *a'b'*, viz. $a'A_2$ equal to ao_1 and $b'B_2$ equal to bo_2, on opposite sides of it. A_2B_2 is the true length of *AB*. ø (its inclination with *a'b'*) is the inclination of *AB* with the V.P. and the point at which A_2B_2 intersects *a'b'* is the V.T. of *AB*.

Problem 10-14. (fig. 10-39): *A line AB, 90 mm long, is inclined at 45° to the H.P. and its top view makes an angle of 60° with the V.P. The end A is in the H.P. and 12 mm in front of the V.P. Draw its front view and find its true inclination with the V.P.*

(i) Mark *a* and *a'*, the projections of the end *A*.

(ii) Assuming *AB* to be parallel to the V.P. and inclined at 45° to the H.P., draw its front view *a'b'* equal to *AB* and making an angle of 45° with *xy*. Project *b'* to *b* so that *ab* the top view is parallel to *xy*. Keeping the end *a* fixed, turn the top view *ab* to a position ab_1 so that it makes an angle of 60° with *xy*. Project b_1 to b'_1 on the locus of *b'*. Join *a'* with b'_1. $a'b'_1$ is the front view of *AB*.

(iii) To find the true inclination with the V.P., draw an arc with *a* as centre and radius equal to *AB*, cutting the locus of b_1 in b_2. Join *a* with b_2. ø is the true inclination of *AB* with the V.P.

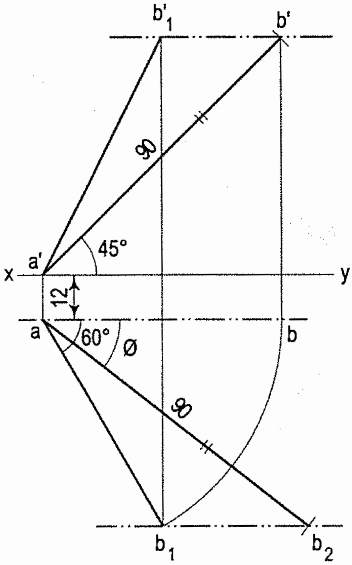


FIG. 10-39

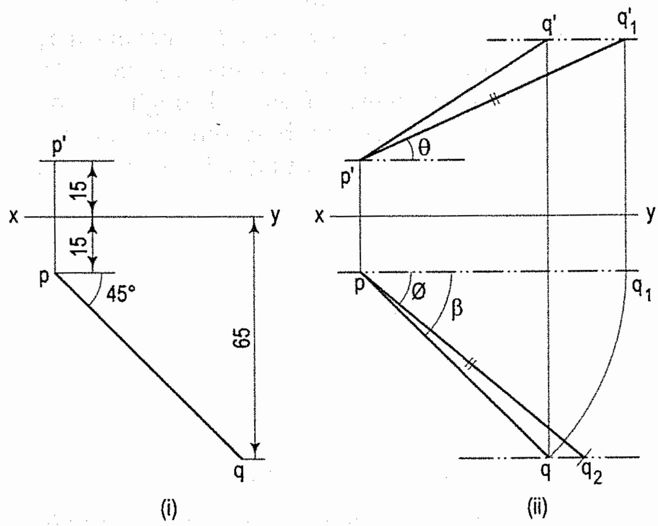


FIG. 10-40

Problem 10-15. (fig. 10-40): *Incomplete projections of a line PQ, inclined at 30° to the H.P. are given in fig. 10-40(i). Complete the projections and determine the true length of PQ and its inclination with the V.P.*

(i) Turn the top view *pq* [fig. 10-40(ii)] to a position pq_1, so that it is parallel to *xy*. Through *p'*, draw a line making an angle θ (equal to 30°) with *xy* and cutting the projector through q_1 at q'_1. $p'q'_1$ is the front view of *PQ*.

(ii) Through q'_1, draw a line parallel to *xy* and cutting the projector through *q* at *q'*. *p'q'* is the front view of *PQ*.

(iii) With *p* as centre and radius equal to $p'\ q'_1$, draw an arc cutting the locus of *q* at q_2.

(iv) Join *p* with q_2. ø is the inclination of *PQ* with the V.P.

Problem 10-16. (fig. 10-41): *The end A of a line AB is 25 mm behind the V.P. and is below the H.P. The end B is 12 mm in front of the V.P. and is above the H.P. The distance between the projectors is 65 mm. The line is inclined at 40° to the H.P. and its H.T. is 20 mm behind the V.P. Draw the projections of the line and determine its true length and the V.T.*

Draw the top view *ab* and mark the H.T. on it, 20 mm above *xy*.

We have seen that the line representing the true length obtained by the trapezoid method, intersects the top view or the top view-produced, at the H.T. at an angle equal to the true inclination of the line with the V.P.

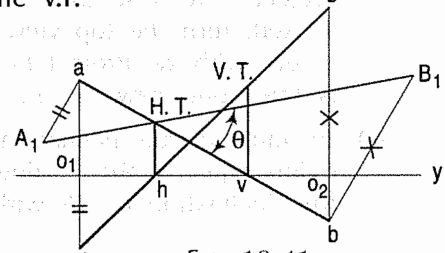


FIG. 10-41

(i) Hence, at the ends a and b, draw perpendiculars to ab on its opposite sides (as one end is below the H.P. and the other end above it). Through the H.T., draw a line making angle θ (equal to 40°) with ab and cutting the perpendiculars at A_1 and B_1, as shown. A_1B_1 is the true length of AB. aA_1 and bB_1 are the distances of the ends A and B respectively from the H.P.

(ii) Project a and b to a' and b', making $a'o_1$ equal to aA_1 and $b'o_2$ equal to bB_1. $a'b'$ is the front view of AB. Through v, the point of intersection between ab and xy, draw a projector cutting $a'b'$ at the V.T. of the line.

Problem 10-17. (fig. 10-42): *A line AB, 90 mm long, is inclined at 30° to the H.P. Its end A is 12 mm above the H.P. and 20 mm in front of the V.P. Its front view measures 65 mm. Draw the top view of AB and determine its inclination with the V.P.*

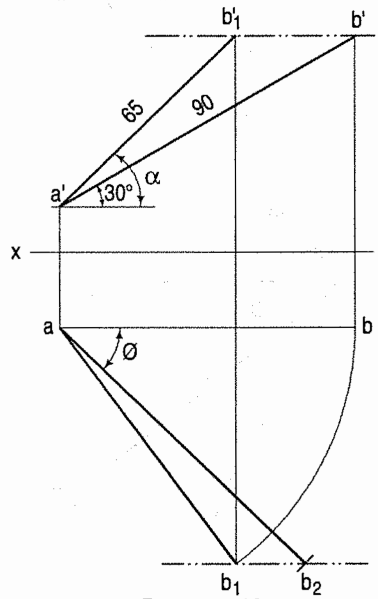


FIG. 10-42

(i) Mark a and a' the projections of the end A. Through a', draw a line ab' 90 mm long and making an angle of 30° with xy.

(ii) With a' as centre and radius equal to 65 mm, draw an arc cutting the path of b' at b'_1. $a'b'_1$ is the front view of AB.

(iii) Project b' to b, so that ab is parallel to xy. ab is the length of AB in the top view.

(iv) With a as centre and radius equal to ab, draw an arc cutting the projector through b'_1 at b_1. Join a with b_1. ab_1 is the required top view.

Determine ø as described in problem 10-14.

Problem 10-18. (fig. 10-43): *The ends of a line PQ are on the same projector. The end P is 30 mm below the H.P. and 12 mm behind the V.P. The end Q is 55 mm above the H.P. and 45 mm in front of the V.P. Determine the true length and traces of PQ and its inclinations with the two planes.*

Note: When the ends of a line are on the same projector or sum of angles of inclinations of a line with xy is 90°, use Method II only.

Draw the projections pq and $p'q'$ as per given positions of the ends P and Q. They will partly coincide with each other.

(i) At the ends p and q of the top view pq, erect perpendiculars, viz. pP_1 equal to $p'o$, and qQ_1 equal to $q'o$ and on opposite sides of pq. P_1Q_1 is the true length of PQ. θ is the inclination of PQ with the H.P. and the point of intersection between P_1Q_1 and pq is the H.T. of PQ.

(ii) Similarly, draw perpendiculars to $p'q'$. viz. $p'P_2$ equal to po and $q'Q_2$ equal to qo and on opposite sides of $p'q'$. P_2Q_2 is the true length. ø is the true inclination of PQ with the V.P. and the point where P_2Q_2 cuts $p'q'$ is the V.T. of PQ.

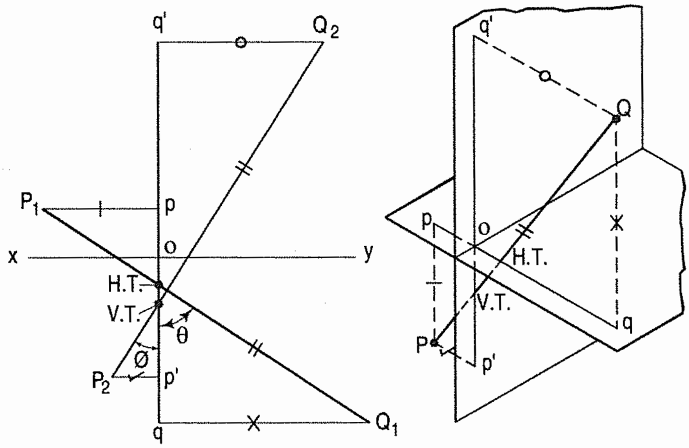


FIG. 10-43

Problem 10-19. (fig. 10-44): *A line AB, inclined at 40° to the V.P., has its ends 50 mm and 20 mm above the H.P. The length of its front view is 65 mm and its V.T. is 10 mm above the H.P. Determine the true length of AB, its inclination with the H.P. and its H.T.*

(i) Draw the front view $a'b'$ as per given positions of A and B and the given length.

(ii) Draw a line parallel to and 10 mm above xy. This line will contain the V.T. Produce $a'b'$ to cut this line at the V.T. Draw a projector through V.T. to v on xy.

(iii) Assuming a' V.T. to be the front view of a line which makes 40° angle with the V.P. and whose one end v is in the V.P., let us determine its true length.

(iv) Keeping V.T. fixed, turn the end a' to a'_1 so that the line becomes parallel to xy. Through v, draw a line making an angle of 40° with xy and cutting the projector through a'_1 at a_1. The line through a_1, drawn parallel to xy, is the locus of A in the top view. Project a' to a on this line. av is the top view of the line, whose front view is a' V.T. and whose true length is equal to a_1v.

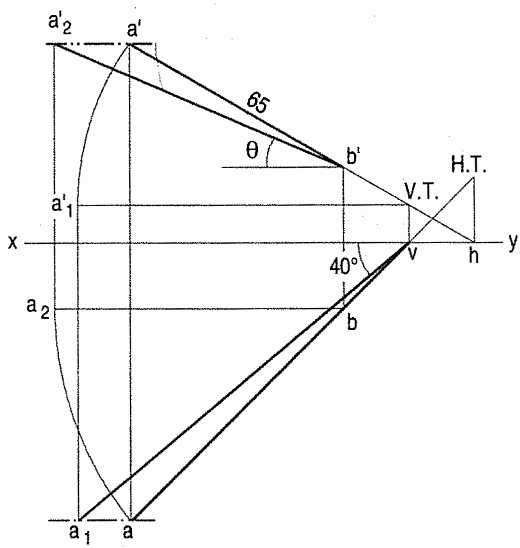


FIG. 10-44

(v) But $a'b'$ is the given front view of AB. Therefore, project b' to b on av. ab is the top view of AB. Obtain the inclination θ with the H.P. by making the top view ab parallel to xy, as shown.

Produce $a'b'$ to meet xy at h. Draw a projector through h to cut ab-produced, at the H.T. of the line.

Problem 10-20. (fig. 10-45): *The front view a'b' and the H.T. of a line AB, inclined at 23° to the H.P. are given in fig. 10-45(i). Determine the true length of AB, its inclination with the V.P. and its V.T.*

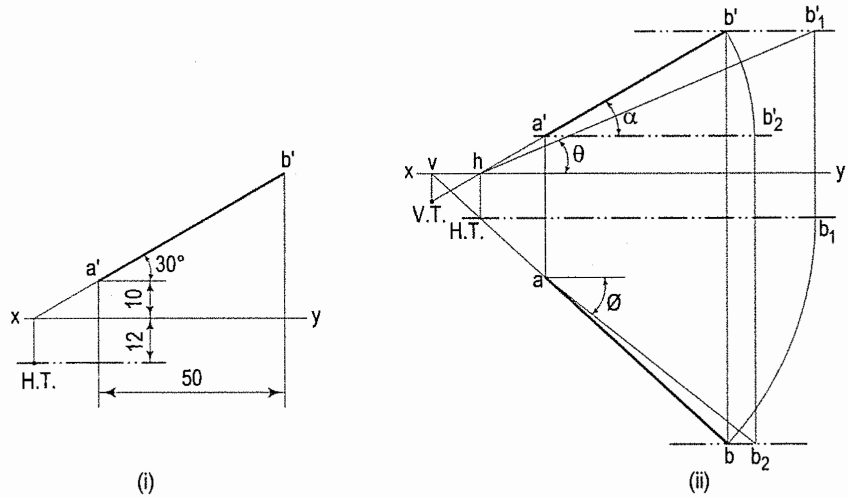


FIG. 10-45

Consider that hb' is the front view of a line inclined at 23° to the H.P. and the top view of whose one end is in H.T.

(i) Through h [fig. 10-45(ii)], draw a line making an angle of $\theta = 23°$ with xy and cutting the locus of B in the front view in b'_1. hb'_1 is the true length of the line whose length in the top view is H.T. b_1.

(ii) With H.T. as centre and radius equal to H.T. b_1, draw an arc cutting the projector through b' at b. H.T. b is the top view and hb' is the front view of a line which contains AB.

(iii) Therefore, through a', draw a projector cutting H.T. b at a. ab is the top view of AB.

Obtain the true length ab_2 (of AB) and its inclination ø with the V.P. by making a' b' parallel to xy.

(iv) Produce ba to meet xy in v. Draw a projector through v to cut $b'a'$-produced, at the V.T. of the line.

Problem 10-21. (fig. 10-46): *A tripod stand rests on the floor. One of its legs is 150 mm long and makes an angle of 70° with the floor. The other two legs are 163 mm and 175 mm long respectively. The upper ends of the legs are attached to the corners of a horizontal equilateral triangular frame of 50 mm side, one side of which is parallel to the V.P. In the top view, the legs appear as lines 120° apart, which if produced, would meet in a point. Draw the projections of the tripod and determine the angle which each of the other two legs makes with the floor. Assume the thickness of the frame and of the legs to be equal to that of the line.*

(i) At any point P on xy, draw a line PA, 150 mm long and making 70° angle with xy. h is the height of the tripod and PA_1 is the length of the leg in the top view.

(ii) Draw an equilateral triangle *abc* of 50 mm side with one side parallel to and below *xy*. Project the front view *a'b'c'* at the height *h* above *xy*. Determine the lengths of the other two legs in the top view as described below.

(iii) With *b'* as centre and radius equal to 163 mm, draw an arc cutting *xy* in q'_1. Similarly with *a'* and *c'* as centre and radius equal to 150 mm and 175 mm, draw an arc cutting *xy* in p'_1 and r'_1 respectively ap_1, bq_1 and cr_1 are the lengths of the three legs in the top view and α and β respectively are their inclinations with the floor (H.P.).

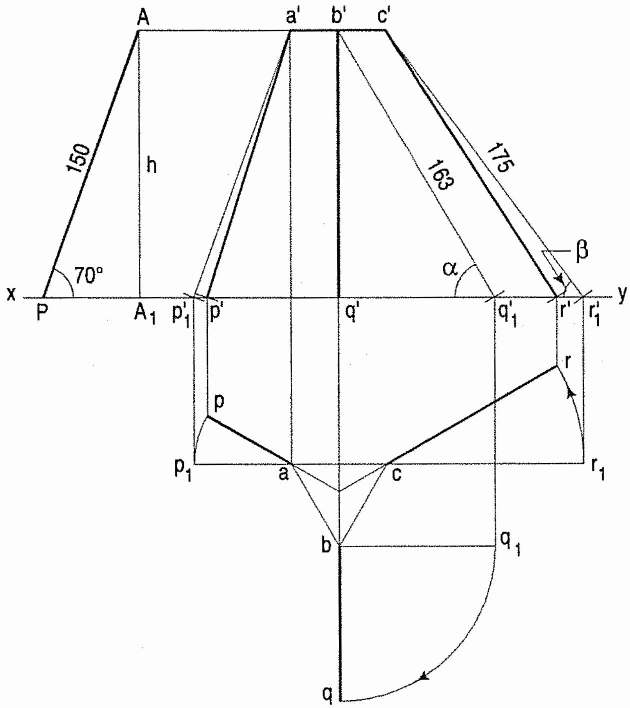


FIG. 10-46

(iv) The legs in the top view are to be inclined at 120° to each other and to meet at a point, if produced. Therefore, draw lines bisecting the angles of the triangle, making *ap* equal to PA_1, *bq* equal to bq_1 and *cr* equal to cr_1, thus completing the top view.

(v) Project *p,q* and *r* to *p',q'* and *r'* respectively on *xy*. Complete the front view by drawing lines *a'p'*, *b'q'* and *c'r'*.

Problem 10-22. (fig. 10-47): *A straight road going uphill from a point A, due east to another point B, is 4 km long and has a slope of 15°. Another straight road from B, due 30° east of north, to a point C is also 4 km long but is on ground level. Determine the length and slope of the straight road joining the points A and C. Scale, 10 mm = 0.4 km.*

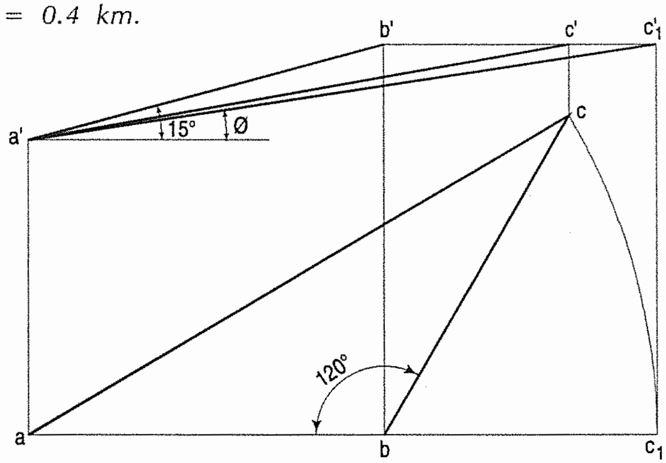


FIG. 10-47

(i) Mark any point a'. Draw a line $a'b'$, 100 mm long, to the right of a' and inclined upwards at 15° to the horizontal (to represent the road from A to B). Project its top view ab keeping it horizontal.

(ii) As the road from B to C is on ground level, the top view bc will be equal to 100 mm and inclined at (90° + 30°) i.e. 120° to ab.

(iii) From b, draw a line bc, 100 mm long and making 120° angle with ab. Project c to c' making $b'c'$ horizontal. $a'c'$ and ac are the front view and the top view respectively of the road from A to C,

Determine the true length $a'c'_1$ and the angle ø as shown, which are respectively the length and slope of the road from A to C.

Problem 10-23. (fig. 10-48): *Two lines AB and AC make an angle of 120° between them in their front view and top view. AB is parallel to both the H.P. and the V.P. Determine the real angle between AB and AC.*

Draw any line $b'a'$ parallel to and above xy, and another line $a'c'$ of any length making 120° angle with $b'a'$. Join b' with c'.

(i) Project the top view ba parallel to xy and the top view ac, making 120° angle with ba. Join b with c. $b'a'$ or ba is the true length of AB. Determine the true lengths of AC and BC, viz. $a'c'_1$ and $b'c'_2$, as shown.

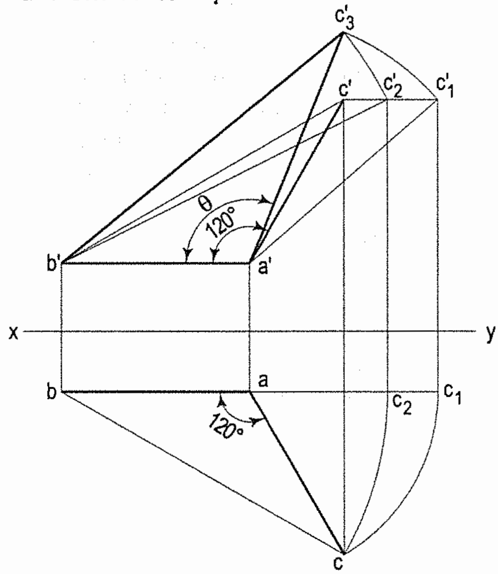


FIG. 10-48

(ii) Draw a triangle $a'b'c'_3$ making $a'c'_3$ equal to $a'c'_1$ and $b'c'_3$ equal to $b'c'_2$. $\angle b'a'c'_3$ is the real angle between AB and AC.

Problem 10-24. (fig. 10-49): *An object O is placed 1.2 m above the ground and in the centre of a room 4.2 m × 3.6 m × 3.6 m high. Determine graphically its distance from one of the corners between the roof and two adjacent walls. Scale, 10 mm = 0.5 m.*

(i) Draw the front view (of the room) $a'b'c'd'$ as seen from the front of, say 3.6 m wall. $a'b'$ is the width of the room and $a'd'$ is the height. The front view o' of the object will be seen 1.2 m above the mid-point of $a'b'$. c' and d' are the top corners of the room. $o'c'$ is the front view of the line joining the object with a top corner.

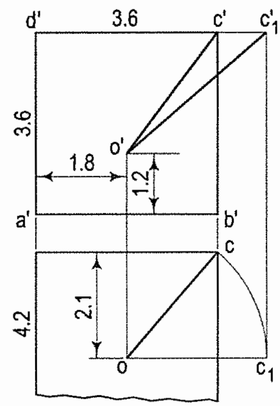


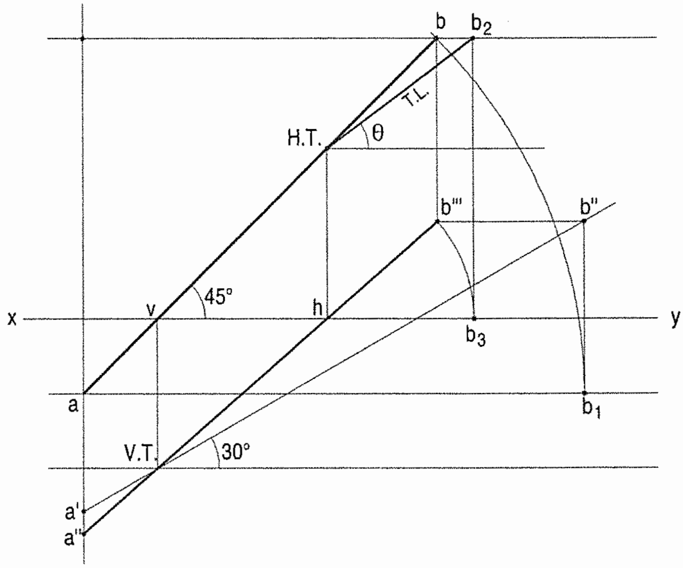
FIG. 10-49

(ii) Draw the top view of the room. It will be a rectangle of sides equal to 3.6 m and 4.2 m. The top view o of the object will be in the centre of the rectangle. oc is the top view of the line joining the object with the top corner.

Determine the true length $o'c'_1$, which will show the distance of the object from one of the top corners of the room.

Problem 10-25. *The straight line AB is inclined at 30° to H.P., while its top view at 45° to a line xy. The end A is 20 mm in front of the V.P. and it is below the H.P. The end B is 75 mm behind the V.P. and it is above the H.P. Draw the projections of the line when its V.T. is 40 mm below. Find the true length of the portion of the straight line which is in the second quadrant and locate its H.T.*

Refer to fig. 10-50.



T.L. = 50 mm. ∠V.P., θ = 37°

FIG. 10-50

(i) Mark the points *a* (top view of *A*) and *b* (top view of *B*) at the distances of 20 mm and 75 mm below and above *xy* respectively.

(ii) Through the point *a*, draw a line at 45° intersecting *xy* and the path of *b* at *v* and *b* respectively as shown.

(iii) Construct a line containing V.T. 40 mm below *xy*. Draw perpendicular from *v* to the line V.T.

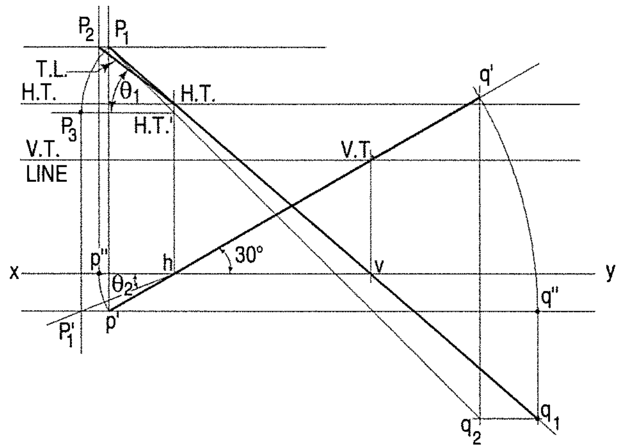
(iv) With *a* as centre and radius equal to *ab*, draw an arc which intersects at b_1 a line drawn from *a* parallel to *xy*. From V.T. draw a line at 30° intersecting the projector of b_1 at *b''*. From *b''*, draw *b''b'''* parallel to *xy* to intersect projector of *b* at *b'''*. Join V.T. *b'''*. Produce it to meet the projector from *a* at *a''*. *a'' b'''* is the required projection. *a''b'''* intersects line *xy* at *h*. From *h* draw the perpendicular to meet *ab*. The intersection point represents H.T.

(v) With *h* as centre and radius equal to *hb'''*, draw an arc intersecting at b_3. Draw projector from b_3 to cut the path of *b* at b_2. Join H.T. b_2. Measure the angle H.T. b_2 with *xy*. This is an angle made by the line with the V.P.

Problem 10-26. (fig. 10-51.): *The front view of a line PQ makes an angle of 30° with xy. The H.T. of the line is 45 mm behind the V.P. While its V.T. is 30 mm above the H.P. The end P of the line is 10 mm below the H.P. and the end Q is in the first quadrant. The line is 150 mm long. Draw the projections of the line and determine the true-length of the portion of the line which is in the second quadrant. Also find the angle of the line with the H.P. and V.P.*

(i) Draw lines containing H.T. and V.T. at 45 mm and 30 mm above *xy* respectively. Mark point *p'* at 10 mm below line *xy*. Draw *p'q'* at 30° from *p'* intersecting *xy* at *h* and line V.T. at V.T. From *h*, draw perpendicular to line H.T. to locate H.T.

(ii) Draw perpendicular from V.T. to intersect *xy* at *v*. Join *v* H.T. and produce to intersect the projector of *p'* at p_1. Draw $p_1 q_1$ of 150 mm representing true length of the line in top view. From *p'*, draw line parallel to *xy* representing front view of the line *PQ*. Draw projector from q_1 to cut the line drawn from *p'* at *q''*.



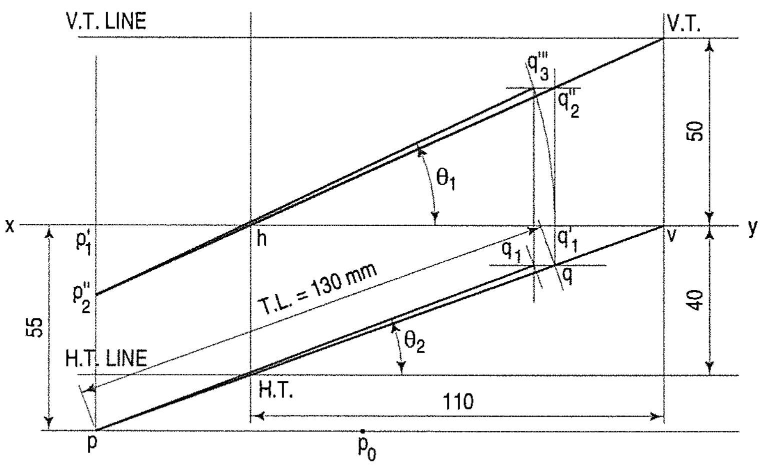
T.L. = 25 mm. ∠V.P., θ_1 = 37° ∠V.P., θ_2 = 22°

FIG. 10-51

(iii) Keeping *p'* fixed, turn *p'q''* such that it cuts the line drawn from *p'* at *q'*. From q_1 draw line parallel to *xy* which intersects the vertical projector drawn from *q'* at q_2. Join $P_1 q_2$. This is the required projection.

(iv) Keeping *h* fixed, rotate *hp'* and make it parallel to *xy*. From *P''* draw projector intersecting the path of p_1 at p_2. H.T. p_2 is true-length of the line. Similarly keeping H.T.' fixed, turn H.T.' p_1 to make it parallel to *xy* as shown. From P_3, draw projector to intersect the horizontal line drawn from $P'P'_1$. Measure angle xhp'_1. This is the required angle with H.P.

Problem 10-27. (fig. 10-52): *The end P of a line PQ 130 mm long, is 55 mm in front of the V.P. The H.T. of the line is 40 mm in front of the V.P. and the V.T. is 50 mm above the H.P. The distance between H.T. and V.T. is 110 mm. Draw the projections of the line PQ and determine its angles with the H.P. and the V.P.*



∠H.P., θ_1 = 25.5°; ∠V.P., θ_2 = 21°

FIG. 10-52

(i) Mark p below xy at a distance of 55 mm. Draw lines containing H.T. and V.T. at 40 mm below and 50 mm above xy respectively. Construct projectors through H.T. and V.T. 110 mm apart intersecting xy at h and v respectively.

(ii) Draw perpendiculars from h and v intersecting the lines containing H.T. and V.T. Join H.T. v and produce to cut the line drawn from point P as shown. Join h V.T. and extend further to intersect the projector drawn from P at P''_2.

(iii) Mark true length 130 mm on H.T.v. Let it be pq. Draw projector from q cutting xy at q'_1. With p'_1 as centre and radius equal to $p'_1q'_1$, draw an arc cutting a horizontal line drawn from q''_2 at q'''_3. Join pq_1 and $p''_2q'''_3$, are the required projections. $\theta_1 = 25.5°$ and $\theta_2 = 21°$ are the measured angles.

Problem 10-28. (fig. 10-53): *The distance between the end projectors of a line AB is 70 mm and the projectors through the traces are 110 mm apart. The end of a line is 10 mm above H.P. If the top view and the front view of the line make 30° and 60° with xy line respectively, draw the projections of the line and determine* (i) *the traces,* (ii) *the angles with the H.P. and the V.P.,* (iii) *the true length of the line. Assume that the line is in the first quadrant.*

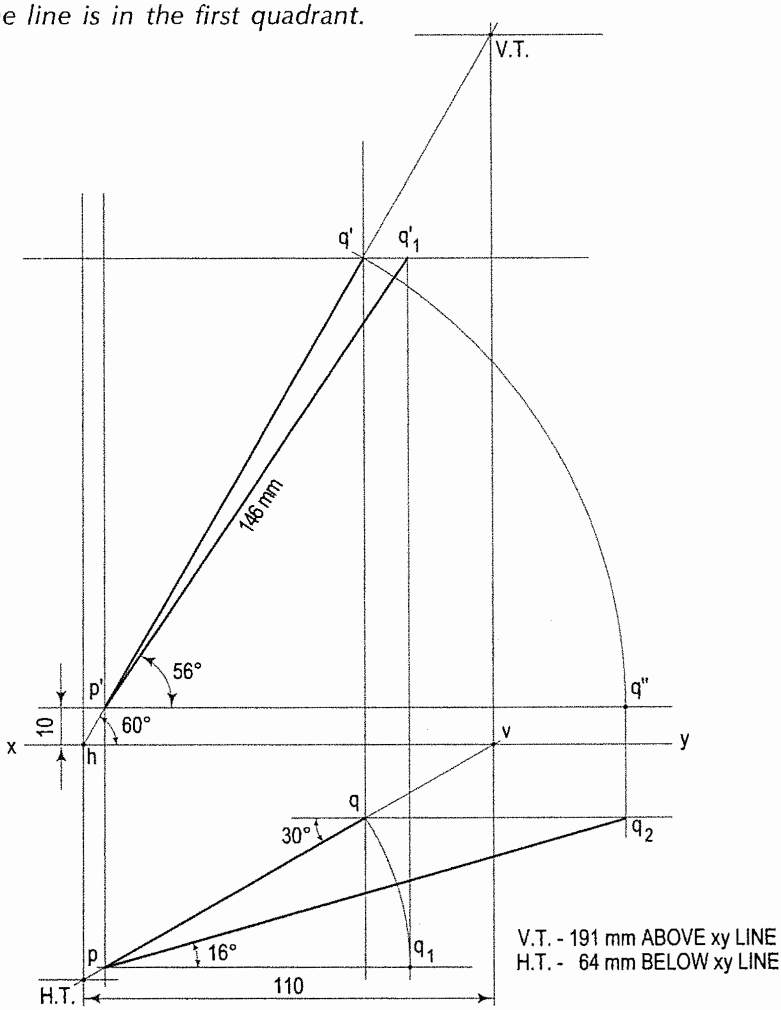


FIG. 10-53

(i) Draw two vertical lines 110 mm apart representing projectors through the traces of the line. Mark intersection of these projectors *h* and *v* on *xy* as shown.

(ii) From *v* and *h*, draw lines at 30° and 60°.

(iii) Mark *p*', 10 mm above *xy*. Draw two vertical projectors at 70 mm apart keeping equal distance from the projectors through traces. *p*'*q*' and *pq* represent front view and top view of the line as shown.

(iv) Keeping *p* fixed, turn *pq* to position pq_1. From q_1, draw a vertical projector intersecting the path of *q*' at q'_1. Join $p'q'_1$. This is the true length of the line. Measure angle $q''p'q'_1$ with the horizontal line as shown. This is the angle made by the line with the H.P. Similarly rotate *p*'*q*' making it parallel to *xy* as shown.

Draw a vertical projector from *q*" to intersect the path of *q* at q_2. Measure the angle q_1pq_2 with the horizontal line. This is the angle made by the line with V.P.

Note: Problem 10-29 and problem 10-31 are solved by using auxiliary plane method.

Problem 10-29. *Two pipes PQ and RS seem to intersect at a' and a in front view and top view as shown in fig. 10-54. The point A is 400 mm above H.P. and 300 mm in front of a wall.*

Neglecting the thickness of the pipes, determine the clearance between the pipes.

Refer to fig. 10-54.

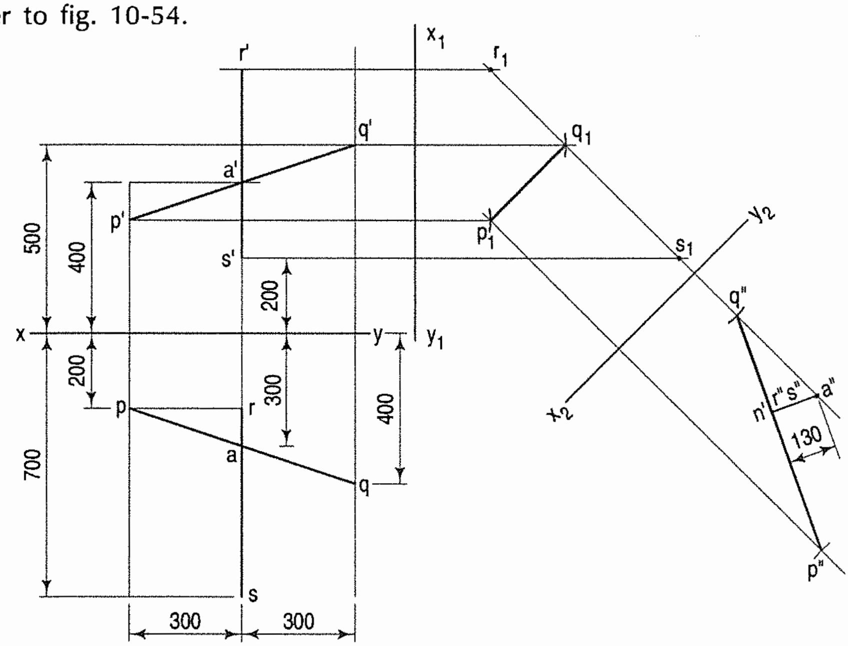


FIG. 10-54

(i) Draw the projections of pipes treating them as lines.

(ii) Draw x_1y_1 perpendicular to the line *xy* and project auxiliary top view. Note that the distances of p_1, q_1, r_1 and s_1 from x_1y_1 are equal to the distances of *p*, *q*, *r* and *s* from *xy*.

(iii) Mark an auxiliary reference line x_2y_2 perpendicular to the line r_1s_1 for obtaining the point view of the line. Draw an auxiliary front view as shown. Note that the distances of p'', q'', r'' and s'' from x_2y_2 are equal to the distances of p', q', r' and s' from x_1y_1.

(iv) a'' n' represents clearance between two pipes which is approximately 130 mm.

Problem 10-30. *The end projectors of line AB are 22 mm apart. A is 12 mm in front of the V.P. and 12 mm above the H.P. The point B 6 mm in front of the V.P. and 40 mm above the H.P. Locate the H.T. and the V.T. of the line and also determine its inclinations with the V.P. and the H.P.*

If the line *AB* is shifted to II, III and IV quadrants as shown in fig. 10-55 (assume that the distances of *A* and *B* from the projection-planes are same as the first quadrant), draw the projections of line and locate the traces.

The solution of first part of problem is shown in pictorial view. For the second part of the problem, the locations of the line in the respective quadrants are shown in fig. 10-55.

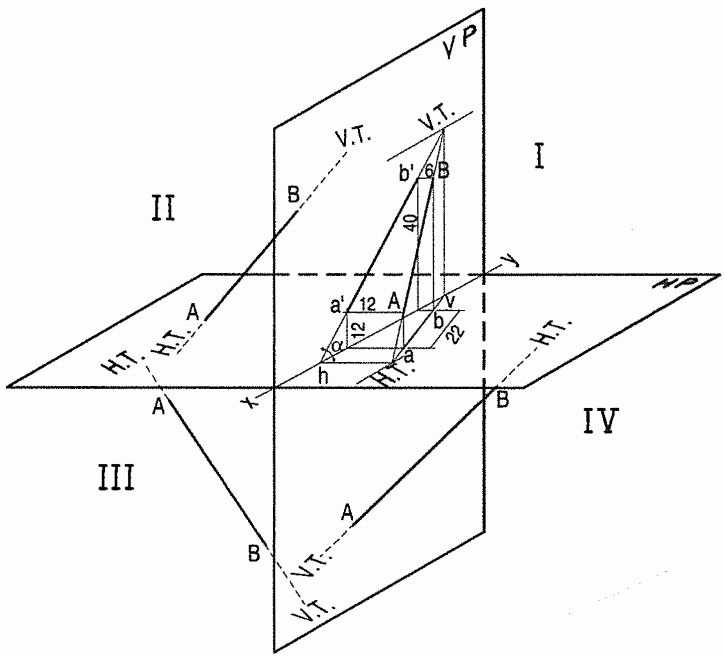


FIG. 10-55

Students are advised to draw the orthographic projections of the line for the respective quadrants.

Problem 10-31. *The end projectors of line PQ are 80 mm apart. The end P is 15 mm in front of the V.P. and 60 mm above the H.P. While Q is 50 mm in front of the V.P. and 10 mm above the H.P. A point R is situated on the projector at a distance of 31 mm from the projector through P measured towards the projector of Q. The point R is 70 mm in front of the V.P. and above the H.P. A perpendicular is drawn from R on PQ. Draw its projections.*

Refer to fig. 10-56.

(i) Draw two end projectors of the line *PQ* at 80 mm apart.

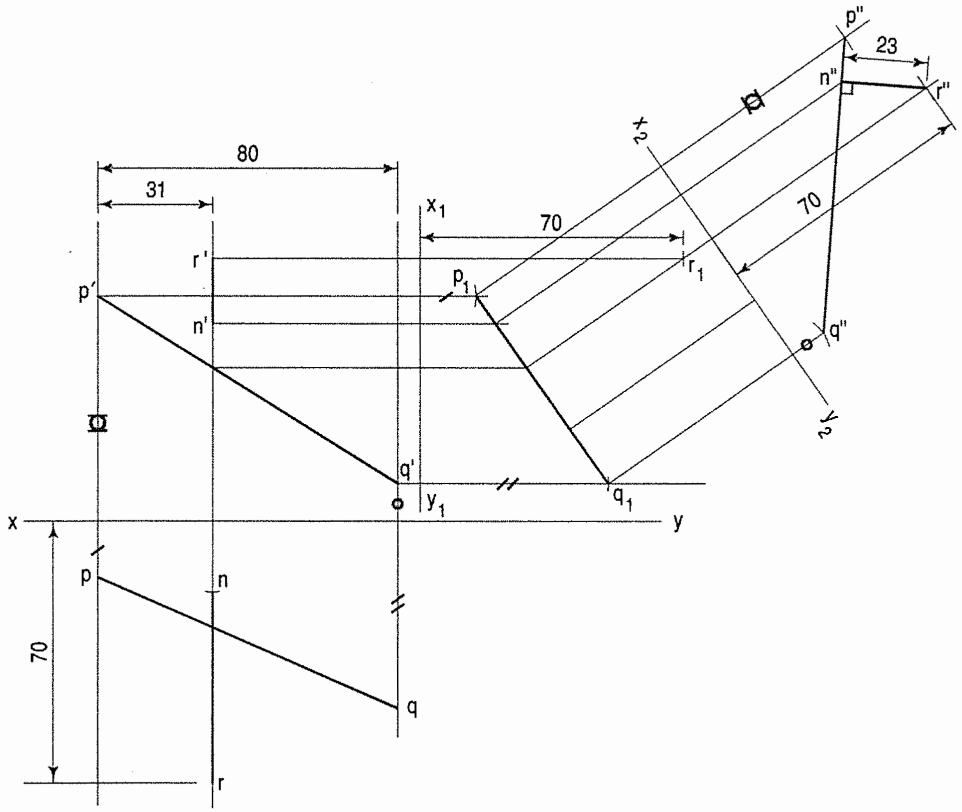
(ii) Mark the front view *p'* and the top view *p* at 60 mm and 15 mm from *xy* on the end projector of *P*. Similarly mark *q'* and *q* at 10 mm and 50 mm from *xy* on the end projector of *Q*.

(iii) Draw a vertical line at 31 mm away from *p'p* towards *q'q*. Mark the position of *R* in the top view *r* and the front view *r'* at 70 mm from *xy* as shown.

(iv) Draw x_1y_1 perpendicular to *xy* as shown. Draw projectors from *p'*, *q'* and *r'* on x_1y_1. Transfer the distances 15 mm, 50 mm and 70 mm of *p*, *q* and *r* from *xy* to the new top view p_1, q_1 and r_1 from x_1y_1.

(v) Draw another reference line x_2y_2 for the new front view parallel to p_1q_1. Transfer the distances 60 mm, 10 mm and 70 mm of *p'*, *q'* and *r'* from *xy* to the new front view *p"*, *q"* and *r"* from x_2y_2.

(vi) From *r"*, draw perpendicular to *p"q"* intersecting at *n"* as shown which is measured as 23 mm.



n"r" = 23 mm

FIG. 10-56

Problem 10-32. *The end projectors of a line PQ are 65 mm apart. P is 25 mm behind the V.P. and 30 mm below the H.P. The point Q is 40 mm above the H.P. and 15 mm in front of the V.P. Find the third point C in the H.P. and in front of the V.P. such that its distance from a point P is 45 mm and that from Q is 60 mm. Determine inclinations of PQ with the H.P. and the V.P.*

Refer to fig. 10-57.

(i) Draw the end projectors of the line *PQ* 65 mm apart. Mark the projection of ends *P* and *Q* according to given distances. *p'q'* and *pq* are the front view and the top view respectively.

(ii) Mark the front view of point *c* in the line *xy* because it is lying in the H.P. Extend the projection line from *c'* to *c* on the top view *pq*.

(iii) Keeping *c* fixed, turn *cp* to cp_2 making parallel to *xy*.

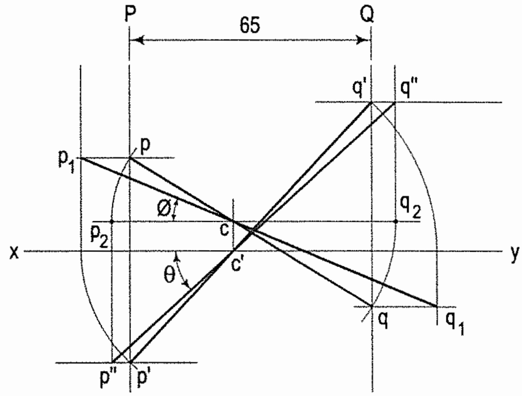


FIG. 10-57

(iv) Project p_2 to *p"*. Join *c'p"*. This is the true distance of the line *cp*. Similarly turn *cq* to cq_2 making it parallel to *xy*. Project q_2 to *q"* join *c'q"*. This represents the true distance of the line *cq*.

(v) Measure angles θ and ø as shown.

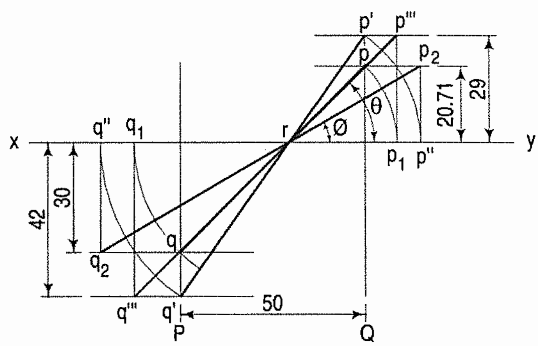
Problem 10-33. (fig. 10-58): *The distance between the end-projectors of a line PQ is 50 mm. A point P is 29 mm above H.P. and 20.71 mm behind V.P. While a point Q is 42 mm below H.P. and 30 mm in front of V.P.*

Draw the projections of the line and determine the true length and the true inclinations of line with H.P. and V.P.

(i) Draw the end-projectors 50 mm apart. Mark *p'* and *p*, the front view and the top view of the end *P* at 29 mm and 20.71 mm respectively. Similarly mark *q* and *q'* at 42 mm and 30 mm on the end-projector *Q* as shown. Join *p'q'* and *pq*. They are intersecting *xy* at *r*. Mark paths of *p'*, *q'*, *p* and *q* parallel to *xy*.

(ii) With centre *r* and radius *rp'*, draw an arc intersecting *xy* at *p"*. Through *p"*, draw projector cutting the path of *p* at p_2. Similarly with the same centre and radius *rq'*, draw an arc intersecting *xy* at *q"*. Through *q"*, draw projector cutting the path of *q* at q_2. Join p_2q_2 which represents true length. Measure ø angle made by p_2q_2 with *xy* at *r*. This is an angle made by the line with V.P.

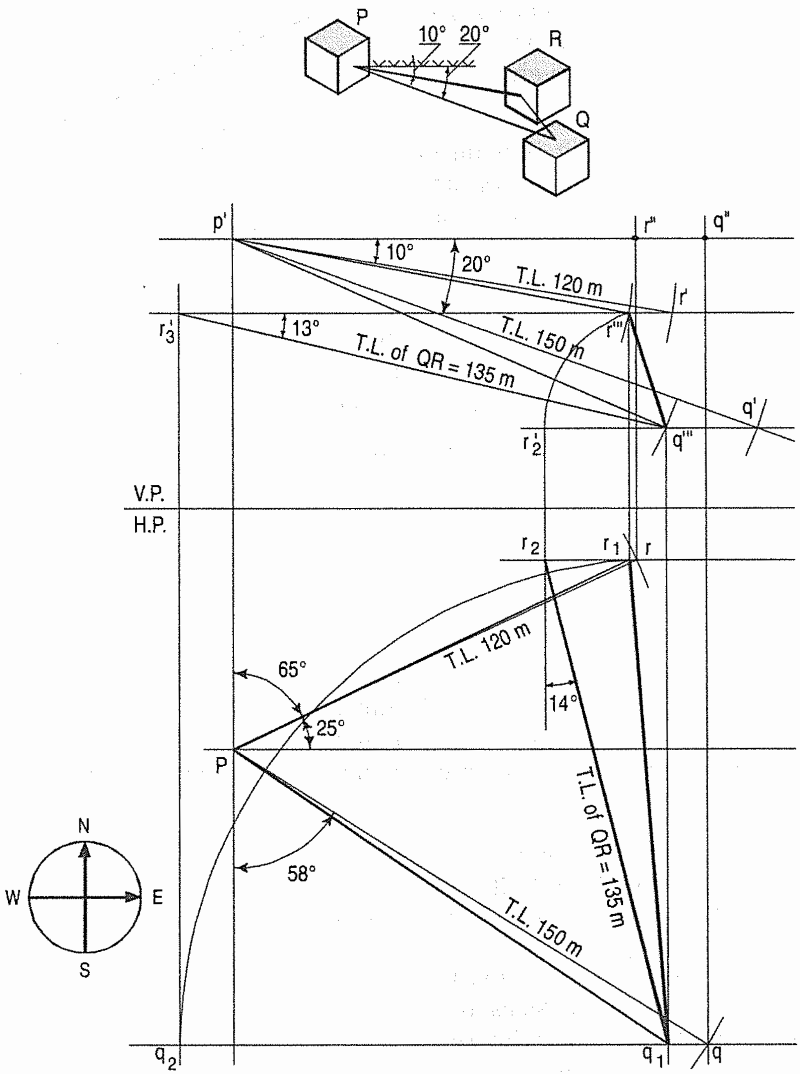
(iii) Similarly centre as *r* and radii *rp* and *rq*, draw arcs intersecting *xy* at p_1 and q_1 respectively. Through p_1 and q_1 draw projectors to intersect the paths of *p'* and *q'* at *p'''* and *q'''*. Join *p'''q'''*. Measure θ angle made by it with *xy*. This is an angle by line with H.P.



∠θ = 45°; ∠ø = 30°, true length = 100 mm

FIG. 10-58

Problem 10-34. (fig. 10-59): *Two pipes emerge from a common tank. The pipe PQ is 150 metre long and bears S 58° E on a downward slope of 20°. The pipe PR is 120 metre long and bears N 65° E on a downward slope of 10°. Determine the length of pipe required to connect Q and R. Take scale 1 mm = 1 m.*



T.L. of QR = 135 m. S 13 E at downward slope of 13°

FIG. 10-59

(i) Mark position of *P* in the top view and the front view as shown.

(ii) At *P*, draw a line *pq* 150 mm at 58° with the vertical measuring anti-clockwise as the angle is required to measure from south to east. Similarly draw a line *pr* 120 mm at 25° measured from east to north.

(iii) Draw the horizontal lines from *q* and *r*.

(iv) From point *p'*, draw a line *p'q'* 150 mm and *p'r'* 120 mm at 20° and 10° with the horizontal lines. Draw horizontal lines from *r'* and *q'*.

(v) Draw a projector from q to intersect the horizontal line drawn from p' at q''. $p'q''$ is the front view of PQ. With p' as centre and radius equal to $p'q''$, draw an arc intersecting the path of q' at q'''. Join $p'q'''$ and draw a projector from q''' cutting the path of q at q_1. Then $p'q'''$ and pq_1 are required projection. Similarly obtain the projection of $p'r'''$ and pr_1 for the line PR as shown. Join $r'''q'''$ and r_1q_1. With centre q''' and a radius $q'''r'''$, draw an arc intersecting the path of q''' at r'_2. Draw a projector from r'_2 cutting the path of r at r_2. Join q_1r_2 and measure its true length and angle.

(vi) With centre q_1 and radius q_1r_1, draw an arc intersecting the path of q at q_2. Draw a projector from q_2 cutting the path of r' at r'_3. Join $q'''r'_3$ and measure its true length and angle.

(vii) QR = 135 m is the measured true length and S 13 E at downward slope of 13° is the measured angle.

Note: Depression or front view angles are seen in front view while bearing angles are seen in top view.

Problem 10-35. (fig. 10-60): *The projectors of two points P and Q are 70 mm apart. The point P is 25 mm behind the V.P. and 30 mm below the H.P. The point Q is 40 mm above the H.P. and 15 mm infront of the V.P. Find the third point S which is in the H.P. and infront of the V.P. such that its distance from point P is 90 mm and that from Q is 60 mm.*

(i) Draw xy line.

(ii) Draw the projectors of P and Q 70 mm apart.

(iii) Mark on the projector of the point p the front view and top view of the point p at 25 mm and 30 mm from xy respectively, say p' and p. Similarly on the projector of the point Q, mark the front view q' and the top view q for given distance from the xy.

(iv) Join $p'q'$ and pq. They are the front view and the top view of the line PQ.

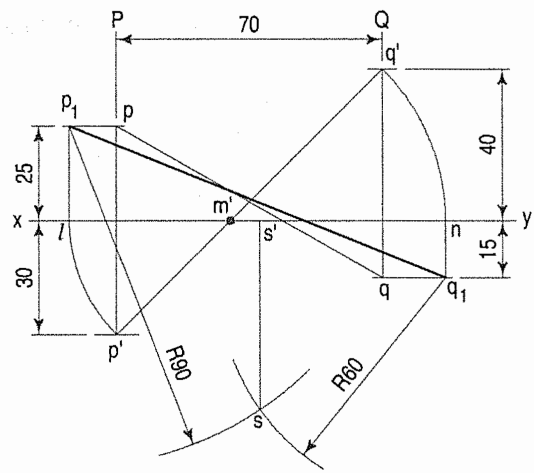


FIG. 10-60

(v) The front view $p'q'$ intersects xy line at m'. Taking m' as centre, $m'p'$ and $m'q'$ as radii rotate so that $p'q'$ becomes parallel to the xy line or intersect xy line at l and n.

(vi) Draw the projectors from l and n to intersect path of the point p and the point q at p_1 and q_1. Join p_1 and q_1, it represents true length of the line PQ.

(vii) Now p_1 as centre and 90 mm radius, draw the arc. Take q_1 as centre and 60 mm as radius, draw the another arc so that it intersects previous arc at s. From s draw the projector to intersect xy line at s', which is front view of s.

Problem 10-36. (fig. 10-61): *Two unequal lines PQ and PR meeting at P makes an angle of 130° between them in their front view and top view. Line PQ is parallel and 6 mm away from both the principal planes. Assume the front view length of PQ and PR 50 mm and 60 mm respectively. Determine real angle between them.*

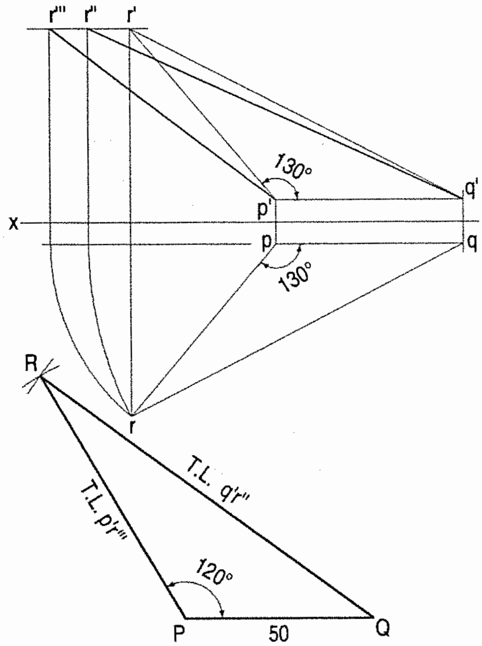


FIG. 10-61

(i) Mark reference line *xy*. Draw at convinent distance above and below two parallel lines to the *xy* representing the front view and top view of *PQ* as shown. At the point *P'* construct angle of 130° with the length of *p'q'* and *p'r'* of 50 mm and 60 mm respectively.

(ii) Draw projector from *r'* to intersect the line at angle of 130° at *p* in the top view of *pq*.

(iii) Make *pr* and *qr* parallel to *xy* line and project above *xy* to intersect the path of *r'* at *r''* and *r'''* respectively.

Join *p'r'''* and *q'r''*. They are true length of the line *PR* and the line *QR*.

(iv) Construct triangle with sides *PQ* = 50 mm, *PR* = *p'r'''* and *QR* = *q'r''* as shown. Measure angle ∠*QPR* equal and it is 120° approximately.

Problem 10-37. (fig. 10-62): *The distance between end projectors of a straight line AB is 80 mm. The point A is 15 mm below the H.P. and 20 mm infront of the V.P. B is 60 mm behind the V.P. Draw projections of the line if it is inclined at 45° to V.P. Determine also true length and inclination with the H.P.*

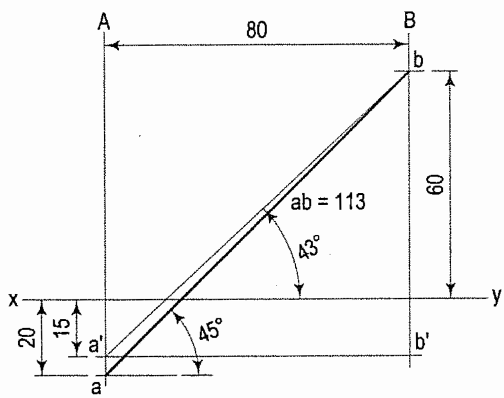


FIG. 10-62

(i) Draw *xy* line and two vertical parallel lines 80 mm apart showing the end projectors of the line *AB*.

(ii) Mark the point a' and the point a 15 mm and 20 mm below *xy* line on the projector of *A*.

(iii) Mark the point *b* above *xy* line at distance of 60 mm on the projector of *B*. Join the point *a* and the point *b*.

(iv) If we measure angle of *ab* with *xy* line it is 45° which is also inclination of the line *AB* with V.P.

(v) Therefore *ab* shows true length. From *a'* draw parallel line to *xy* to intersect the projector of *B* at *b'*. It is front view of the line *AB*. Measure angle of a'*b* with *xy* line. It is 43° with *xy* line.

Problem 10-38. (fig.10-63): *Two mangoes on a tree are respectively 2.0 m and 3.5 m above the ground and 1.5 m and 2.0 m away from 0.2 m thick compound wall, but on the opposite sides of it. The distance between the mangoes, measured along the ground and parallel to the wall is 2.7 m. Determine the real distance between the mangoes. Take scale 1 m = 10 mm.*

(i) Pictorial view is shown for understanding purpose.

(ii) Draw reference line (ground line) *xy*. Mark two parallel lines as end-projectors at 2.7 m (27 mm) apart.

(iii) Let *P* be mango behind the wall and *Q* be infront of the wall.

(iv) Mark *P*' and *P* along projector *P* to given distances. Similarly mark *q*' and *q* for given distances on the projector *Q*.

(v) Join the point *p* and the point *q*, the point *p*' and the point *q*'. Then *pq* and *p*'*q*' are projections of *PQ*.

(vi) Rotates *pq* taking *q* as centre, make it parallel to the ground line *xy*, intersecting at the poin P_1. Draw the projector from p_1. Draw line parallel to the ground line *xy* from *p*', intersecting at *p*".

(vii) Join *p*"*q*'. The line *p*"*q*' is true distance between mangoes *P* and *Q*. It is approximately 4.8 m.

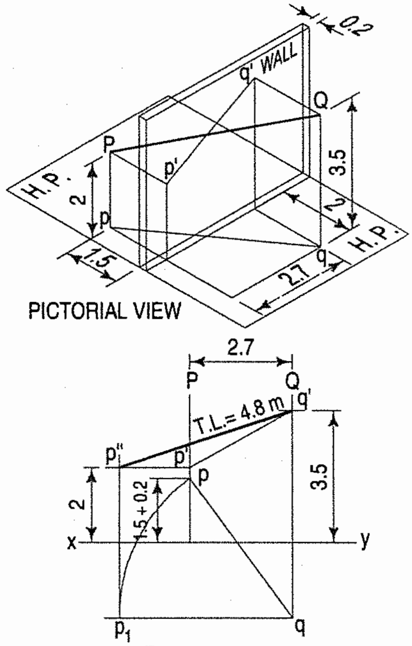


FIG. 10-63

Problem 10-39. (fig. 10-64): *The front view of straight line AB is 60 mm long and is inclined at 60° to the reference line xy. The end point A is 15 mm above H.P. and 20 mm infront of V.P. Draw the projections of a line AB if it is inclined at 45° to the V.P. and is situated in the first quadrant (Dihedral angle). Determine its true length, and inclination with the H.P.*

See fig. 10-64 which is self explanatory.

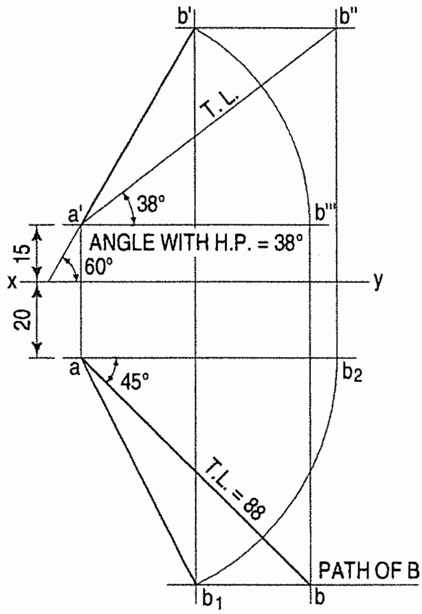


FIG. 10-64

Problem 10-40. (fig. 10-65): *A room 6 m × 5 m × 4 m high has a light-bracket above the centre of the longer wall and 1 m below the ceiling. The light bulb is 0.3 m away from the wall. The switch for the light is on an adjacent wall, 1.5 m above the floor and 1 m from the other longer wall. Determine graphically the shortest distance between the bulb and the switch.*

(i) Draw to scale 1 m = 10 mm front view and a top view of the room.

(ii) Mark mid point of longer wall (i.e. 6 m), say 6 mm.

(iii) Mark the point in the V.P. at distance 1 m (= 10 mm) from midpoint *m*. It is the front view of the light bulb, say *b*'.

(iv) From *xy* line at 0.3 m (i.e. 3 mm) on the projector passing through *b*' mark the point *b* (top view).

(v) Consider adjacent wall right side. Mark *s*' front view of the switch at 1.5 m (i.e. 15 mm) above *xy* line and on the same line mark *s* (top view of switch) at 4 m away from *xy*.

(vi) Join *b*'*s*' and *bs* which are the front view and the top view of the line.

(vii) *s* as centre and *bs* as radius, draw the arc to intersect a parallel line passing through *s* at b_1. From b_1, draw projector to intersect a parallel line to *xy* drawn from *b*' at *b*". Join *s*'*b*".

(viii) Line *s*'*b*" shows shortest distance between the switch and the bulb. Here it is approximately 5 m.

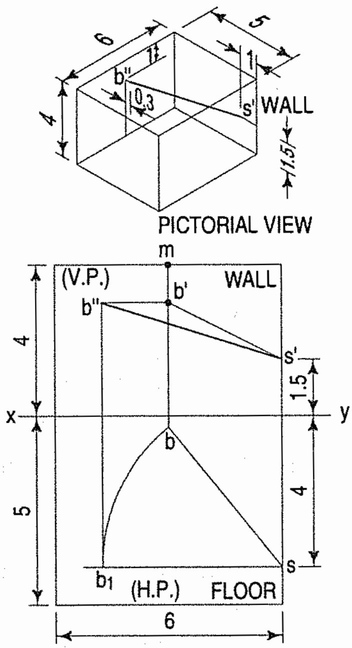
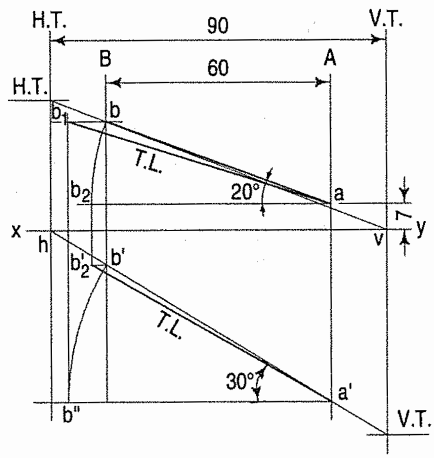


FIG. 10-65

Problem 10-41. (fig. 10-66): *The distance between end projectors of a line AB are 60 mm apart, while the projectors passing from H.T. and V.T. are 90 mm apart. The H.T. is 35 mm behind the V.P., the V.T. is 55 mm below the H.P. The point A is 7 mm behind the V.P. Find graphically true length of the line and inclinations with the H.P. and the V.P.*

See fig. 10-66 which is self-explanatory.



True length = 74;
Angle with H.P. = 30°;
Angle with V.P. = 20°
FIG. 10-66

Problem 10-42. (fig. 10-67): *A line CD is inclined at 30° to the H.P. and it is in the first quadrant. The end C is 15 mm above the H.P. while the end D is in the V.P. The mid point M of of the line is 40 mm above H.P. The distance between the end projectors of the line is 70 mm. Draw the projections of the line CD and the mid point M. Determine graphically the length of front view and top view and true length of the line. Also determine inclination of the line with the V.P.*

(i) Draw *xy* line.

(ii) Draw two projectors 70 mm, apart.

(iii) On the projector of *C*, mark *c*' at 15 mm above *xy* line.

(iv) Draw a line parallel to *xy* at 40 mm, to represent the path of mid-point *M*.

(v) From *c*' draw a 30° inclined line t cut the path of mid-point at *m*'. *c*'*m*' is half true length. With *m*' as centre and radius equal to *c*'*m*', draw an arc cutting the 30° inclined line at *d*'. *c*'*d*' is true length. From *d*', draw a line parallel to *xy*, to represent path of *D* in front view.

(vi) The path of D will intersect the projector of D at d''. Join $c'd''$. It is front view of CD.

(vii) With c' as centre and radius equal to $c'd''$, draw an arc cutting a line drawn parallel to xy from c' at d'''. Project d''' to cut a line drawn with c as centre and $c'd'$ (true length) as radius, at d_3. From d_3, draw a line parallel to xy, repressenting path of D in top view.

(viii) Project d'' to cut path of D in top view at d_4. Join cd_4. It is top view of CD.

(ix) The results are shown in fig. 10-67.

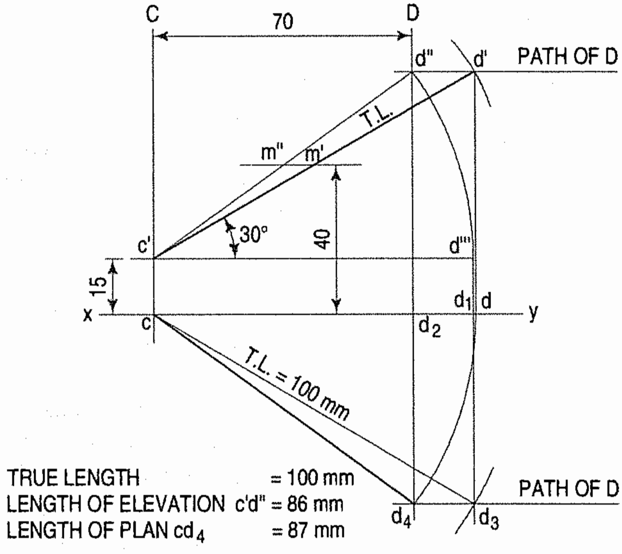


FIG. 10-67

EXERCISES 10(b)

1. A line AB, 75 mm long, is inclined at 45° to the H.P. and 30° to the V.P. Its end B is in the H.P. and 40 mm in front of the V.P. Draw its projections and determine its traces.
2. Draw the projections of a line AB, 90 mm long, its mid-point M being 50 mm above the H.P. and 40 mm in front of the V.P. The end A is 20 mm above the H.P. and 10 mm in front of the V.P. Show the traces and the inclinations of the line with the H.P. and the V.P.
3. The front view of a 125 mm long line PQ measures 75 mm and its top view measures 100 mm. Its end Q and the mid-point M are in the first quadrant, M being 20 mm from both the planes. Draw the projections of the line PQ.
4. A line AB, 75 mm long is in the second quadrant with the end A in the H.P. and the end B in the V.P. The line is inclined at 30° to the H.P. and at 45° to the V.P. Draw the projections of AB and determine its traces.
5. The end A of a line AB is in the H.P. and 25 mm behind the V.P. The end B is in the V.P. and 50 mm above the H.P. The distance between the end projectors is 75 mm. Draw the projections of AB and determine its true length, traces and inclinations with the two planes.
6. The top view of a 75 mm long line CD measures 50 mm. C is 50 mm in front of the V.P. and 15 mm below the H.P. D is 15 mm in front of the V.P. and is above the H.P. Draw the front view of CD and find its inclinations with the H.P. and the V.P. Show also its traces.
7. A line PQ, 100 mm long, is inclined at 45° to the H.P. and at 30° to the V.P. Its end P is in the second quadrant and Q is in the fourth quadrant. A point R on PQ, 40 mm from P is in both the planes. Draw the projections of PQ.
8. A line AB, 65 mm long, has its end A in the H.P. and 15 mm in front of the V.P. The end B is in the third quadrant. The line is inclined at 30° to the H.P. and at 60° to the V.P. Draw its projections.

9. The front view of a line *AB* measures 65 mm and makes an angle of 45° with *xy*. *A* is in the H.P. and the V.T. of the line is 15 mm below the H.P. The line is inclined at 30° to the V.P. Draw the projections of *AB* and find its true length and inclination with the H.P. Also locate its H.T.
10. A room is 4.8 m × 4.2 m × 3.6 m high. Determine graphically the distance between a top corner and the bottom corner diagonally opposite to it.
11. A line *AB* is in the first quadrant. Its end *A* and *B* are 20 mm and 60 mm in front of the V.P. respectively. The distance between the end projectors is 75 mm. The line is inclined at 30° to the H.P. and its H.T. is 10 mm above *xy*. Draw the projections of *AB* and determine its true length and the V.T.
12. Two oranges on a tree are respectively 1.8 m and 3 m above the ground, and 1.2 m and 2.1 m from a 0.3 m thick wall, but on the opposite sides of it. The distance between the oranges, measured along the ground and parallel to the wall is 2.7 m. Determine the real distance between the oranges.
13. Draw an isosceles triangle *abc* of base *ab* 40 mm and altitude 75 mm with *a* in *xy* and *ab* inclined at 45° to *xy*. The figure is the top view of a triangle whose corners *A*, *B* and *C* are respectively 75 mm, 25 mm and 50 mm above the H.P. Determine the true shape of the triangle and the inclination of the side *AB* with the two planes.
14. Three points *A*, *B* and *C* are 7.5 m above the ground level, on the ground level and 9 m below the ground level respectively. They are connected by roads with each other and are seen at angles of depression of 10°, 15° and 30° respectively from a point *O* on a hill 30 m above the ground level. A is due north-east, *B* is due north and *C* is due south-east of *O*. Find the lengths of the connecting roads.
15. A pipe-line from a point *A*, running due north-east has a downward gradient of 1 in 5. Another point *B* is 12 m away from and due east of *A* and on the same level. Find the length and slope of a pipe-line from *B* which runs due 15° east of north and meets the pipe-line from *A*.
16. The guy ropes of two poles 12 m apart, are attached to a point 15 m above the ground on the corner of a building. The points of attachment on the poles are 7.5 m and 4.5 m above the ground and the ropes make 45° and 30° respectively with the ground. Draw the projections and find the distances of the poles from the building and the lengths of the guy ropes.
17. A plate chimney, 18 m high 0.9 m diameter is supported by two sets of three guy wires each, as shown in fig. 10-68.

    One set is attached at 3 m from the top and anchored 6 m above the ground level. The other set is fixed to the chimney at its mid-height and anchored on the ground. Determine the length and slope with the ground, of one of the wires from each set.

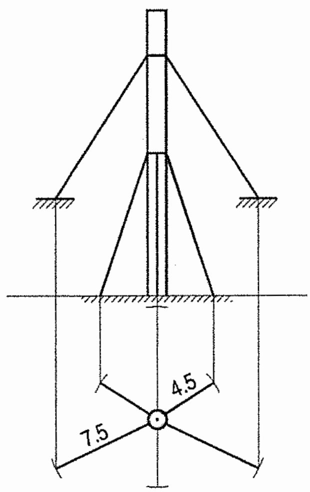


FIG. 10-68

18. The projectors drawn from the H.T. and the V.T. of a straight line *AB* are 80 mm apart while those drawn from its ends are 50 mm apart. The H.T. is 35 mm in front of the V.P., the V.T. is 55 mm above the H.P. and the end *A* is 10 mm above the H.P. Draw the projections of *AB* and determine its length and inclinations with the reference planes.

19. Three guy ropes *AB*, *CD* and *EF* are tied at points *A*, *C* and *E* on a vertical post 15 m long at heights of 14 m, 12 m and 10 m respectively from the ground. The lower ends of the ropes are tied to hooks at points *B*, *D* and *F* on the ground level. If the points *B*, *D* and *F* lie at the corners of an equilateral triangle of 9 m long sides and if the post is situated at the centre of this triangle, determine graphically the length of each rope and its inclination with the ground. Assume the thickness of the post and the ropes to be equal to that of a line.

20. A line *AB*, 80 mm long, makes an angle of 60° with the H.P. and lies in an auxiliary vertical plane (A.V.P.), which makes an angle of 45° with the V.P. Its end *A* is 10 mm away from both the H.P. and the V.P.

    Draw the projections of *AB* and determine (i) its true inclination with the V.P. and (ii) its traces.

21. A line *PQ* is 75 mm long and lies in an auxiliary inclined plane (A.I.P.) which makes an angle of 45° with the H.P. The front view of the line measures 55 mm and the end *P* is in the V.P. and 20 mm above the H.P.

    Draw the projections of *PQ* and find (i) its inclinations with both the planes and (ii) its traces.

22. A line *AB*, 80 mm long, makes an angle of 30° with the V.P. and lies in a plane perpendicular to both the H.P. and the V.P. Its end *A* is in the H.P. and the end *B* is in the V.P. Draw its projections and show its traces.

23. The front view of a line makes an angle of 30° with xy. The H.T. of the line is 45 mm in front of the V.P., while its V.T. is 30 mm below the H.P. One end of the line is 10 mm above the H.P. and the other end is 100 mm in front of the V.P.

    Draw the projections of the line and determine (i) its true length, and (ii) its inclinations with the H.P. and the V.P.

24. A room is 6 m × 5 m × 3.5 m high. An electric bracket light is above the centre of the longer wall and 1 m below the ceiling. The bulb is 0.3 m away from the wall. The switch for the light is on an adjacent wall, 1.5 m above the floor and 1 m away from the other longer wall. Find graphically the shortest distance between the bulb and the switch.

25. Three lines *oa*, *ob* and *oc* are respectively 25 mm, 45 mm and 65 mm long, each making 120° angles with the other two and the shortest line being vertical. The figure is the top view of the three rods *OA*, *OB* and *OC* whose ends *A*, *B* and *C* are on the ground, while *O* is 100 mm above it. Draw the front view and determine the length of each rod and its inclination with the ground.

26. The projectors of the ends of a line *PQ* are 90 mm apart. *P* is 20 mm above the H.P. while *Q* is 45 mm behind the V.P. The H.T. and the V.T. of the line coincide with each other on *xy*, between the two end projectors and 35 mm away from the projector of the end *P*. Draw the projections of *PQ* and determine its true length and inclinations with the two planes.

27. A person on the top of a tower 30 m high, which rises from a horizontal plane, observes the angles of depression (below the horizon) of two objects *H* and *K* on the plane to be 15° and 25°, the direction of *H* and *K* from the tower being due north and due west respectively. Draw the top view to a scale of 1 mm = 0.5 m showing the relative positions of the person and the two objects. Measure and state in metres the distance between *H* and *K*.

28. Two pegs *A* and *B* are fixed in each of the two adjacent side walls (of a rectangular room) which meet in a corner. Peg *A* is 1.5 m above the floor, 1.2 m from the side wall and is protruding 0.3 m from the wall. Peg *B* is 2 m above the floor, 1 m from the other side wall and is also protruding 0.3 m from the wall. Find the distance between the ends of the pegs.
29. Two objects *A* and *B*, 10 m above and 7 m below the ground level respectively, are observed from the top of a tower 35 m high from the ground. Both the objects make an angle of depression of 45° with the horizon. The horizontal distance between *A* and *B* is 20 m. Draw to scale 1:250, the projections of the objects and the tower and find (a) the true distance between *A* and *B*, and (b) the angle of depression of another object *C* situated on the ground midway between *A* and *B*.
30. A room measures 8 m long, 5 m wide and 4 m high. An electric point hangs in the centre of the ceiling and 1 m below it. A thin straight wire connects the point to a switch kept in one of the corners of the room and 2 m above the floor. Draw the projections of the wire, and find the length of the wire and its slope-angle with the floor.
31. A rectangular tank 4 m high is strengthened by four stay rods one at each corner, connecting the top corner to a point in the bottom 0.7 m and 1.2 m from the sides of the tank. Find graphically the length of the rod required and the angle it makes with the surface of the tank.
32. Three vertical poles *AB*, *CD* and *EF* are respectively 5, 8 and 12 metres long. Their ends *B*, *D* and *F* are on the ground and lie at the corners of an equilateral triangle of 10 metres long sides. Determine graphically the distance between the top ends of the poles, viz. *AC*, *CE* and *EA*.
33. The front view of a line *AB* measures 70 mm and makes an angle of 45° with *xy*. *A* is in the H.P. and the V.T. of the line is 15 mm below the H.P. The line is inclined at 30° to the V.P. Draw the projections of *AB*, and find its true length, inclination with the H.P. and its H.T.
34. A line *AB* measures 100 mm. The projectors through its V.T. and the end *A* are 40 mm apart. The point *A* is 30 mm below the H.P. and 20 mm behind the V.P. The V.T. is 10 mm above the H.P. Draw the projections of the line and determine its H.T. and inclinations with the H.P. and the V.P.
35. A horizontal wooden platform is 3.5 m long and 2 m wide. It is suspended from a hook by means of chains attached at its four corners. The hook is situated vertically above the centre of the platform and at a distance of 5 m above it. Determine graphically the length of each chain and the angle which it makes with the platform. Assume the thickness of the platform and the chain to be equal to that of a line. Scale: 10 mm = 0.5 m.
36. A picture frame 2 m wide and 1 m high is to be fixed on a wall railing by two straight wires attached to the top corners. The frame is to make an angle of 40° with the wall and the wires are to be fixed to a hook on the wall on the centre line of the frame and 1.5 m above the railing. Find the length of the wires and the angle between them.
37. The top view of line *AB* measures 60 mm and inclined to reference line at 60°. The end point *A* is 15 mm above the H.P. and 30 mm infront of the V.P. Draw the projections of the line when it is inclined at 45° to the H.P. and is situated in the first quadrant. Find true length and inclination of the line with the V.P. and traces.

Chapter 12

PROJECTIONS OF PLANES

12-0. INTRODUCTION

Plane figures or surfaces have only two dimensions, viz. length and breadth. They do not have thickness. A plane figure may be assumed to be contained by a plane, and its projections can be drawn, if the position of that plane with respect to the principal planes of projection is known.

In this chapter, we shall discuss the following topics:

1. Types of planes and their projections.
2. Traces of planes.

12-1. TYPES OF PLANES

Planes may be divided into two main types:

(1) Perpendicular planes.

(2) Oblique planes.

(1) Perpendicular planes: These planes can be divided into the following sub-types:

(i) Perpendicular to both the reference planes.

(ii) Perpendicular to one plane and parallel to the other.

(iii) Perpendicular to one plane and inclined to the other.

(i) ***Perpendicular to both the reference planes*** (fig. 12-1): A square *ABCD* is perpendicular to both the planes. Its H.T. and V.T. are in a straight line perpendicular to *xy*.

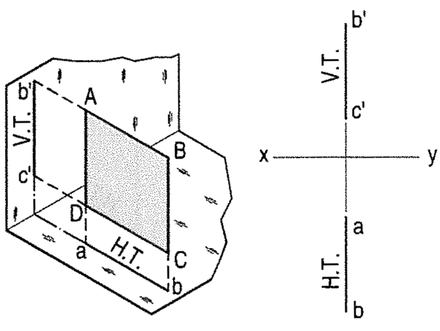


FIG. 12-1

The front view *b'c'* and the top view *ab* of the square are both lines coinciding with the V.T. and the H.T. respectively.

(ii) *Perpendicular to one plane and parallel to the other plane:*

(a) Plane, perpendicular to the H.P. and parallel to the V.P. [fig. 12-2(i)].

A triangle *PQR* is perpendicular to the H.P. and is parallel to the V.P. Its H.T. is parallel to *xy*. It has no V.T.

The front view *p'q'r'* shows the exact shape and size of the triangle. The top view *pqr* is a line parallel to *xy*. It coincides with the H.T.

(b) Plane, perpendicular to the V.P. and parallel to the H.P. [fig. 12-2(ii)].

A square *ABCD* is perpendicular to the V.P. and parallel to the H.P. Its V.T. is parallel to *xy*. It has no H.T.

The top view *abcd* shows the true shape and true size of the square. The front view *a'b'* is a line, parallel to *xy*. It coincides with the V.T.

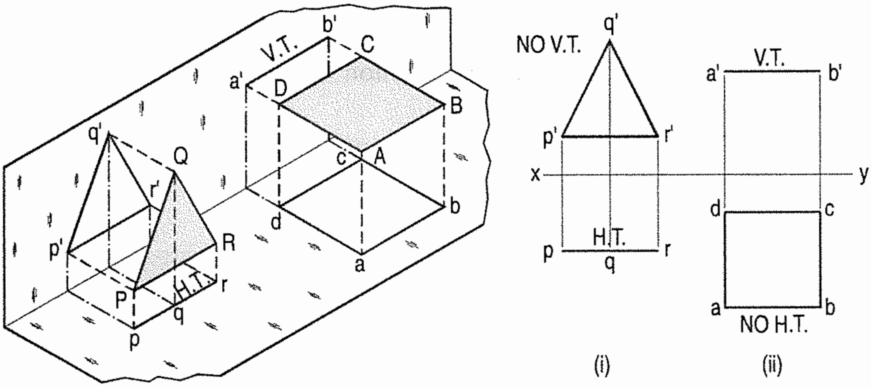


FIG. 12-2

(iii) *Perpendicular to one plane and inclined to the other plane:*

(a) Plane, perpendicular to the H.P. and inclined to the V.P. (fig. 12-3).

A square *ABCD* is perpendicular to the H.P. and inclined at an angle ø to the V.P. Its V.T. is perpendicular to *xy*. Its H.T. is inclined at ø to xy.

Its top view *ab* is a line linclined at ø to *xy*. The front view *a'b'c'd'* is smaller than *ABCD*.

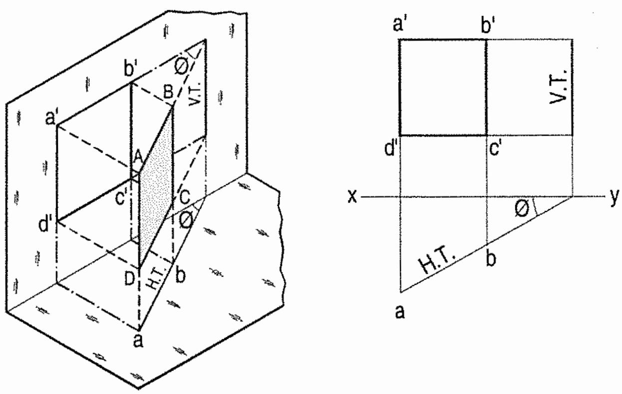


FIG. 12-3

(b) Plane, perpendicular to the V.P. and inclined to the H.P. (fig. 12-4).

A square *ABCD* is perpendicular to the V.P. and inclined at an angle θ to the H.P. Its H.T. is perpendicular to *xy*. Its V.T. makes the angle θ with *xy*. Its front view *a'b'* is a line inclined at θ to *xy*. The top view *abcd* is a rectangle which is smaller than the square *ABCD*.

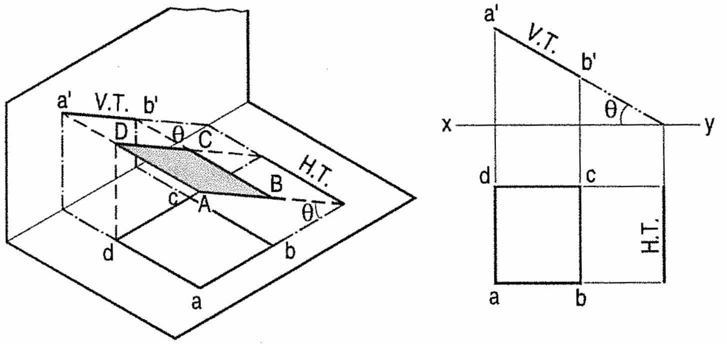


FIG. 12-4

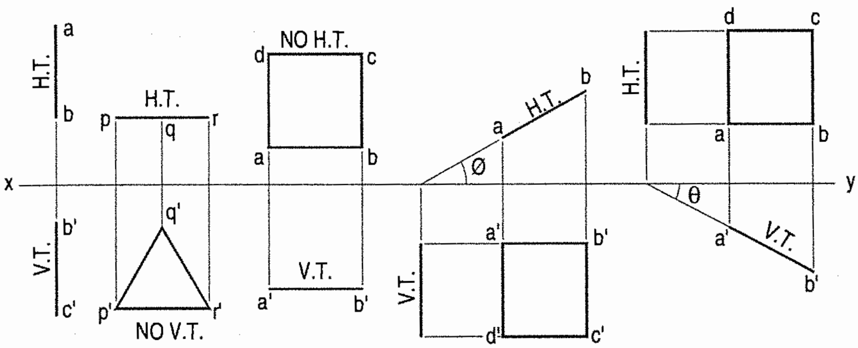


FIG. 12-5

Fig. 12-5 shows the projections and the traces of all these perpendicular planes by third-angle projection method.

(2) Oblique planes: Planes which are inclined to both the reference planes are called *oblique planes*. Representation of oblique planes by their traces is too advanced to be included in this book.

A few problems on the projections of plane figures inclined to both the reference planes are however, illustrated at the end of the chapter. They will prove to be of great use in dealing with the projections of solids.

12-2. TRACES OF PLANES

A plane, extended if necessary, will meet the reference planes in lines, unless it is parallel to any one of them.

These lines are called the *traces* of the plane. The line in which the plane meets the H.P. is called the *horizontal trace* or the H.T. of the plane. The line in which it meets the V.P. is called its *vertical trace* or the V.T. A plane is usually represented by its traces.

12-3. GENERAL CONCLUSIONS

(1) Traces:

(a) When a plane is perpendicular to both the reference planes, its traces lie on a straight line perpendicular to *xy*.

(b) When a plane is perpendicular to one of the reference planes, its trace upon the other plane is perpendicular to *xy* (except when it is parallel to the other plane).

(c) When a plane is parallel to a reference plane, it has no trace on that plane. Its trace on the other reference plane, to which it is perpendicular, is parallel to *xy*.

(d) When a plane is inclined to the H.P. and perpendicular to the V.P., its inclination is shown by the angle which its V.T. makes with *xy*. When it is inclined to the V.P. and perpendicular to the H.P., its inclination is shown by the angle which its H.T. makes with *xy*.

(e) When a plane has two traces, they, produced if necessary, intersect in *xy* (except when both are parallel to *xy* as in case of some oblique planes).

(2) Projections:

(a) When a plane is perpendicular to a reference plane, its projection on that plane is a straight line.

(b) When a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.

(c) When a plane is perpendicular to one of the reference planes and inclined to the other, its inclination is shown by the angle which its projection on the plane to which it is perpendicular, makes with *xy*. Its projection on the plane to which it is inclined, is smaller than the plane itself.

Problem 12-1. *Show by means of traces, each of the following planes:*

(a) *Perpendicular to the H.P. and the V.P.*

(b) *Perpendicular to the H.P. and inclined at 30° to the V.P.*

(c) *Parallel to and 40 mm away from the V.P.*

(d) *Inclined at 45° to the H.P. and perpendicular to the V.P.*

(e) *Parallel to the H.P. and 25 mm away from it.*

Fig. 12-6 and fig. 12-7 show the various traces.

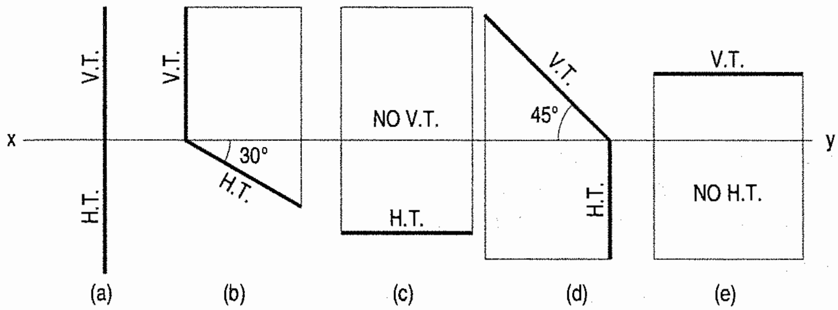
(a) The H.T. and the V.T. are in a line perpendicular to xy.

(b) The H.T. is inclined at 30° to *xy*; the V.T. is normal to *xy*; both the traces intersect in *xy*.

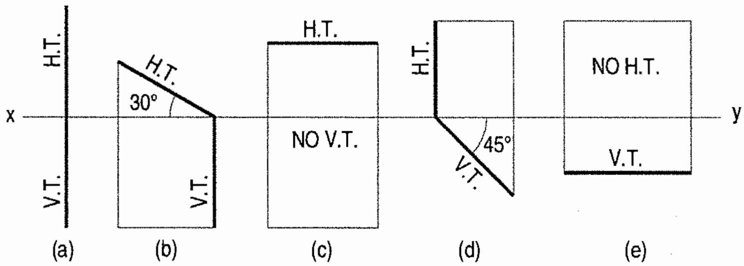
(c) The H.T. is parallel to and 40 mm away from *xy*. It has no V.T.

(d) The H.T. is perpendicular to *xy*; the V.T. makes 45° angle with *xy*; both intersect in *xy*.

(e) The V.T. is parallel to and 25 mm away from *xy*. It has no H.T.



(First-angle projection)
FIG. 12-6



(Third-angle projection)
FIG. 12-7

12-4. PROJECTIONS OF PLANES PARALLEL TO ONE OF THE REFERENCE PLANES

The projection of a plane on the reference plane parallel to it will show its true shape. Hence, beginning should be made by drawing that view. The other view which will be a line, should then be projected from it.

(1) When the plane is parallel to the H.P.: The top view should be drawn first and the front view projected from it.

Problem 12-2. (fig. 12-8): *An equilateral triangle of 50 mm side has its V.T. parallel to and 25 mm above xy. It has no H.T. Draw its projections when one of its sides is inclined at 45° to the V.P.*

As the V.T. is parallel to *xy* and as there is no H.T. the triangle is parallel to the H.P. Therefore, begin with the top view.

(i) Draw an equilateral triangle *abc* of 50 mm side, keeping one side, say *ac*, inclined at 45° to *xy*.

(ii) Project the front view, parallel to and 25 mm *above* *xy*, as shown.

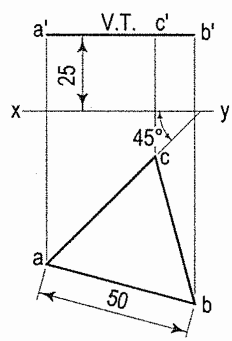


FIG. 12-8

(2) When the plane is parallel to the V.P.: Beginning should be made with the front view and the top view projected from it.

Problem 12-3. (fig. 12-9): *A square ABCD of 40 mm side has a corner on the H.P. and 20 mm in front of the V.P. All the sides of the square are equally inclined to the H.P. and parallel to the V.P. Draw its projections and show its traces.*

As all the sides are parallel to the V.P., the surface of the square also is parallel to it. The front view will show the true shape and position of the square.

(i) Draw a square *a'b'c'd'* in the front view with one corner in *xy* and all its sides inclined at 45° to *xy*.

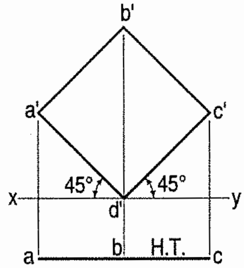


FIG. 12-9

(ii) Project the top view keeping the line *ac* parallel to *xy* and 30 mm below it. The top view is its H.T. It has no V.T.

12-5. PROJECTIONS OF PLANES INCLINED TO ONE REFERENCE PLANE AND PERPENDICULAR TO THE OTHER

When a plane is inclined to a reference plane, its projections may be obtained in two stages. In the initial stage, the plane is assumed to be parallel to that reference plane to which it has to be made inclined. It is then tilted to the required inclination in the second stage.

(1) Plane, inclined to the H.P. and perpendicular to the V.P.: When the plane is inclined to the H.P. and perpendicular to the V.P., in the initial stage, it is assumed to be parallel to the H.P. Its top view will show the true shape. The front view will be a line parallel to *xy*. The plane is then tilted so that it is inclined to the H.P. The new front view will be inclined to *xy* at the true inclination. In the top view the corners will move along their respective paths (parallel to *xy*).

Problem 12-4. (fig. 12-10): *A regular pentagon of 25 mm side has one side on the ground. Its plane is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections and show its traces.*

Assuming it to be parallel to the H.P.

(i) Draw the pentagon in the top view with one side perpendicular to *xy* [fig. 12-10(i)]. Project the front view. It will be the line *a'c'* contained by *xy*.

(ii) Tilt the front view about the point *a'*, so that it makes 45° angle with *xy*.

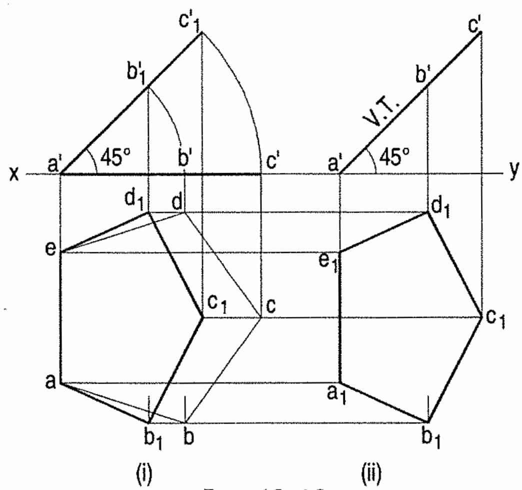


FIG. 12-10

(iii) Project the new top view $ab_1c_1d_1e$ upwards from this front view and horizontally from the first top view. It will be more convenient if the front view is reproduced in the new position separately and the top view projected from it, as shown in fig. 12-10(ii). The V.T. coincides with the front view and the H.T. is perpendicular to *xy*, through the point of intersection between *xy* and the front view-produced.

(2) Plane, inclined to the V.P. and perpendicular to the H.P.: In the initial stage, the plane may be assumed to be parallel to the V.P. and then tilted to the required position in the next stage. The projections are drawn as illustrated in the next problem.

Problem 12-5. (fig. 12-11): *Draw the projections of a circle of 50 mm diameter, having its plane vertical and inclined at 30° to the V.P. Its centre is 30 mm above the H.P. and 20 mm in front of the V.P. Show also its traces.*

A circle has no corners to project one view from another. However, a number of points, say twelve, equal distances apart, may be marked on its circumference.

(i) Assuming the circle to be parallel to the V.P., draw its projections. The front view will be a circle [fig. 12-11(i)], having its centre 30 mm above *xy*. The top view will be a line, parallel to and 20 mm below *xy*.

(ii) Divide the circumference into twelve equal parts (with a 30°-60° set-square) and mark the points as shown. Project these points in the top view. The centre *O* will coincide with the point 4.

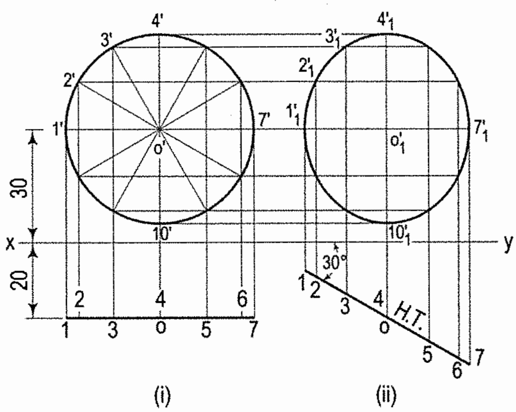


FIG. 12-11

(iii) When the circle is tilted, so as to make 30° angle with the V.P., its top view will become inclined at 30° to *xy*. In the front view all the points will move along their respective paths (parallel to *xy*). Reproduce the top view keeping the centre *o* at the same distance, viz. 20 mm from *xy* and inclined at 30° to *xy* [fig. 12-11(ii)].

(iv) For the final front view, project all the points upwards from this top view and horizontally from the first front view. Draw a freehand curve through the twelve points $1'_1$, $2'_1$ etc. This curve will be an ellipse.

12-6. PROJECTIONS OF OBLIQUE PLANES

When a plane has its surface inclined to one plane and an edge or a diameter or a diagonal parallel to that plane and inclined to the other plane, its projections are drawn in three stages.

(1) If the surface of the plane is inclined to the H.P. and an edge (or a diameter or a diagonal) is parallel to the H.P. and inclined to the V.P.,

  (i) in the initial position the plane is assumed to be parallel to the H.P. and an edge perpendicular to the V.P.

  (ii) It is then tilted so as to make the required angle with the H.P. As already explained, its front view in this position will be a line, while its top view will be smaller in size.

  (iii) In the final position, when the plane is turned to the required inclination with the V.P., only the position of the top view will change. Its shape and size will not be affected. In the final front view, the corresponding distances of all the corners from *xy* will remain the same as in the second front view.

If an edge is in the H.P. or on the ground, in the initial position, the plane is assumed to be lying in the H.P. or on the ground, with the edge perpendicular to the V.P. *If a corner is in the H.P. or on the ground, the line joining that corner with the centre of the plane is kept parallel to the V.P.*

(2) Similarly, if the surface of the plane is inclined to the V.P. and an edge (or a diameter or a diagonal) is parallel to the V.P. and inclined to the H.P.,

(i) in the initial position, the plane is assumed to be parallel to the V.P. and an edge perpendicular to the H.P.

(ii) It is then tilted so as to make the required angle with the V.P. Its top view in this position will be a line, while its front view will be smaller in size.

(iii) When the plane is turned to the required inclination with the H.P., only the position of the front view will change. Its shape and size will not be affected. In the final top view, the corresponding distances of all the corners from *xy* will remain the same as in the second top view.

If an edge is in the V.P., in the initial position, the plane is assumed to be lying in the V.P. with an edge perpendicular to the H.P. *If a corner is in the V.P., the line joining that corner with centre of the plane is kept parallel to the H.P.*

Problem 12-6. (fig. 12-12): *A square ABCD of 50 mm side has its corner A in the H.P., its diagonal AC inclined at 30° to the H.P. and the diagonal BD inclined at 45° to the V.P. and parallel to the H.P. Draw its projections.*

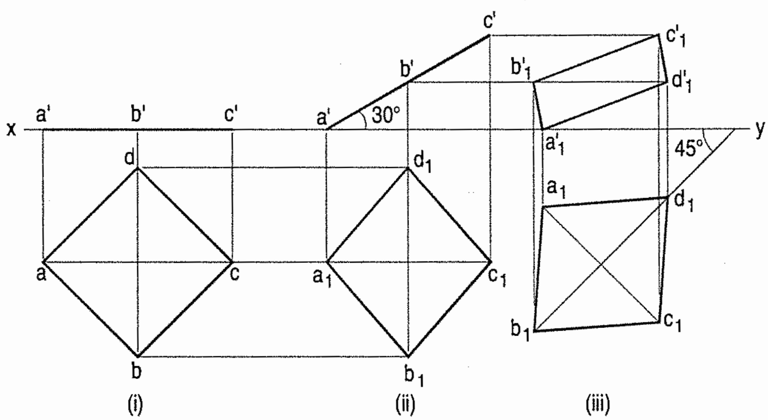


FIG. 12-12

In the initial stage, assume the square to be lying in the H.P. with *AC* parallel to the V.P.

(i) Draw the top view and the front view. When the square is tilted about the corner *A* so that *AC* makes 30° angle with the H.P., *BD* remains perpendicular to the V.P. and parallel to the H.P.

(ii) Draw the second front view with *a'c'* inclined at 30° to *xy*, keeping *a'* or *c'* in *xy*. Project the second top view. The square may now be turned so that *BD* makes 45° angle with the V.P. and remains parallel to the H.P. Only the position of the top view will change. Its shape and size will remain the same.

(iii) Reproduce the top view so that b_1d_1 is inclined at 45° to *xy*. Project the final front view upwards from this top view and horizontally from the second front view.

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 27 for the following problem.

Problem 12-7. (Fig. 12-13): *A rectangular plane surface of size L × W is positioned in the first quadrant and is inclined at an angle of 60° with the H.P. and 30° with the V.P. Draw its projections.*

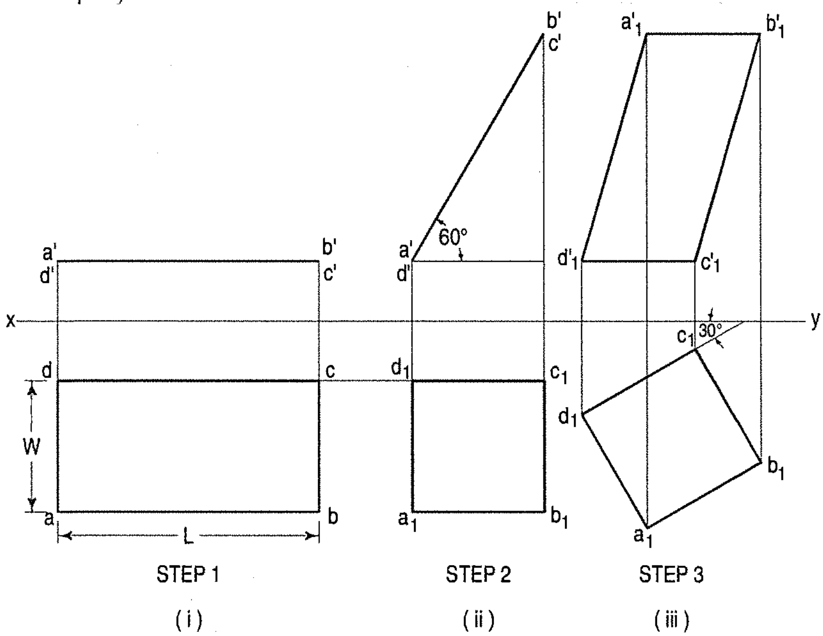


FIG. 12-13

(i) The plane is first assumed to be parallel to H.P. with its shorter edge perpendicular to V.P. In this position, true shape and size of the plane is given by its projection on H.P. The front view will be a true line parallel to the reference line *xy*.

(ii) Rotate the front view projection by 60° (the angle of inclination of plane with H.P.) as shown in Step 2 of fig. 12-13(ii). Draw vertical lines from the ends of line *a'd'* and *b'c'* to intersect horizontal lines drawn from the top view *abcd* (step 1) at points b_1, c_1, d_1 and a_1. Join $a_1b_1c_1d_1$ to obtain the top view of the plane in this inclined position.

(iii) Now rotate the edge d_1c_1 of the top view (step 2) by 30° (the angle of inclination of plane with V.P.) and reproduce it as shown in step 3 of the fig. 12-13(iii). Draw projections from a_1, b_1, c_1 and d_1 to intersect the horizontal projections from *a'd'* and *b'c'* to get the points a'_1, b'_1, c'_1 and d'_1. Join the lines $a'_1b'_1c'_1d'_1$ to obtain the final front view of the given plane surface.

Problem 12-8. (fig. 12-14): *Draw the projections of a regular hexagon of 25 mm side, having one of its sides in the H.P. and inclined at 60° to the V.P., and its surface making an angle of 45° with the H.P.*

(i) Draw the hexagon in the top view with one side perpendicular to *xy*. Project the front view *a'c'* in *xy*.

(ii) Draw *a'c'* inclined at 45° to *xy* keeping *a'* or *c'* in *xy* and project the second top view.

(iii) Reproduce this top view making a_1f_1 inclined at 60° to *xy* and project the final front view.

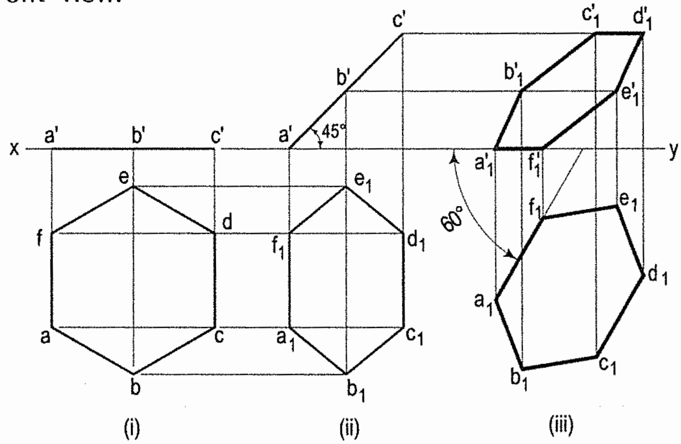


FIG. 12-14

Problem 12-9. (fig. 12-15): *Draw the projections of a circle of 50 mm diameter resting in the H.P. on a point A on the circumference, its plane inclined at 45° to the H.P. and*

(a) the top view of the diameter AB making 30° angle with the V.P.;

(b) the diameter AB making 30° angle with the V.P.

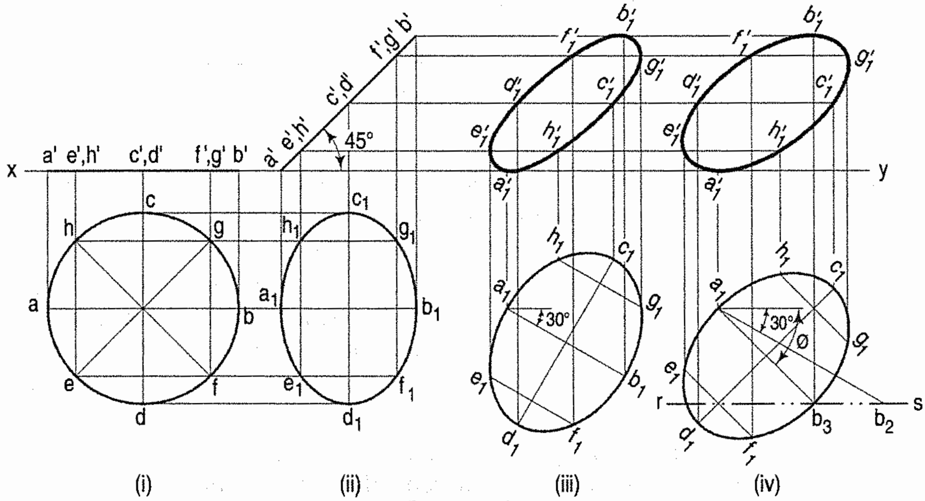


FIG. 12-15

Draw the projections of the circle with *A* in the H.P. and its plane inclined at 45° to the H.P. and perpendicular to the V.P. [fig. 12-15(i) and fig. 12-15(ii)].

(a) In the second top view, the line a_1b_1 is the top view of the diameter *AB*. Reproduce this top view so that a_1b_1 makes 30° angle with *xy* [fig. 12-15(iii)]. Project the required front view.

(b) If the diameter AB, which makes 45° angle with the H.P., is inclined at 30° to the V.P. also, its top view a_1b_1 will make an angle greater than 30° with xy. This apparent angle of inclination is determined as described below.

Draw any line a_1b_2 equal to AB and inclined at 30° to xy [fig. 12-15(iv)]. With a_1 as centre and radius equal to the top view of AB, viz. a_1b_1, draw an arc cutting rs (the path of B in the top view) at b_3. Draw the line joining a_1 with b_3, and around it, reproduce the second top view. Project the final front view. It is evident that a_1b_3 is inclined to xy at an angle ø which is greater than 30°.

Problem 12-10. (fig. 12-16): *A thin 30°-60° set-square has its longest edge in the V.P. and inclined at 30° to the H.P. Its surface makes an angle of 45° with the V.P. Draw its projections.*

In the initial stage, assume the set-square to be in the V.P. with its hypotenuse perpendicular to the H.P.

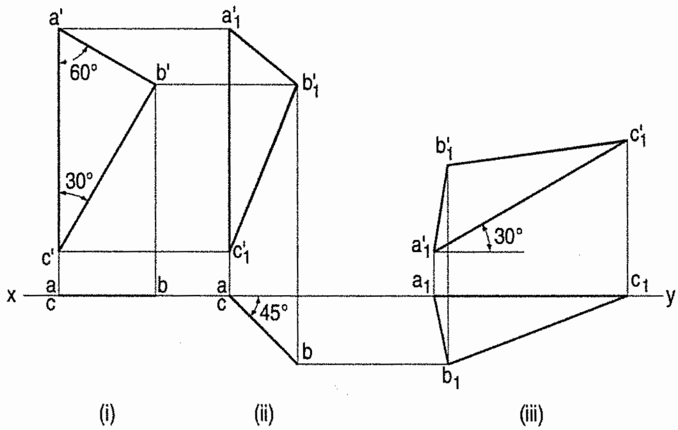


FIG. 12-16

(i) Draw the front view $a'b'c'$ and project the top view ac in xy.

(ii) Tilt ac around the end a so that it makes 45° angle with xy and project the front view $a'_1b'_1c'_1$.

(iii) Reproduce the second front view $a'_1b'_1c'_1$ so that $a'_1b'_1$ makes an angle of 30° with xy. Project the final top view $a_1b_1c_1$.

Problem 12-11. (fig. 12-17): *A thin rectangular plate of sides 60 mm × 30 mm has its shorter side in the V.P. and inclined at 30° to the H.P. Project its top view if its front view is a square of 30 mm long sides.*

As the front view of the plate is a square, its surface must be inclined to the V.P. Hence, assume the plate to be in the V.P. with its shorter edge perpendicular to the H.P.

(i) Draw the front view $a'b'c'd'$ and project the top view ab in xy [fig. 12-17(i)].

(ii) The line ab should be so inclined to xy that the front view becomes a square. Therefore, draw the square $a'_1b'_1c'_1d'_1$ of side equal to $a'd'$. With a as centre and radius equal to ab draw an arc cutting the projector through b'_1 at b. Then ab is the new top view.

(iii) Reproduce the second front view in such a way that $a'_1d'_1$ makes 30° angle with xy. Project the final top view as shown.

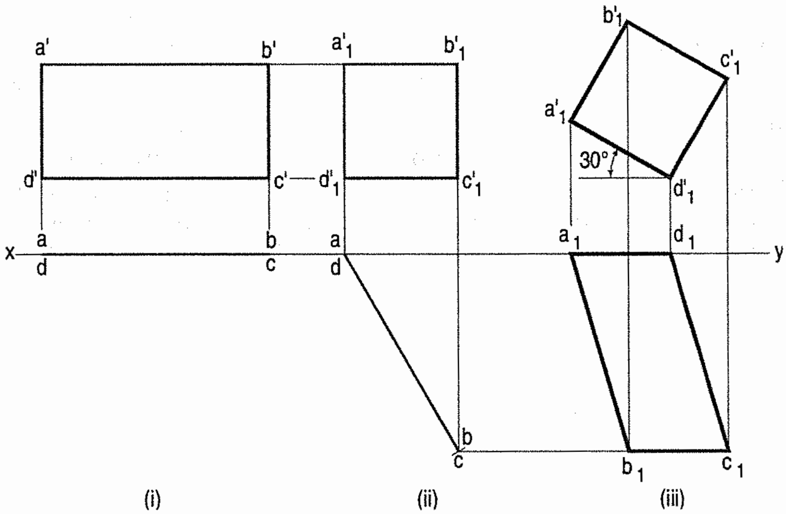
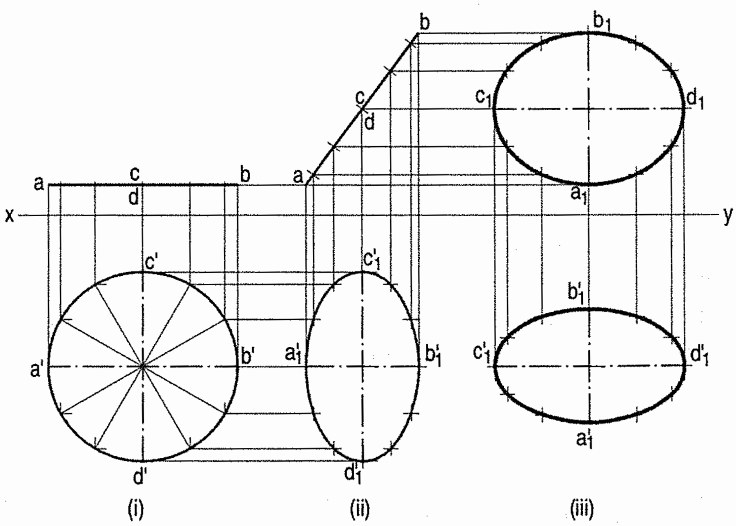


FIG. 12-17

Problem 12-12. (fig. 12-18): *A circular plate of negligible thickness and 50 mm diameter appears as an ellipse in the front view, having its major axis 50 mm long and minor axis 30 mm long. Draw its top view when the major axis of the ellipse is horizontal.*

As the plate is seen as an ellipse in the front view, its surface must be inclined to the V.P.



(Third-angle projection)
FIG. 12-18

(i) Therefore, assume it to be parallel to the V.P. and draw its front view and the top view.

(ii) Turn the line *ab* so that its length in the front view becomes 30 mm, and project the front view. It will be an ellipse whose major axis is vertical.

(iii) Reproduce this view so that the major axis $c'_1d'_1$ is horizontal, and project the required top view.

Problem 12-13. *Fig. 12-19 shows a thin plate of negligible thickness. It rests on its PQ edge with its plane perpendicular to V.P. and inclined 40° to the H.P. Draw its projections.*

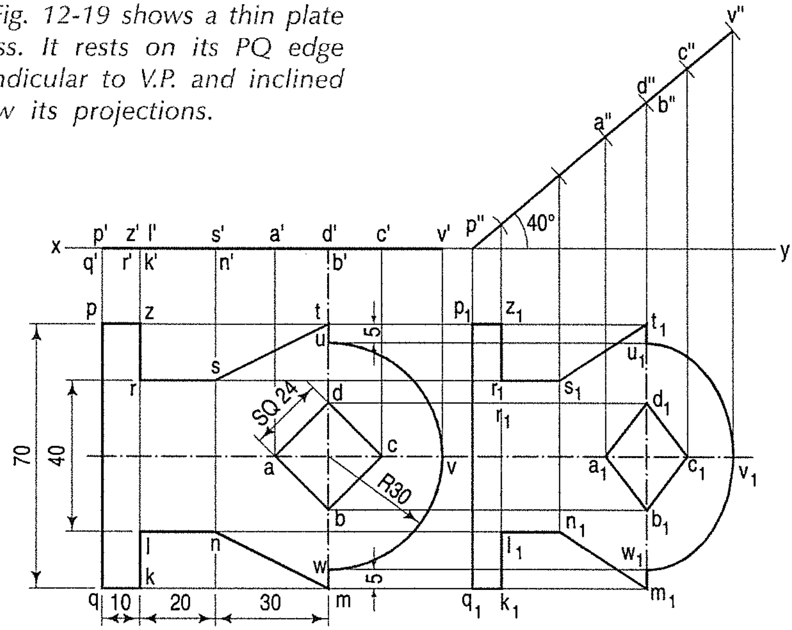


FIG. 12-19

(i) Keep the plane of plate in the H.P. and draw the projections as shown.

(ii) Tilt front view at *p*" making an angle of 40°.

(iii) Project *p*", *a*", *d*", *c*", *v*" etc. in the top view. Draw horizontal projectors intersecting previously drawn projectors from the front view. Join by smooth curve to complete the top view.

Problem 12-14. (fig. 12-20): *A pentagonal plate of 45 mm side has a circular hole of 40 mm diameter in its centre. The plane stands on one of its sides on the H.P. with its plane perpendicular to V.P. and 45° inclined to the H.P. Draw the projections.*

(i) Keep the plane of plate in the horizontal plane.

(ii) Draw top view and front view as shown.

(iii) Tilt the front view *a*" .....*d*" at *a*" making an angle of 45°. Draw the projectors from various points *a*" ....*d*".

(iv) Draw horizontal projectors from the top view *abcd* as shown. Join the intersection points and complete new top view a_1, b_1, c_1, d_1, e_1, as shown.

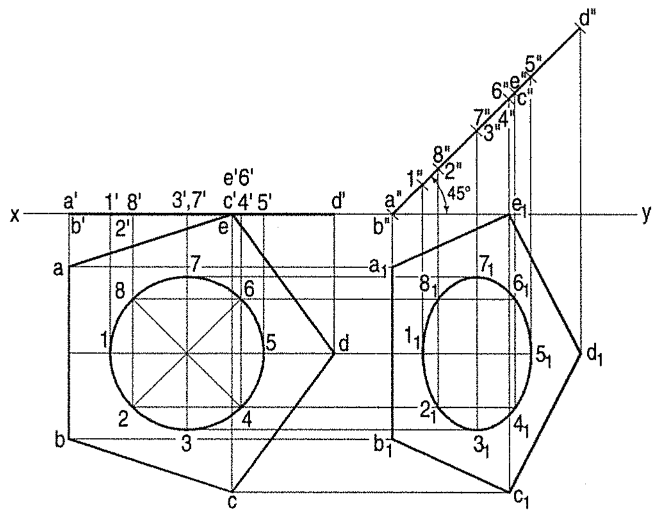


FIG. 12-20

Problem 12-15. (fig. 12-21): *A thin circular plate of 70 mm diameter is resting on its circumference such that its plane is inclined 60° to the H.P. and 30° to the V.P. Draw the projections of the plate.*

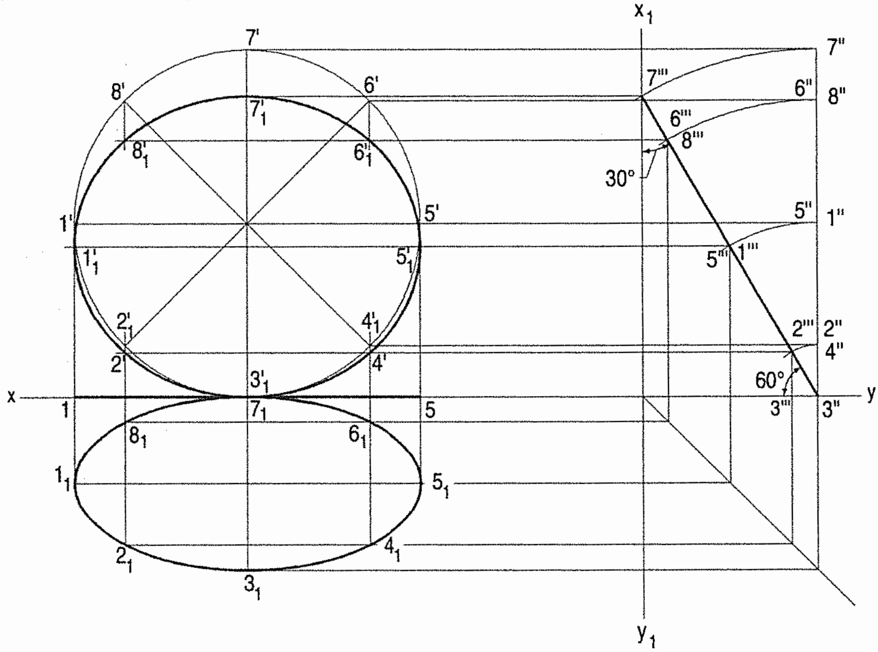


FIG. 12-21

(i) Draw the projection of the plate keeping its plane parallel to the V.P. as shown in fig. 12-21.

(ii) Mark a reference line x_1y_1 perpendicular to *xy* line to represent the auxiliary plane which is at right angle to both the H.P. and the V.P.

(iii) Divide the front view in eight parts and mark the points 1', 2' ....8'. Project these points on the side view as 1", 2" ....8".

(iv) Tilt the side view 3" 7" such that it touches the x_1y_1 line and also makes 60° with the *xy* line.

(v) Complete the projection as shown in fig. 12-21.

Problem 12-16. (fig. 12-22): *PQRS is a rhombus having diagonal PR = 60 mm and QS = 40 mm and they are perpendicular to each other. The plane of the rhombus is inclined with H.P. such that its top view appears to be square. The top view of PR makes 30° with the V.P. Draw its projections and determine inclination of the plane with the H.P.*

(i) Assume that the rhombus is lying in H.P. with its longest diagonal parallel to *xy* line.

(ii) Draw the plans of diagonals *PR* = 60 mm and *QS* = 40 mm (true length) perpendicular each other as shown.

(iii) Join points *p*, *q*, *r* and *s*. It is top view of the rhombus. Project the points *p*, *q*, *r* and *s* in the *xy* line. It is front view of the rhombus points *p'*, *q'*, *r'* and *s'* in the *xy* line as the plane of rhombus is perpendicular to the V.P.

(iv) *PR* and *QS* are lying in H.P. *pr* and *qs* are true length. As the plane of the rhombus is inclined to H.P., the top view of the rhombus is going to be a square. But diagonal *qs* does not change in the length as it is perpendicular to V.P.

(v) Draw the projectors from the points *p*, *q* and *r*, *s* parallel to the *xy*. From q_1 and s_1 draw square $p_1\ q_1\ r_1\ s_1$ such that $s_1\ q_1 = p_1\ r_1$ as shown in fig. 12-22.

(vi) Draw vertical projectors from p_1, q_1, r_1 and s_1.

(vii) Projector of p_1 intersects at p'' in xy line. Taking p'' as centre and the radius equal to 60 mm ($p'r'$), draw the arc to intersect the vertical projectors of r_1 at r''. Join $p''r''$. Measure angle of $p''r''$ with *xy* line.

(viii) Tilt diagonal p_1r_1 at 30° with xy and reproduce square $p_2q_2r_2s_2$. Draw vertical projectors from p_2, q_2, r_2 and s_2 to intersect the horizontal projectors from p'', q'', r'' and s'' at p''', q''', r''' and s'''. Join the points p''', q''', r''' and s''' as shown.

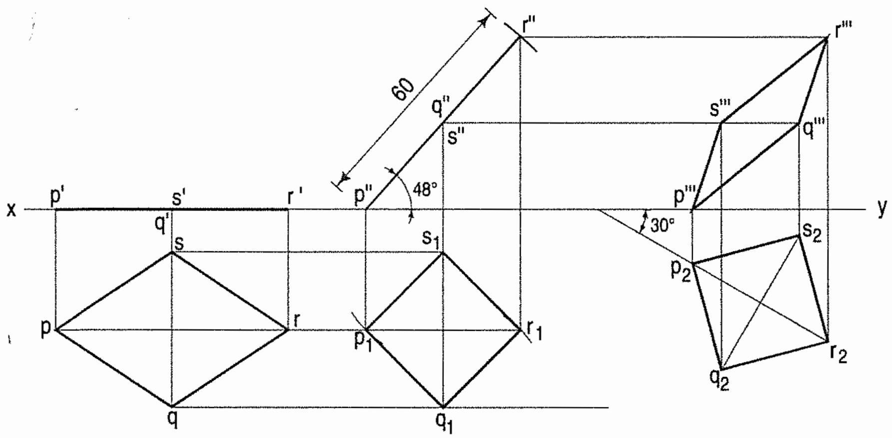


FIG. 12-22

EXERCISES 12

1. Draw an equilateral triangle of 75 mm side and inscribe a circle in it. Draw the projections of the figure, when its plane is vertical and inclined at 30° to the V.P. and one of the sides of the triangle is inclined at 45° to the H.P.

2. A regular hexagon of 40 mm side has a corner in the H.P. Its surface is inclined at 45° to the H.P. and the top view of the diagonal through the corner which is in the H.P. makes an angle of 60° with the V.P. Draw its projections.

3. Draw the projections of a regular pentagon of 40 mm side, having its surface inclined at 30° to the H.P. and a side parallel to the H.P. and inclined at an angle of 60° to the V.P.

4. Draw the projections of a rhombus having diagonals 125 mm and 50 mm long, the smaller diagonal of which is parallel to both the principal planes, while the other is inclined at 30° to the H.P.

5. Draw a regular hexagon of 40 mm side, with its two sides vertical. Draw a circle of 40 mm diameter in its centre. The figure represents a hexagonal plate with a hole in it and having its surface parallel to the V.P. Draw its projections when the surface is vertical and inclined at 30° to the V.P. Assume the thickness of the plate to be equal to that of a line.

6. Draw the projections of a circle of 75 mm diameter having the end *A* of the diameter *AB* in the H.P., the end *B* in the V.P., and the surface inclined at 30° to the H.P. and at 60° to the V.P.

7. A semi-circular plate of 80 mm diameter has its straight edge in the V.P. and inclined at 45° to the H.P. The surface of the plate makes an angle of 30° with the V.P. Draw its projections.

8. The top view of a plate, the surface of which is perpendicular to the V.P. and inclined at 60° to the H.P. is a circle of 60 mm diameter. Draw its three views.

9. A plate having shape of an isosceles triangle has base 50 mm long and altitude 70 mm. It is so placed that in the front view it is seen as an equilateral triangle of 50 mm sides and one side inclined at 45° to *xy*. Draw its top view.

10. Draw a rhombus of diagonals 100 mm and 60 mm long, with the longer diagonal horizontal. The figure is the top view of a square of 100 mm long diagonals, with a corner on the ground. Draw its front view and determine the angle which its surface makes with the ground.

11. A composite plate of negligible thickness is made-up of a rectangle 60 mm × 40 mm, and a semi-circle on its longer side. Draw its projections when the longer side is parallel to the H.P. and inclined at 45° to the V.P., the surface of the plate making 30° angle with the H.P.

12. A 60° set-square of 125 mm longest side is so kept that the longest side is in the H.P. making an angle of 30° with the V.P. and the set-square itself inclined at 45° to the H.P. Draw the projections of the set-square.

13. A plane figure is composed of an equilateral triangle *ABC* and a semi-circle on *AC* as diameter. The length of the side *AB* is 50 mm and is parallel to the V.P. The corner *B* is 20 mm behind the V.P. and 15 mm below the H.P. The plane of the figure is inclined at 45° to the H.P. Draw the projections of the plane figure.

14. An equilateral triangle *ABC* having side length as 50 mm is suspended from a point *O* on the side *AB* 15 mm from *A* in such a way that the plane of the triangle makes an angle of 60° with the V.P. The point *O* is 20 mm below the H.P. and 40 mm behind the V.P. Draw the projections of the triangle.

15. *PQRS* and *ABCD* are two square thin plates with their diagonals measuring 30 mm and 60 mm. They are touching the H.P. with their corners *P* and *A* respectively, and touching each other with their corresponding opposite corners *R* and *C*. If the plates are perpendicular to each other and perpendicular to V.P. also, draw their projections and determine the length of their sides.

Chapter 13

PROJECTIONS OF SOLIDS

13-0. INTRODUCTION

A solid has three dimensions, viz. length, breadth and thickness. To represent a solid on a flat surface having only length and breadth, at least two orthographic views are necessary. Sometimes, additional views projected on auxiliary planes become necessary to make the description of a solid complete.

This chapter deals with the following topics:

1. Types of solids.
2. Projections of solids in simple positions.
   (a) Axis perpendicular to the H.P.
   (b) Axis perpendicular to the V.P.
   (c) Axis parallel to both the H.P. and the V.P.
3. Projections of solids with axes inclined to one of the reference planes and parallel to the other.
   (a) Axis inclined to the V.P. and parallel to the H.P.
   (b) Axis inclined to the H.P. and parallel to the V.P.
4. Projections of solids with axes inclined to both the H.P. and the V.P.
5. Projections of spheres.

13-1. TYPES OF SOLIDS

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 28 for the types of solids.

Solids may be divided into *two* main groups:

(1) Polyhedra
(2) Solids of revolution.

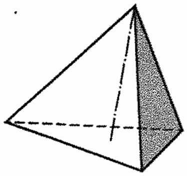
(1) Polyhedra: A polyhedron is defined as a solid bounded by planes called *faces*. When all the faces are equal and regular, the polyhedron is said to be regular.

There are *seven* regular polyhedra which may be defined as stated below:

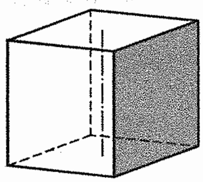
(i) *Tetrahedron* (fig. 13-1): It has four equal faces, each an equilateral triangle.

(ii) *Cube or hexahedron* (fig. 13-2): It has six faces, all equal squares.

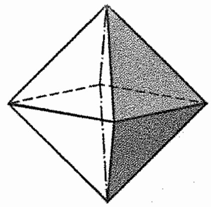
(iii) *Octahedron* (fig. 13-3): It has eight equal equilateral triangles as faces.



Tetrahedron
FIG. 13-1



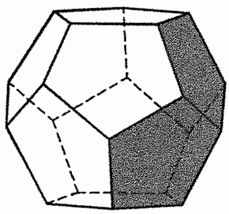
Cube
FIG. 13-2



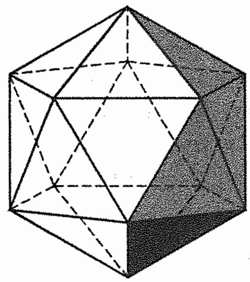
Octahedron
FIG. 13-3

(iv) *Dodecahedron* (fig. 13-4): It has twelve equal and regular pentagons as faces.

(v) *Icosahedron* (fig. 13-5): It has twenty faces, all equal equilateral triangles.



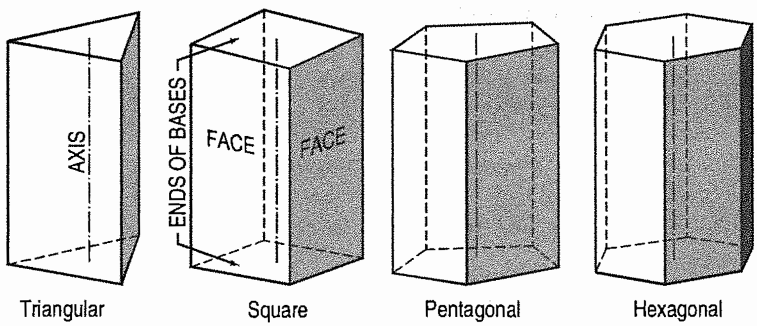
Dodecahedron
FIG. 13-4



Icosahedron
FIG. 13-5

(vi) *Prism:* This is a polyhedron having two equal and similar faces called its ends or bases, parallel to each other and joined by other faces which are parallelograms. The imaginary line joining the centres of the bases is called the axis.

A right and regular prism (fig. 13-6) has its axis perpendicular to the bases. All its faces are equal rectangles.



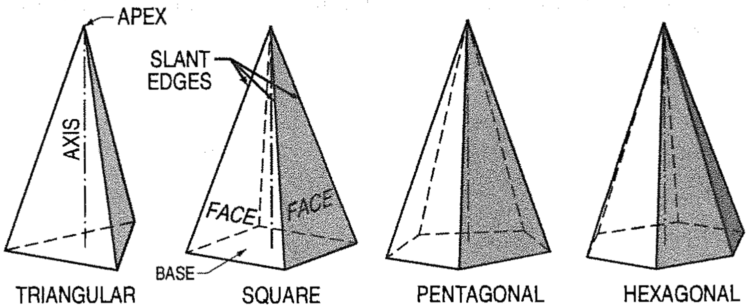
Prisms
FIG. 13-6

(vii) *Pyramid:* This is a polyhedron having a plane figure as a base and a number of triangular faces meeting at a point called the vertex or apex. The imaginary line joining the apex with the centre of the base is its axis.

A right and regular pyramid (fig. 13-7) has its axis perpendicular to the base which is a regular plane figure. Its faces are all equal isosceles triangles.

Oblique prisms and pyramids have their axes inclined to their bases.

Prisms and pyramids are named according to the shape of their bases, as triangular, square, pentagonal, hexagonal etc.



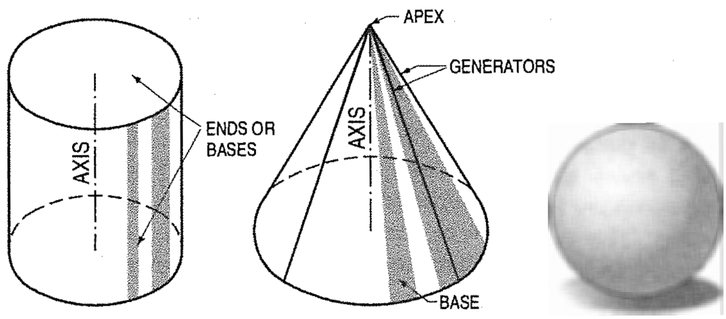
Pyramids
FIG. 13-7

(2) Solids of revolution:

(i) *Cylinder* (fig. 13-8): A *right circular cylinder* is a solid generated by the revolution of a rectangle about one of its sides which remains fixed. It has two equal circular bases. The line joining the centres of the bases is the axis. It is perpendicular to the bases.

(ii) *Cone* (fig. 13-9): A *right circular cone* is a solid generated by the revolution of a right-angled triangle about one of its perpendicular sides which is fixed.

It has one circular base. Its axis joins the apex with the centre of the base to which it is perpendicular. Straight lines drawn from the apex to the circumference of the base-circle are all equal and are called *generators* of the cone. The length of the generator is the slant height of the cone.



Cylinder
FIG. 13-8

Cone
FIG. 13-9

Sphere
FIG. 13-10

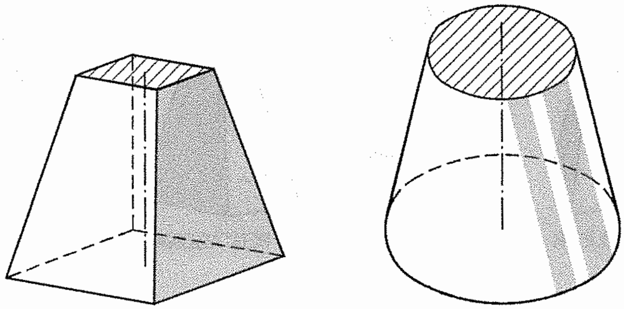
(iii) *Sphere* (fig. 13-10): A *sphere* is a solid generated by the revolution of a semi-circle about its diameter as the axis. The mid-point of the diameter is the centre of the sphere. All points on the surface of the sphere are equidistant from its centre.

Oblique cylinders and cones have their axes inclined to their bases.

(iv) *Frustum*: When a pyramid or a cone is cut by a plane parallel to its base, thus removing the top portion, the remaining portion is called its *frustum* (fig. 13-11).

(v) *Truncated*: When a solid is cut by a plane inclined to the base it is said to be *truncated.*

In this book mostly right and regular solids are dealt with. Hence, when a solid is named without any qualification, it should be understood as being right and regular.



Frustums
FIG. 13-11

13-2. PROJECTIONS OF SOLIDS IN SIMPLE POSITIONS

A solid in simple position may have its axis perpendicular to one reference plane or parallel to both. When the axis is perpendicular to one reference plane, it is parallel to the other. Also, when the axis of a solid is perpendicular to a plane, its base will be parallel to that plane. We have already seen that when a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.

Therefore, the projection of a solid on the plane to which its axis is perpendicular, will show the true shape and size of its base.

Hence, when the axis is perpendicular to the ground, i.e. to the H.P., the top view should be drawn first and the front view projected from it.

When the axis is perpendicular to the V.P., beginning should be made with the front view. The top view should then be projected from it.

When the axis is parallel to both the H.P. and the V.P., neither the top view nor the front view will show the actual shape of the base. In this case, the projection of the solid on an auxiliary plane perpendicular to both the planes, viz. the side view must be drawn first. The front view and the top view are then projected from the side view. The projections in such cases may also be drawn in two stages.

(1) Axis perpendicular to the H.P.:

Problem 13-1. (fig. 13-12): *Draw the projections of a triangular prism, base 40 mm side and axis 50 mm long, resting on one of its bases on the H.P. with a vertical face perpendicular to the V.P.*

(i) As the axis is perpendicular to the ground i.e. the H.P. begin with the top view. It will be an equilateral triangle of sides 40 mm long, with one of its

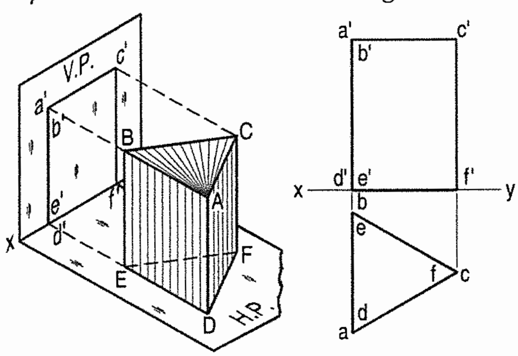


FIG. 13-12

sides perpendicular to *xy*. Name the corners as shown, thus completing the top view. The corners *d*, *e* and *f* are hidden and coincide with the top corners *a*, *b* and *c* respectively.

(ii) Project the front view, which will be a rectangle. Name the corners. The line *b'e'* coincides with *a'd'*.

Problem 13-2. (fig. 13-13): *Draw the projections of a pentagonal pyramid, base 30 mm edge and axis 50 mm long, having its base on the H.P. and an edge of the base parallel to the V.P. Also draw its side view.*

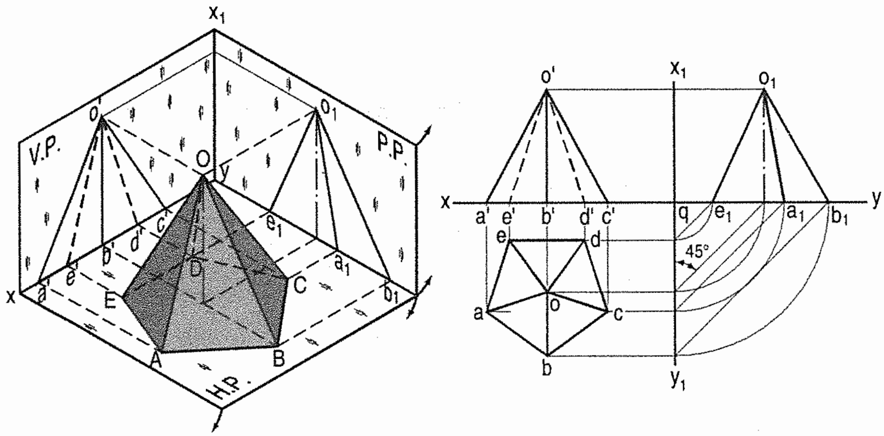


FIG. 13-13

(i) Assume the side *DE* which is nearer the V.P., to be parallel to the V.P. as shown in the pictorial view.

(ii) In the top view, draw a regular pentagon *abcde* with *ed* parallel to and nearer *xy*. Locate its centre *o* and join it with the corners to indicate the slant edges.

(iii) Through *o*, project the axis in the front view and mark the apex *o'*, 50 mm above *xy*. Project all the corners of the base on *xy*. Draw lines *o'a'*, *o'b'* and *o'c'* to show the visible edges. Show *o'd'* and *o'e'* for the hidden edges as dashed lines.

(iv) For the side view looking from the left, draw a new reference line x_1y_1 perpendicular to *xy* and to the right of the front view. Project the side view on it, horizontally from the front view as shown. The respective distances of all the points in the side view from x_1y_1, should be equal to their distances in the top view from *xy*. This is done systematically as explained below:

(v) From each point in the top view, draw horizontal lines upto x_1y_1. Then draw lines inclined at 45° to x_1y_1 (or *xy*) as shown. Or, with *q*, the point of intersection between *xy* and x_1y_1 as centre, draw quarter circles. Project up all the points to intersect the corresponding horizontal lines from the front view and complete the side view as shown in the figure. Lines o_1d_1 and o_1c_1 coincide with o_1e_1 and o_1a_1 respectively.

Problem 13-3. (fig. 13-14): *Draw the projections of (i) a cylinder, base 40 mm diameter and axis 50 mm long, and (ii) a cone, base 40 mm diameter and axis 50 mm long, resting on the H.P. on their respective bases.*

(i) Draw a circle of 40 mm diameter in the top view and project the front view which will be a rectangle [fig. 13-14(ii)].

(ii) Draw the top view [fig. 13-14(iii)]. Through the centre o, project the apex o', 50 mm above *xy*. Complete the triangle in the front view as shown.

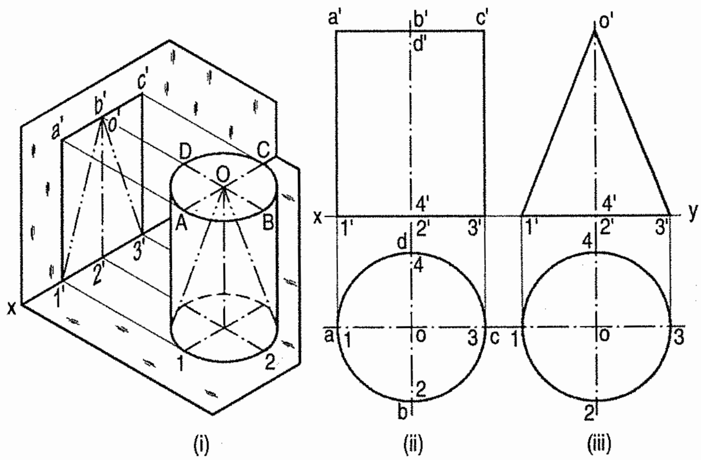


FIG. 13-14

In the pictorial view [fig. 13-14(i)], the cone is shown as contained by the cylinder.

Problem 13-4. (fig. 13-15): *A cube of 50 mm long edges is resting on the H.P. with its vertical faces equally inclined to the V.P. Draw its projections.*

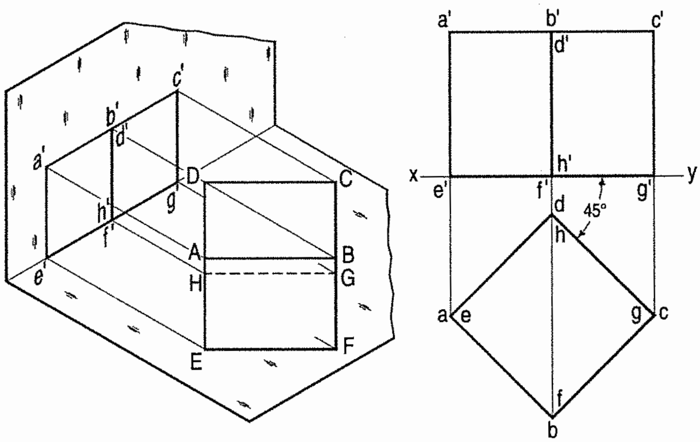


FIG. 13-15

Begin with the top view.

(i) Draw a square *abcd* with a side making 45° angle with *xy*.

(ii) Project up the front view. The line *d' h'* will coincide with *b' f'*.

Problem 13-5. (fig. 13-16): *Draw the projections of a hexagonal pyramid, base 30 mm side and axis 60 mm long, having its base on the H.P. and one of the edges of the base inclined at 45° to the V.P.*

(i) In the top view, draw a line *af* 30 mm long and inclined at 45° to *xy*. Construct a regular hexagon on *af*. Mark its centre *o* and complete the top view by drawing lines joining it with the corners.

(ii) Project up the front view as described in problem 13-2, showing the line *o'e'* and *o'f'* for hidden edges as dashed lines.

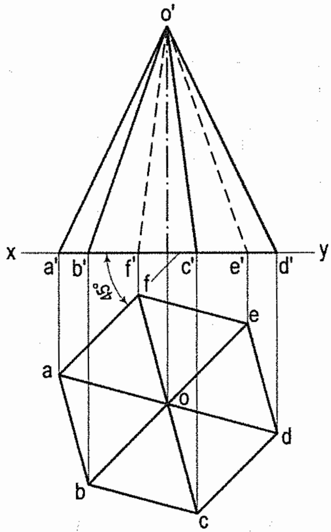


FIG. 13-16

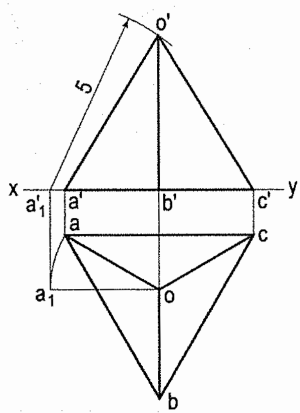


FIG. 13-17

Problem 13-6. (fig. 13-17): *A tetrahedron of 5 cm long edges is resting on the H.P. on one of its faces, with an edge of that face parallel to the V.P. Draw its projections and measure the distance of its apex from the ground.*

All the four faces of the tetrahedron are equal equilateral triangles of 5 cm side.

(i) Draw an equilateral triangle *abc* in the top view with one side, say *ac*, parallel to *xy*. Locate its centre *o* and join it with the corners.

(ii) In the front view, the corners *a'*, *b'* and *c'* will be in *xy*. The apex *o'* will lie on the projector through *o* so that its true distance from the corners of the base is equal to 5 cm.

(iii) To locate *o'*, make *oa* (or *ob* or *oc*) parallel to *xy*. Project a_1 to a'_1 on *xy*. With a'_1 as centre and radius equal to 5 cm cut the projector through *o* in *o'*. Draw lines *o'a'*, *o'b'* and *o'c'* to complete the front view. *o'b'* will be the distance of the apex from the ground.

(2) Axis perpendicular to the V.P.:

Problem 13-7. (fig. 13-18): *A hexagonal prism has one of its rectangular faces parallel to the H.P. Its axis is perpendicular to the V.P. and 3.5 cm above the ground.*

Draw its projections when the nearer end is 2 cm in front of the V.P. Side of base 2.5 cm long; axis 5 cm long.

(i) Begin with the front view. Construct a regular hexagon of 2.5 cm long sides with its centre 3.5 cm above *xy* and one side parallel to it.

(ii) Project down the top view, keeping the line for nearer end, viz. 1-4, 2 cm below *xy*.

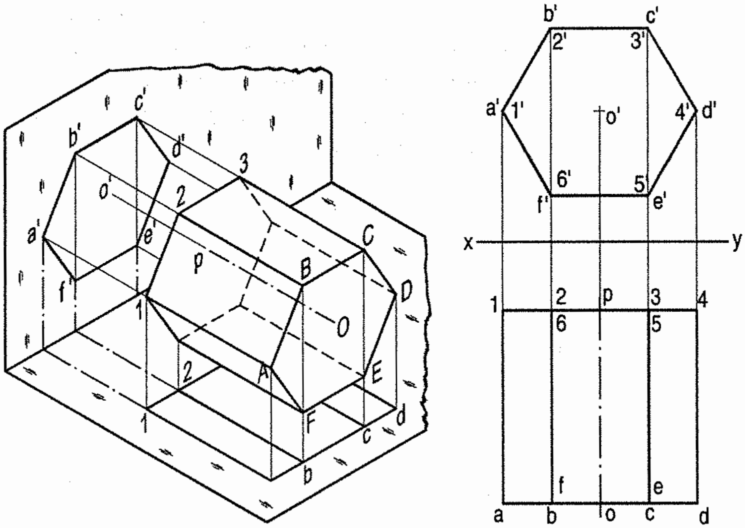


FIG. 13-18

Problem 13-8. (fig. 13-19): *A square pyramid, base 40 mm side and axis 65 mm long, has its base in the V.P. One edge of the base is inclined at 30° to the H.P. and a corner contained by that edge is on the H.P. Draw its projections.*

(i) Draw a square in the front view with the corner *d*' in *xy* and the side *d*'*c*' inclined at 30° to it. Locate the centre *o*' and join it with the corners of the square.

(ii) Project down all the corners in *xy* (because the base is in the V.P.). Mark the apex *o* on a projector through *o*'. Draw lines for the slant edges and complete the top view.

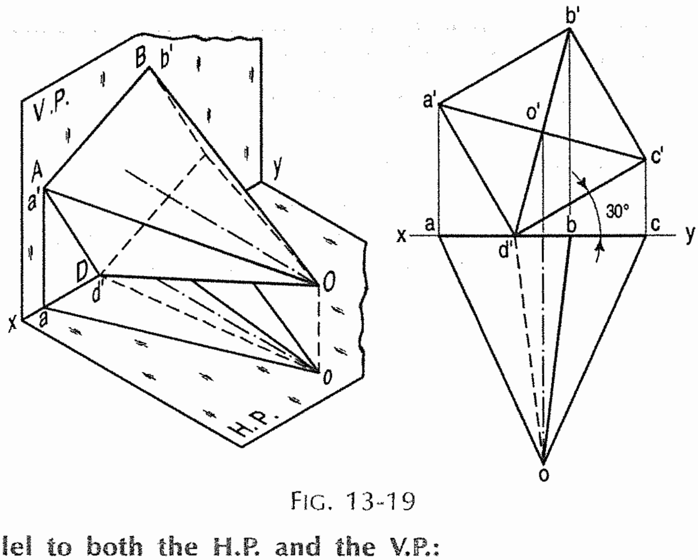


FIG. 13-19

(3) Axis parallel to both the H.P. and the V.P.:

Problem 13-9. (fig. 13-20): *A triangular prism, base 40 mm side and height 65 mm is resting on the H.P. on one of its rectangular faces with the axis parallel to the V.P. Draw its projections.*

As the axis is parallel to both the planes, begin with the side view.

(i) Draw an equilateral triangle representing the side view, with one side in *xy*.

(ii) Project the front view horizontally from this triangle.

(iii) Project down the top view from the front view and the side view, as shown.

This problem can also be solved in two stages as explained in the next article.

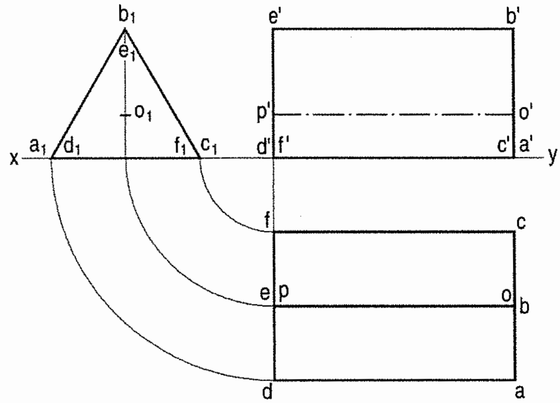


FIG. 13-20

EXERCISES 13(a)

Draw the projections of the following solids, situated in their respective positions, taking a side of the base 40 mm long or the diameter of the base 50 mm long and the axis 65 mm long.

1. A hexagonal pyramid, base on the H.P. and a side of the base parallel to and 25 mm in front of the V.P.
2. A square prism, base on the H.P., a side of the base inclined at 30° to the V.P. and the axis 50 mm in front of the V.P.
3. A triangular pyramid, base on the H.P. and an edge of the base inclined at 45° to the V.P.; the apex 40 mm in front of the V.P.
4. A cylinder, axis perpendicular to the V.P. and 40 mm above the H.P., one end 20 mm in front of the V.P.
5. A pentagonal prism, a rectangular face parallel to and 10 mm above the H.P., axis perpendicular to the V.P. and one base in the V.P.
6. A square pyramid, all edges of the base equally inclined to the H.P. and the axis parallel to and 50 mm away from both the H.P. and the V.P.
7. A cone, apex in the H.P. axis vertical and 40 mm in front of the V.P.
8. A pentagonal pyramid, base in the V.P. and an edge of the base in the H.P.

13-3. PROJECTIONS OF SOLIDS WITH AXES INCLINED TO ONE OF THE REFERENCE PLANES AND PARALLEL TO THE OTHER

When a solid has its axis inclined to one plane and parallel to the other, its projections are drawn in two stages.

(1) In the initial stage, the solid is assumed to be in simple position, i.e. *its axis perpendicular to one of the planes*.

If the axis is to be inclined to the ground, i.e. the H.P., it is assumed to be perpendicular to the H.P. in the initial stage. Similarly, if the axis is to be inclined to the V.P., it is kept perpendicular to the V.P. in the initial stage.

Moreover

(i) if the solid has an edge of its base parallel to the H.P. or in the H.P. or on the ground, that edge should be kept perpendicular to the V.P.; if the edge of the base is parallel to the V.P. or in the V.P., it should be kept perpendicular to the H.P.

(ii) If the solid has a corner of its base in the H.P. or on the ground, the sides of the base containing that corner should be kept equally inclined to the V.P.; if the corner is in the V.P., they should be kept equally inclined to the H.P.

(2) Having drawn the projections of the solid in its simple position, the final projections may be obtained by one of the following *two* methods:

(i) **Alteration of position:** The position of one of the views is altered as required and the other view projected from it.

(ii) **Alteration of reference line or auxiliary plane:** A new reference line is drawn according to the required conditions, to represent an auxiliary plane and the final view projected on it.

In the first method, the reproduction of a view accurately in the altered position is likely to take considerable time, specially, when the solid has curved surfaces or too many edges and corners. In such cases, it is easier and more convenient to adopt the second method. Sufficient care must however be taken in transferring the distances of various points from their respective reference lines.

After determining the positions of all the points for the corners in the final view, difficulty is often felt in completing the view correctly. The following sequence for joining the corners may be adopted:

(a) Draw the lines for the edges of the visible base. The base, which (compared to the other base) is further away from *xy* in one view, will be fully visible in the other view.

(b) Draw the lines for the longer edges. The lines which pass through the figure of the visible base should be dashed lines.

(c) Draw the lines for the edges of the other base.

It should always be remembered that, when two lines representing the edges cross each other, one of them must be hidden and should therefore be drawn as a dashed line.

13-3-1. AXIS INCLINED TO THE V.P. AND PARALLEL TO THE H.P.

CHAROTAR COGNIFRONT

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 29 for the following problem.

Problem 13-10. (fig. 13-21): *Draw the projections of a pentagonal prism, base 25 mm side and axis 50 mm long, resting on one of its rectangular faces on the H.P., with the axis inclined at 45° to the V.P.*

In the simple position, assume the prism to be on one of its faces on the ground with the axis perpendicular to the V.P.

Draw the pentagon in the front view with one side in *xy* and project the top view [fig. 13-21(i)].

The shape and size of the figure in the top view will not change, so long as the prism has its face on the H.P. The respective distances of all the corners in the front view from *xy* will also remain constant.

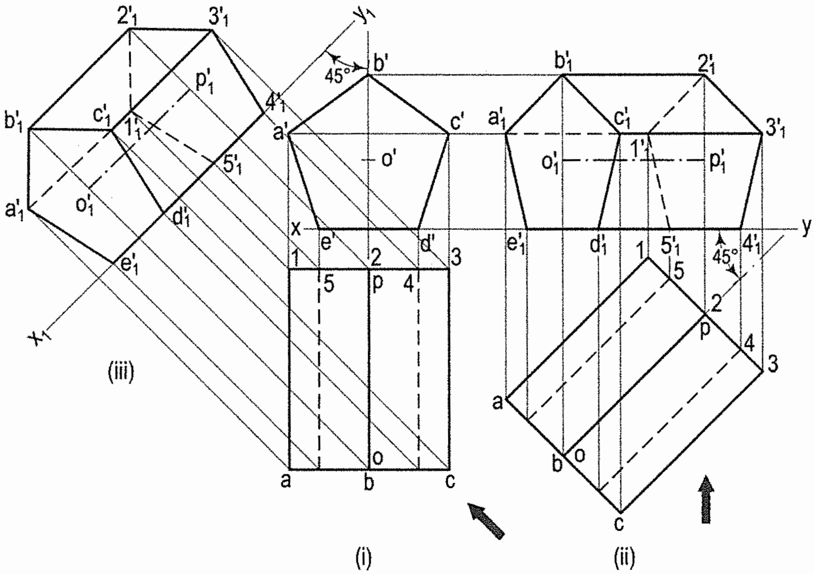


FIG. 13-21

Method I: [fig. 13-21(ii)]:

(i) Alter the position of the top view, i.e. reproduce it so that the axis is inclined at 45° to *xy*. Project all the points upwards from this top view and horizontally from the first front view, e.g. a vertical from *a* intersecting *a* horizontal from *a'* at a point a'_1.

(ii) Complete the pentagon $a'_1b'_1c'_1d'_1e'_1$ for the fully visible end of the prism. Next, draw the lines for the longer edges and finally, draw the lines for the edges of the other end. Note carefully that the lines $a'_11'_1$, $1'_12'_1$ and $1'_15'_1$ are dashed lines. $e'_15'_1$ is also hidden but it coincides with other visible lines.

Method II: [fig. 13-21(iii)]:

(i) Draw a new reference line x_1y_1, making 45° angle with the top view of the axis, to represent an auxiliary vertical plane.

(ii) Draw projectors from all the points in the top view perpendicular to x_1y_1 and on them, mark points keeping the distance of each point from x_1y_1 equal to its distance from *xy* in the front view. Join the points as already explained. The auxiliary front view and the top view are the required projections.

Problem 13-11. (fig. 13-22): *Draw the projections of a cylinder 75 mm diameter and 100 mm long, lying on the ground with its axis inclined at 30° to the V.P. and parallel to the ground.*

Adopt the same methods as in the previous problem. The ellipses for the ends should be joined by common tangents. Note that half of the ellipse for the hidden base will be drawn as dashed line.

Fig. 13-22(iii) shows the front view obtained by the method II.

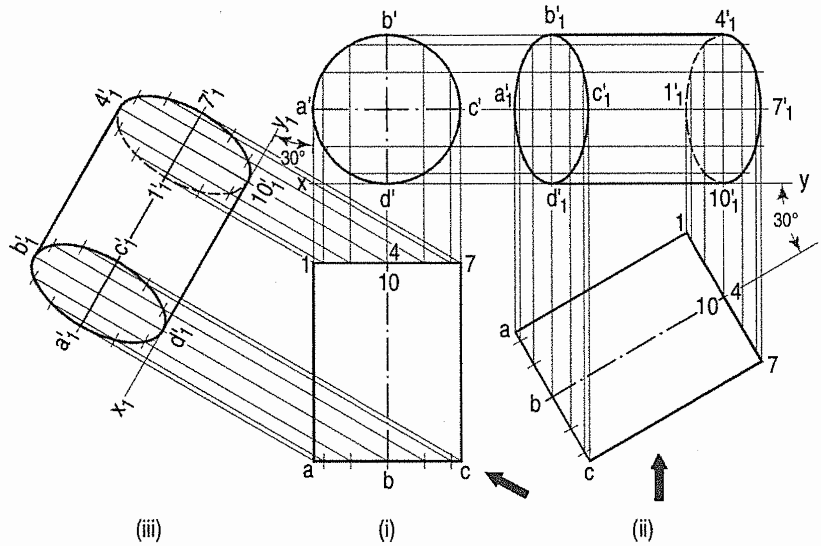


FIG. 13-22

13-3-2. AXIS INCLINED TO THE H.P. AND PARALLEL TO THE V.P.

Problem 13-12. (fig. 13-23): *A hexagonal pyramid, base 25 mm side and axis 50 mm long, has an edge of its base on the ground. Its axis is inclined at 30° to the ground and parallel to the V.P. Draw its projections.*

In the initial position assume the axis to be perpendicular to the H.P.

Draw the projections with the base in *xy* and its one edge perpendicular to the V.P. [fig. 13-23(i)].

If the pyramid is now tilted about the edge *AF* (or *CD*) the axis will become inclined to the H.P. but will remain parallel to the V.P. The distances of all the corners from the V.P. will remain constant.

The front view will not be affected except in its position in relation to *xy*. The new top view will have its corners at same distances from *xy*, as before.

Method I: [fig. 13-23(ii)]:

(i) Reproduce the front view so that the axis makes 30° angle with *xy* and the point *a*' remains in *xy*.

(ii) Project all the points vertically from this front view and horizontally from the first top view. Complete the new top view by drawing (a) lines joining the apex o'_1 with the corners of the base and (b) lines for the edges of the base.

The base will be partly hidden as shown by dashed line a_1b_1, e_1f_1 and f_1a_1. Similarly o_1f_1 and o_1a_1 are also dashed lines.

Method II: [fig. 13-23(iii)]:

(i) Through *a*′ draw a new reference line x_1y_1 inclined at 30° to the axis, to represent an auxiliary inclined plane.

(ii) From the front view project the required top view on x_1y_1, keeping the distance of each point from x_1y_1 equal to the distance of its first top view from *xy*, viz. $e_1q = eb'$ etc.

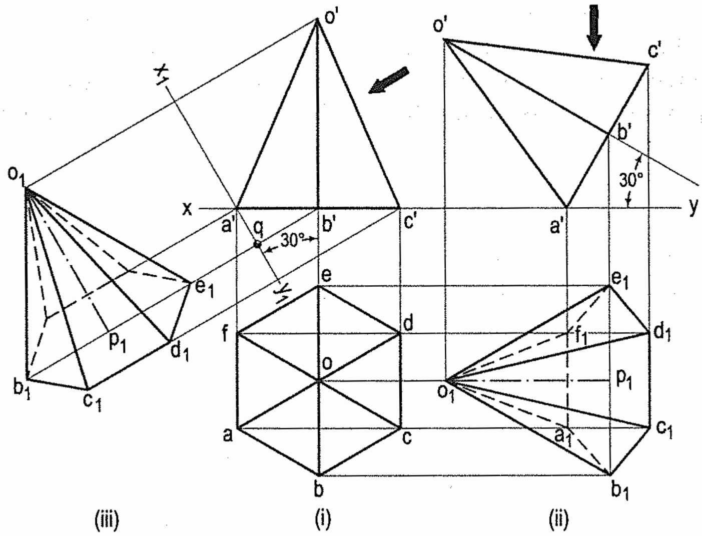


FIG. 13-23

Problem 13-13. (fig. 13-24): *Draw the projections of a cone, base 75 mm diameter and axis 100 mm long, lying on the H.P. on one of its generators with the axis parallel to the V.P.*

(i) Assuming the cone to be resting on its base on the ground, draw its projections.

(ii) Re-draw the front view so that the line $o'7'$ (or $o'1'$) is in xy. Project the required top view as shown. The lines from o_1 should be tangents to the ellipse.

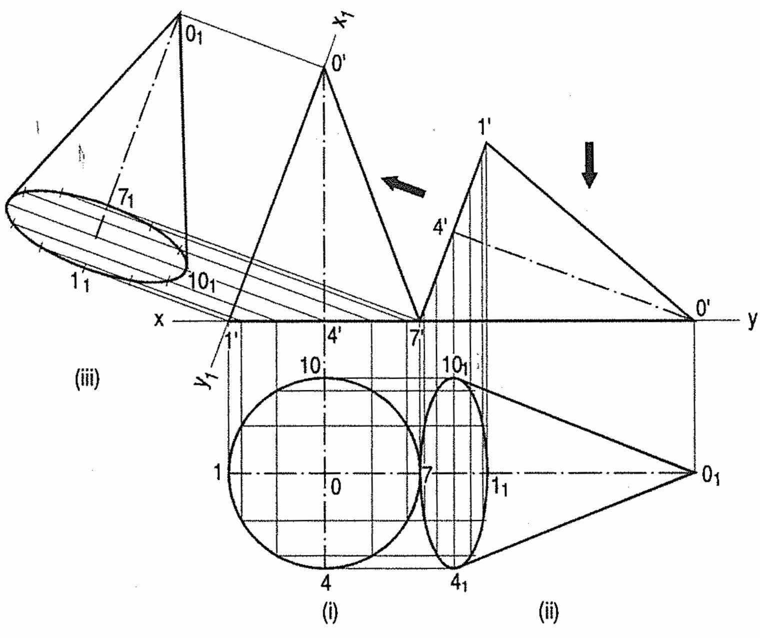


FIG. 13-24

The top view obtained by auxiliary-plane method is shown in fig. 13-24(iii). The new reference line x_1y_1 is so drawn as to contain the generator o'1' instead of o'7' (for sake of convenience). The cone is thus lying on the generator o'1'. Note that $1'1_1$ = 1'1, $o'o_1$ = 4'o etc. Also note that the base is fully visible in both the methods.

Problem 13-14. *The projections of a cylinder resting centrally on a hexagonal prism are given in fig. 13-25(i). Draw its auxiliary front view on a reference line inclined at 60° to xy.*

See fig. 13-25(ii) which is self-explanatory.

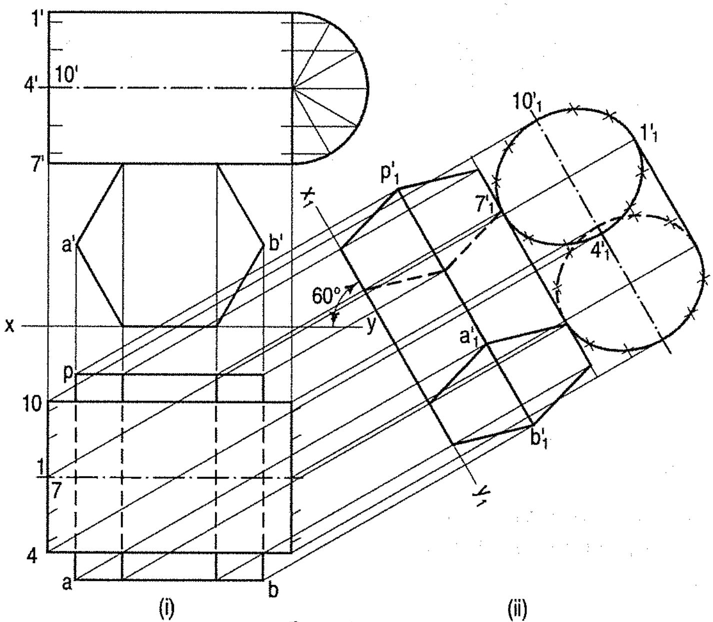


FIG. 13-25

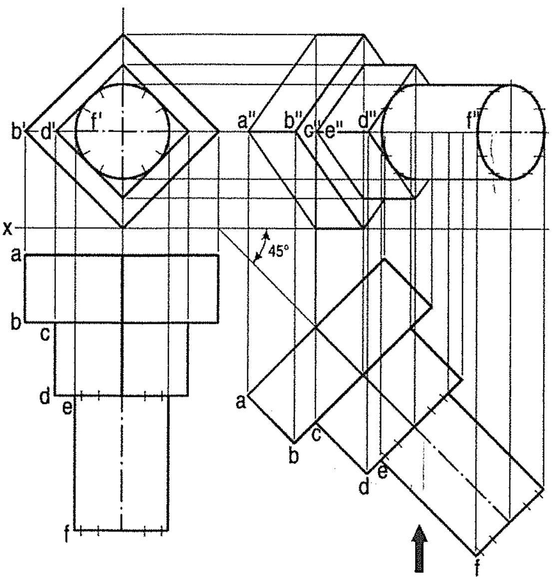


FIG. 13-26

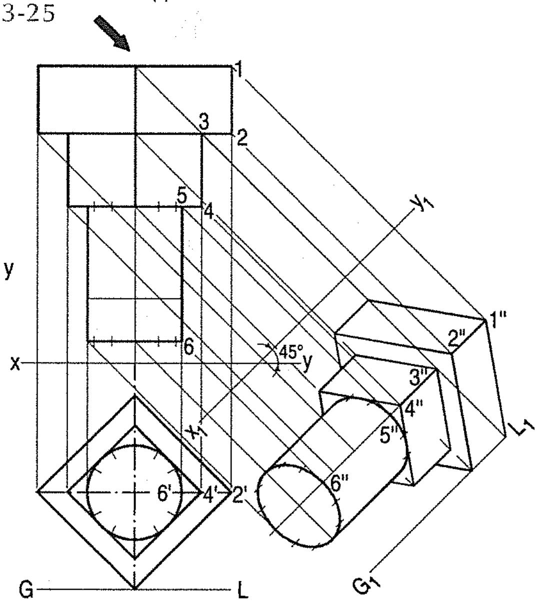


FIG. 13-27

Problem 13-15. (fig. 13-26 and fig. 13-27): *A square-headed bolt 25 mm diameter, 125 mm long and having a square neck has its axis parallel to the H.P. and inclined at 45° to the V.P.*

All the faces of the square head are equally inclined to the H.P. Draw its projections neglecting the threads and chamfer.

See fig. 13-26. The projections are obtained by the change-of-position method. The length of the bolt is taken shorter.

Fig. 13-27 shows the views in third-angle projection, obtained by the auxiliary-plane *method*.

Problem 13-16. (fig. 13-28): *A hexagonal prism, base 40 mm side and height 40 mm has a hole of 40 mm diameter drilled centrally through its ends. Draw its projections when it is resting on one of its corners on the H.P. with its axis inclined at 60° to the H.P. and two of its faces parallel to the V.P.*

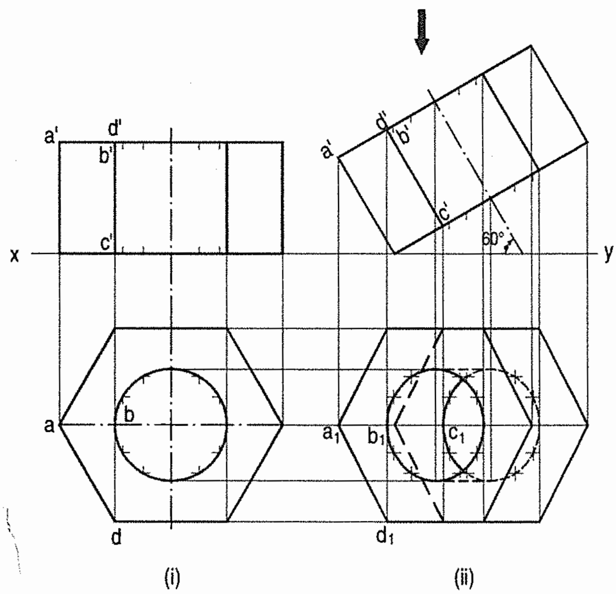


FIG. 13-28

(i) Begin with the top view and project up the front view assuming the axis to be vertical.

(ii) Tilt the front view, and project the required top view. Note that a part of the ellipse for the lower end of the hole will be visible.

Problem 13-17. (fig. 13-29): *The projections of a hopper made of tin sheet are given. Project another top view on an auxiliary inclined plane making 45° angle with the H.P.*

(i) Draw a new reference line x_1y_1 inclined at 45° to xy and project the required top view on it, from the front view.

(ii) Show carefully, the visible ellipses for the outer as well as the inner parts of the hopper rings.

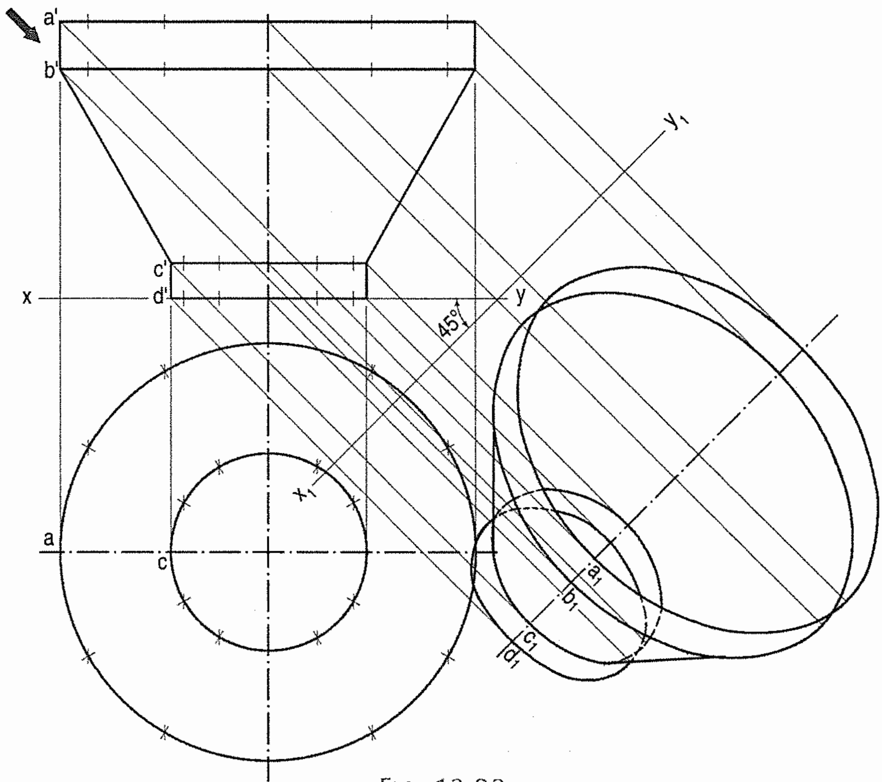


FIG. 13-29

13-4. PROJECTIONS OF SOLIDS WITH AXES INCLINED TO BOTH THE H.P. AND THE V.P.

The projections of a solid with its axis inclined to both the planes are drawn in three stages:

(i) Simple position

(ii) Axis inclined to one plane and parallel to the other

(iii) Final position.

The second and final positions may be obtained either by the alteration of the positions of the solid, i.e. the views, or by the alteration of reference lines.

Problem 13-18. *A square prism, base 40 mm side and height 65 mm, has its axis inclined at 45° to the H.P. and has an edge of its base, on the H.P and inclined at 30° to the V.P. Draw its projections.*

Method I: (fig. 13-30):

(i) Assuming the prism to be resting on its base on the ground with an edge of the base perpendicular to the V.P., draw its projections.

Assume the prism to be tilted about the edge which is perpendicular to the V.P., so that the axis makes 45° angle with the H.P.

(ii) Hence, change the position of the front view so that the axis is inclined at 45° to *xy* and *f'* (or *e'*) is in *xy*. Project the second top view.

Again, assume the prism to be turned so that the edge on which it rests, makes an angle of 30° with the V.P., keeping the inclination of the axis with the ground constant. The shape and size of the second top view will remain the same; only its position will change. In the front view, the distances of all the corners from xy will remain the same as in the second front view.

(iii) Therefore, reproduce the second top view making f_1g_1 inclined at 30° to xy. Project the final front view upwards from this top view and horizontally from the second front view, e.g. a vertical from a_1 and a horizontal from a' intersecting at a'_1. As the top end is further away from xy in the top view it will be fully visible in the front view. Complete the front view showing the hidden edges by dashed lines.

(iv) The second top view may be turned in the opposite direction as shown. In this position, the lower end of the prism, viz. $e'_1f'_1g'_1h'_1$ will be fully visible in the front view.

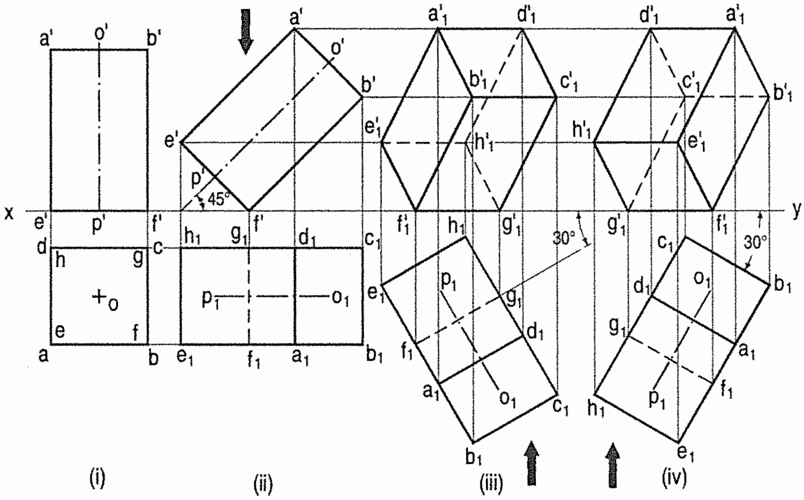


FIG. 13-30

Method II: (fig. 13-31):

(i) Draw the top view and the front view in simple position.

(ii) Through f', draw a new reference line x_1y_1 making 45° angle with the axis. On it, project the auxiliary top view.

(iii) Draw another reference line x_2y_2 inclined at 30° to the line f_1g_1. From the auxiliary top view, project the required front view, keeping the distance of each point from x_2y_2, equal to its distance (in the first front view) from x_1y_1 i.e. $a'_1q_1 = a'q$ etc. The problem is thus solved by change-of-reference line method only.

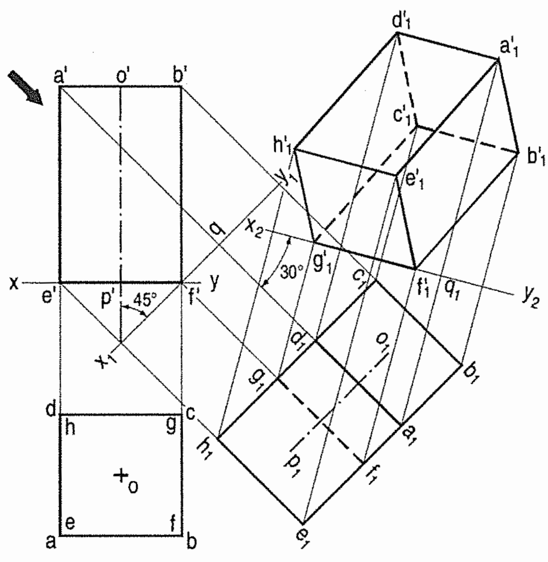


FIG. 13-31

Note: The new reference line satisfying the required conditions may be drawn in various positions, as explained in chapter 11.

Problem 13-19. (fig. 13-32): *Draw the projections of a cone, base 45 mm diameter and axis 50 mm long, when it is resting on the ground on a point on its base circle with (a) the axis making an angle of 30° with the H.P. and 45° with the V.P.; (b) the axis making an angle of 30° with the H.P. and its top view making 45° with the V.P.*

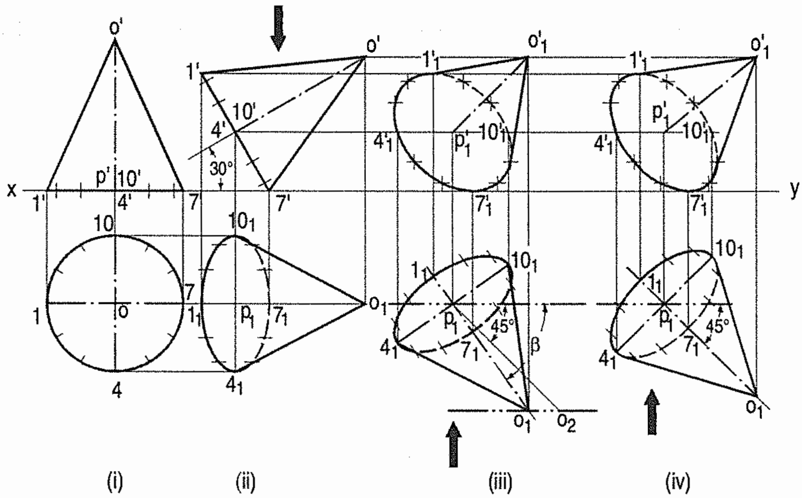


FIG. 13-32

(i) Draw the top view and the front view of the cone with the base on the ground.

(ii) Tilt the front view so that the axis makes 30° angle with *xy*. Project the second top view.

(a) In order that the axis may make an angle of 45° with the V.P., let us determine the apparent angle of inclination which the top view of the axis, viz. o_1p_1 should make with *xy* and which will be greater than 45°.

(iii) Mark any point p_1 below *xy*. Draw a line p_1o_2 equal to the true length of the axis, viz, *o'p'*, and inclined at 45° to *xy*. With p_1 as centre and radius equal to p_1o_1 (the length of the top view of the axis) draw an arc cutting the locus of o_2 at o_1. Then β is the apparent angle of inclination and is greater than 45°. Around p_1o_1 as axis, reproduce the second top view and project the final front view as shown.

Note that the base of the cone is not visible in the front view because it is nearer *xy* in the top view.

(b) When the top view of the axis is to make 45° angle with the V.P., it is evident that p_1o_1 should be inclined at 45° to *xy*. Hence, reproduce the top view accordingly and project the required front view [fig. 13-32(iv)].

Problem 13-20. (fig. 13-33): *A pentagonal pyramid, base 25 mm side and axis 50 mm long has one of its triangular faces in the V.P. and the edge of the base contained by that face makes an angle of 30° with the H.P. Draw its projections.*

(i) In the initial position, assume the pyramid as having its base in the V.P. and an edge of the base perpendicular to the H.P. The front view will have to be drawn first and the top view projected from it.

(ii) Change the position of the top view so that the line o1 (for the face o1 5) is in *xy*. Project the second front view.

(iii) Tilt this front view so that the line $1'_1 5'_1$ makes 30° angle with *xy*. Project the final top view. Note that the base is not visible in the top view as it is nearer *xy* in the front view.

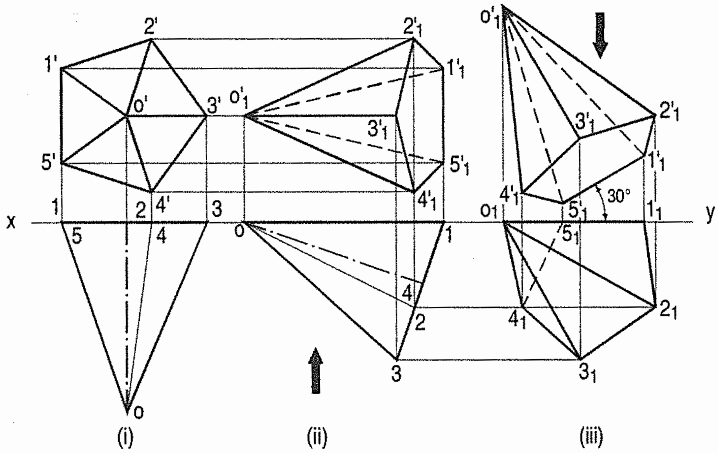


FIG. 13-33

Problem 13-21. (fig. 13-34): *A square pyramid, base 38 mm side and axis 50 mm long, is freely suspended from one of the corners of its base. Draw its projections, when the axis as a vertical plane makes an angle of 45° with the V.P. When a pyramid is suspended freely from a corner of its base, the imaginary line joining that corner with the centre of gravity of the pyramid will be vertical.*

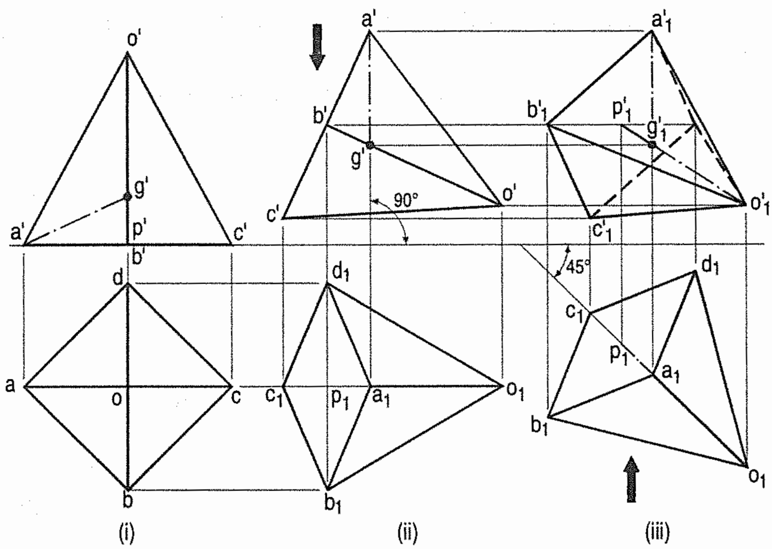


FIG. 13-34

The centre of gravity of a pyramid lies on its axis and at a distance equal to ¼ of the length of the axis from the base.

Assume the pyramid to be suspended from the corner *A* of the base.

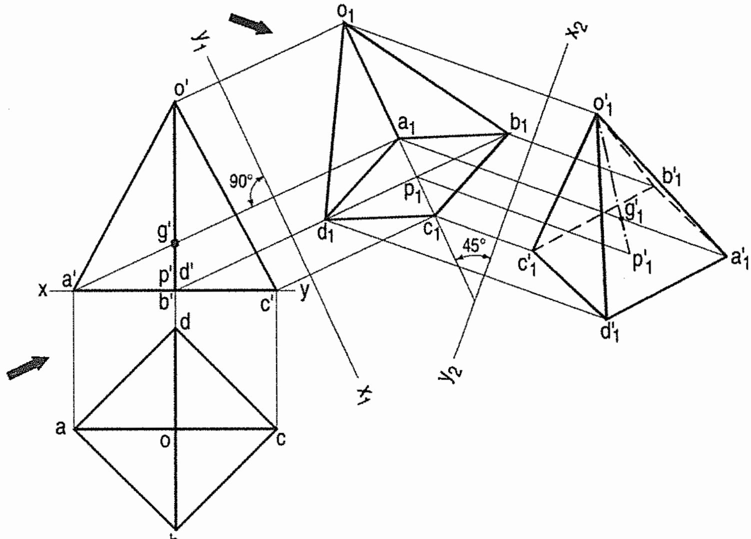


FIG. 13-35

In the initial position, the pyramid should be kept with its base on the ground and the line joining *A* with the centre of gravity *G*, parallel to the V.P. In the top view, *g* will coincide with *o* the top view of the axis.

(i) Draw a square *abcd* (in the top view) with *ag*, i.e. *ao* parallel to *xy*. Project the front view. Making *g*' at a distance equal to ¼ of the axis from *xy*. Join *a*' with *g*'.

(ii) Tilt the front view so that *a'g*' is perpendicular to *xy* and project the top view. The axis will still remain parallel to the V.P.

(iii) Reproduce this top view so that o_1p_1 (the top view of the axis) is inclined at 45° to *xy*. The axis as a vertical plane will thus be making 45° angle with the V.P. Project the final front view.

Fig. 13-35 shows the projections obtained by the change-of-reference-line method.

CHAROTAR COGNIFRONT This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 30 for the following problem.

Problem 13-22. (fig. 13-36): *A hexagonal pyramid, base 25 mm side and axis 55 mm long, has one of its slant edges on the ground. A plane containing that edge and the axis is perpendicular to the H.P. and inclined at 45° to the V.P. Draw its projections when the apex is nearer the V.P. than the base.*

Assume the pyramid to be resting on the ground on its base with a slant edge parallel to the V.P.

(i) Draw the top view of the pyramid with a side of the hexagon parallel to *xy*. The lines *ao* and *do* for the slant edges will also be parallel to *xy*. Project the front view.

(ii) Tilt this front view so that *a'o*' or *d'o*' is in *xy*. Project the second top view.

(iii) Draw a new reference line x_1y_1 making 45° angle with o_1d_1 (the top view of the axis) and project the final front view.

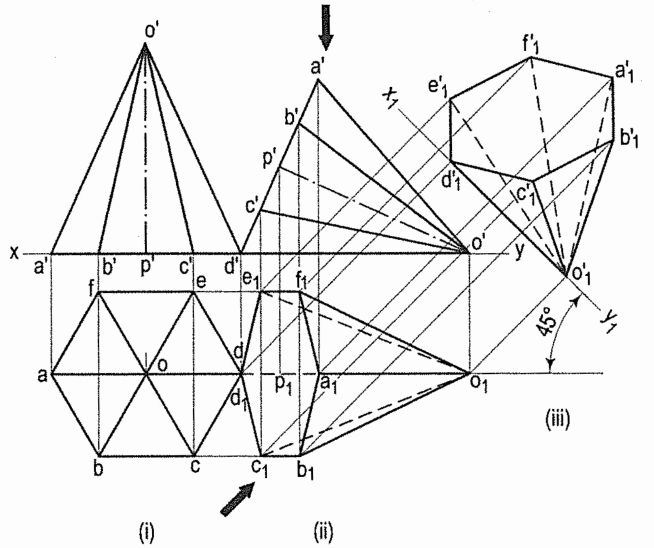


FIG. 13-36

The problem is thus solved by combination of the change-of-position and change-of-reference-line methods.

Problem 13-23. (fig. 13-37): *Draw the projections of a cube of 25 mm long edges resting on the H.P on one of its corners with a solid diagonal perpendicular to the V.P.*

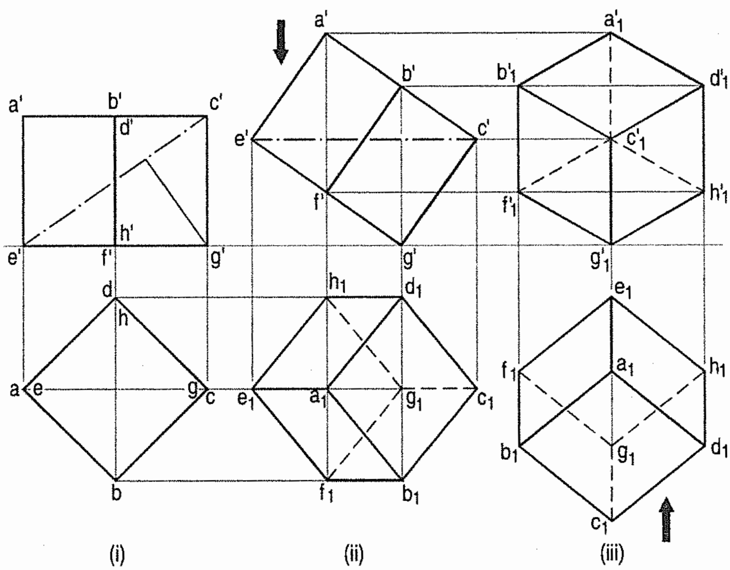


FIG. 13-37

Assume the cube to be resting on one of its faces on the H.P. with a solid diagonal parallel to the V.P.

(i) Draw a square *abcd* in the top view with its sides inclined at 45° to *xy*. The line *ac* representing the solid diagonals *AG* and *CE* is parallel to *xy*. Project the front view.

(ii) Tilt the front view about the corner g' so that the line $e'c'$ becomes parallel to xy. Project the second top view. The solid diagonal CE is now parallel to both the H.P. and the V.P.

(iii) Reproduce the second top view so that the top view of the solid diagonal, viz. e_1c_1 is perpendicular to xy. Project the required front view.

Problem 13-24. (fig. 13-38): *A triangular prism, base 40 mm side and axis 50 mm long, is lying on the H.P. on one of its rectangular faces with the axis perpendicular to the V.P. A cone, base 40 mm diameter and axis 50 mm long, is resting on the H.P. and is leaning centrally on a face of the prism, with its axis parallel to the V.P. Draw the projections of the solids and project another front view on a reference line making 60° angle with xy.*

It will first be necessary to draw the cone with its base on the H.P. to determine the length of its generator and to project the top view.

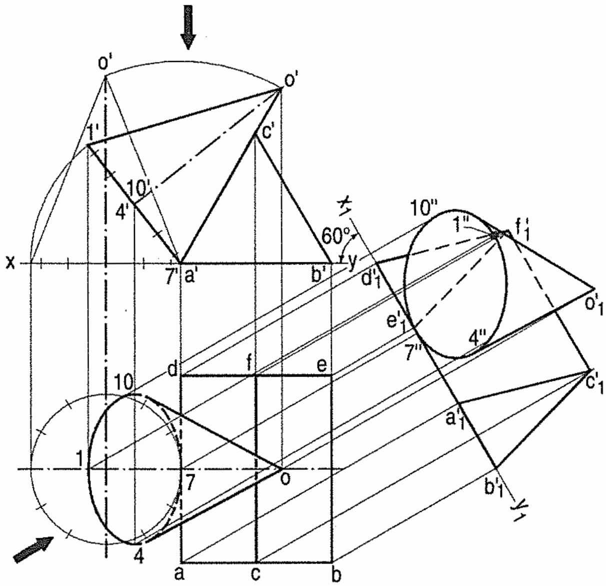


FIG. 13-38

Next, draw a triangle $a'b'c'$ for the prism and a triangle $o'1'7'$ for the cone as shown by the construction lines. Project the top view. Draw a reference line x_1y_1 and project the required front view as shown.

Problem 13-25. *A pentagonal prism is resting on one of the corners of its base on the H.P. The longer edge containing that corner is inclined at 45° to the H.P. The axis of the prism makes an angle of 30° to the V.P. Draw the projections of the solid.*

Also, draw the projections of the solid when the top view of axis is inclined at 30° to xy. Take the side of base 45 mm and height 70 mm.

(i) Assuming the prism to be resting on its base on the horizontal plane, draw its projections keeping one of the sides of its base perpendicular to xy.

(ii) Redraw the front view so that the edge $c'3'$ is inclined at 45° to xy. Project the required top view as shown in fig. 13-39(i).

(iii) Determine the apparent angle of inclination which the top view of the axis should make with xy when the axis makes an angle of 30° with the V.P.

(iv) Mark any point p_1 below xy. Draw a line p_1o_2 equal to the true length of the axis (70 mm) and inclined at 30° to xy. With p_1 as centre and radius equal to p_1o_1 (the length of the top view of the axis) draw an arc cutting the locus of o_2 at o_1. Then β is the required apparent angle of inclination. Considering p_1o_1 as axis, reproduce the second top view and project the final front view as shown in fig. 13-39(i).

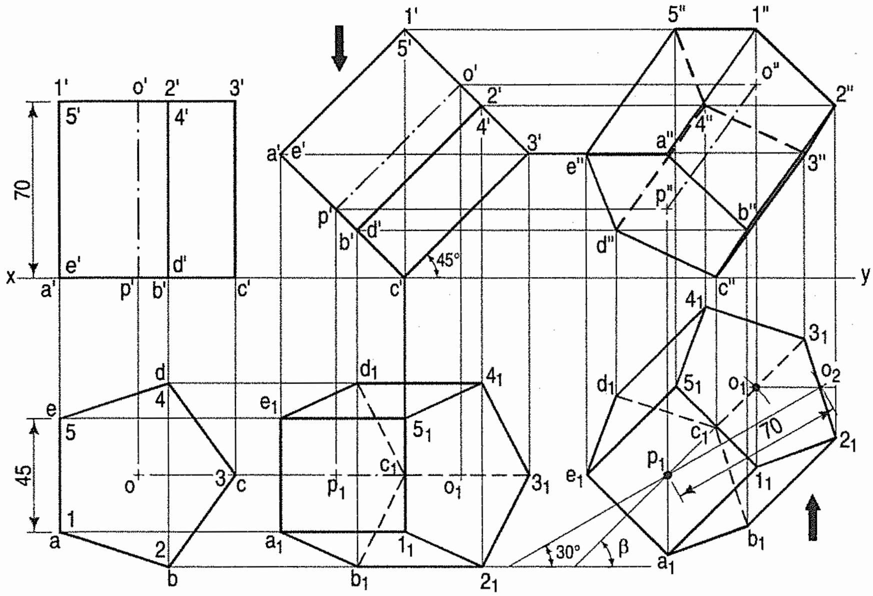


FIG. 13-39(i)

(v) When the top view of axis makes an angle of 30° with the V.P., it is evident that p_1o_1 is inclined at 30° to xy. Hence, reproduce the top view and the front view as shown in fig. 13-39(ii).

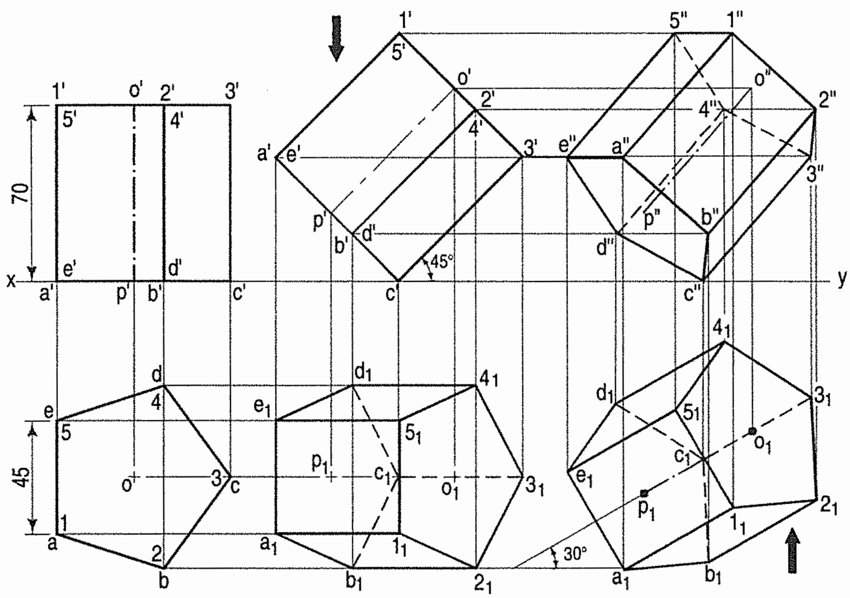


FIG. 13-39(ii)

Problem 13-26 (fig 13-40)**:** *A square prism, with the side of its base 40 mm and axis 70 mm long is lying on one of its base edges on the H.P. in such a way that this base edge makes an angle of 45° with the V.P. and the axis is inclined at 30° to the H.P. Draw the projections of the solid using the 'auxiliary plane method'.*

(i) In the initial position assume the axis of the prism to be perpendicular to the H.P. Draw the projections as shown.

(ii) Draw a new reference line x_1y_1 making an angle of 30° with the front view of the axis, to represent an auxiliary horizontal plane. Draw projectors from *a*', *b*' *c*', *d*' and 1', 2', 3', 4' perpendicular to x_1y_1 and on them, mark these points keeping the distance of each point from x_1y_1 equal to its distance from *xy* in the top view. Join the points as shown.

(iii) Draw another reference line x_2y_2 inclined at 45° to the line *a*1 (or *b*2). From the auxiliary top view, project the required new front view, keeping the distance of each point from x_2y_2, equal to its distance from x_1y_1, i.e. *q*'1" = *q*1' etc. Join the points as shown. Note that the view is obtained by observing the auxiliary top view from the top, along the projectors.

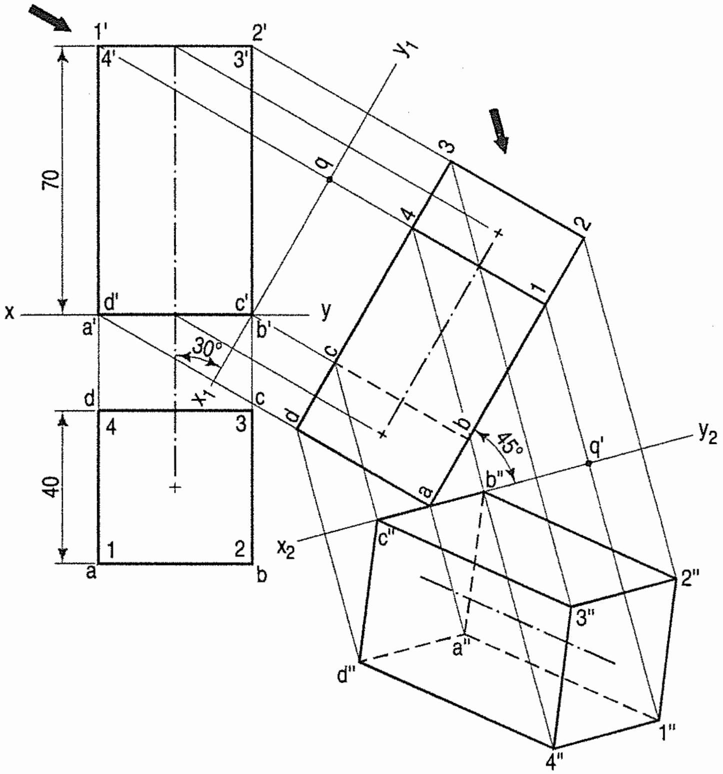


FIG. 13-40

Problem 13-27. *A hexagonal prism, with the side of the hexagon 30 mm and height of 70 mm is resting on the H.P. on one of the edges of its hexagonal base in such a way that, the edge is at 60° to the V.P. and the base is at 30° to the H.P. Draw to scale 1:1, the view from the front and the view from the top.*

Refer to fig. 13-41.

(i) Draw the top view and the front view in simple position keeping the axis perpendicular to the H.P.

(ii) Draw a new reference line x_1y_1 making 60° angle with the axis. On it, project the auxiliary top view.

(iii) Draw another reference line x_2y_2 inclined at 60° to the edge of base c_1d_1. From the auxiliary top view, project the required new front view, keeping the distance of each point from x_2y_2, equal to its distance from x_1y_1 i.e. $q3' = q'3''$ etc. Join the points as shown. It should be noted that the edge of base away from x_2y_2 will be observed as full lines and nearest lines from x_2y_2 will be dotted lines. i.e. $c''d''$, $d''e''$ and $e''f''$ are full lines while $f''a''$, $a''b''$ and $b''c''$ are dotted lines. Note that the view is drawn by observing the auxiliary top view from the top along the projectors.

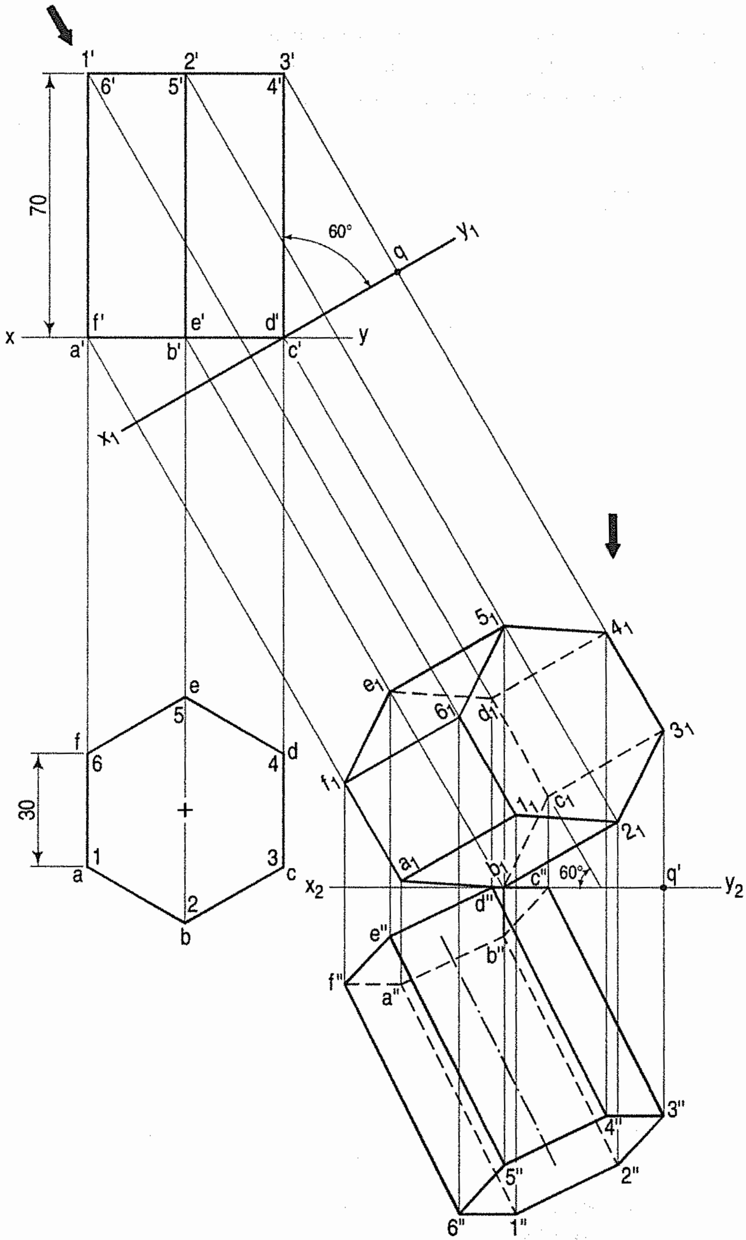


FIG. 13-41

Problem 13-28. *A regular pentagonal prism lies with its axis inclined at 60° to the H.P. and 30° to the V.P. The prism is 60 mm long and has a face width of 25 mm. The nearest corner is 10 mm away from the V.P. and the farthest shorter edge is 100 mm from the H.P. Draw the projections of the solid.*

(i) Draw initial position of the prism as shown in fig. 13-42.

(ii) With 4' as centre and radius equal to 100 mm, draw an arc. Mark tangent to the arc making 60° with the axis as shown. This is a new reference line x_1y_1. Project the required new top view.

(iii) Draw another reference line x_2y_2 inclined at 30° angle to the axis of new top view. Project the various points to obtain new front view as shown in fig. 13-42. Observe the auxiliary top view from the top along the projectors.

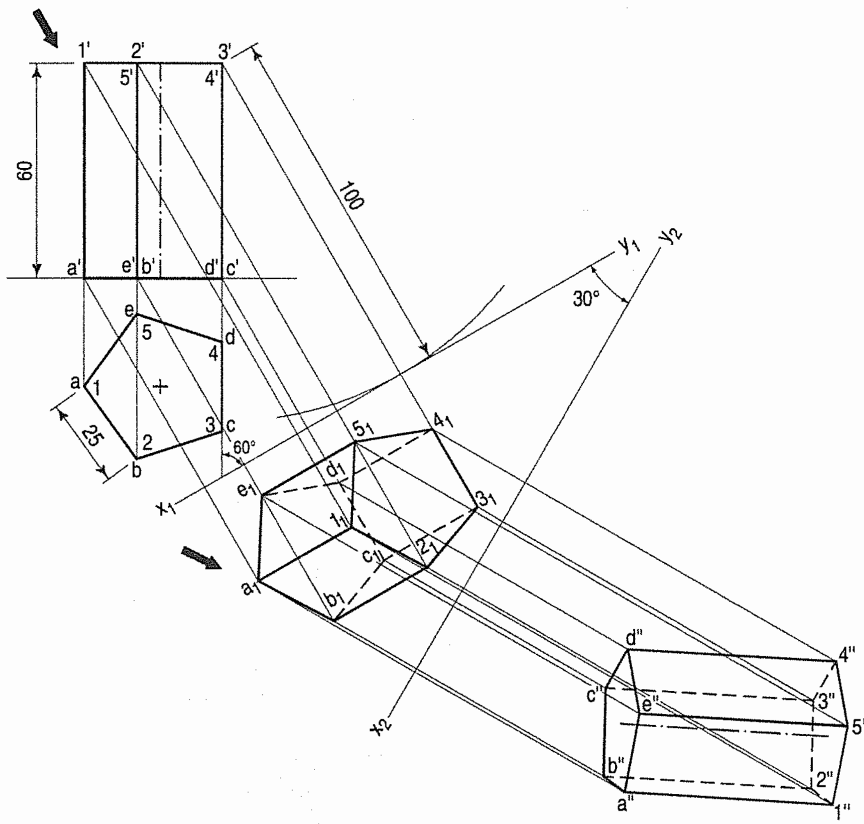


FIG. 13-42

Problem 13-29. *A square pyramid of 50 mm side of base and 50 mm length of axis is resting on one of its triangular faces on the H.P. having a slant edge containing that face parallel to the V.P. Draw the projections of the pyramid.*

(i) Assuming the axis of pyramid perpendicular to the H.P., draw the front view and the top view as shown in fig. 13-43.

(ii) Draw new reference line x_1y_1 coinciding with o'c' in the front view. Project new top view, keeping the distance of a_1, b_1...o_1 from x_1y_1 equal to the distance of *a*, *b*,...*o* from *xy*. Join these points.

(iii) Draw another reference line x_2y_2 parallel to the slant edge o_1c_1 or o_1b_1. Project new front view as shown. Observe auxiliary top view from the base a, b, c, d, o along projectors.

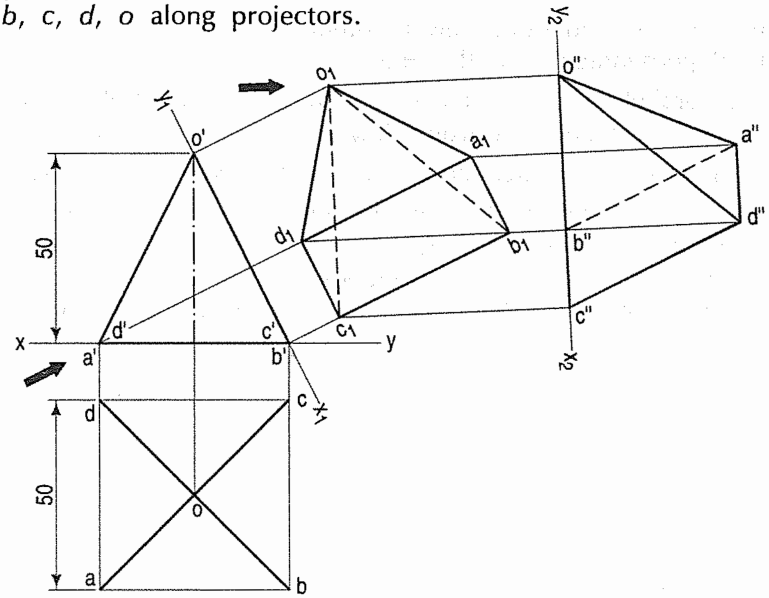


FIG. 13-43

Problem 13-30. *A regular pentagonal pyramid, base 30 mm side and height 80 mm rests on one edge of its base on the ground so that the highest point in the base is 30 mm above the ground. Draw its projection when the axis is parallel to the V.P.*

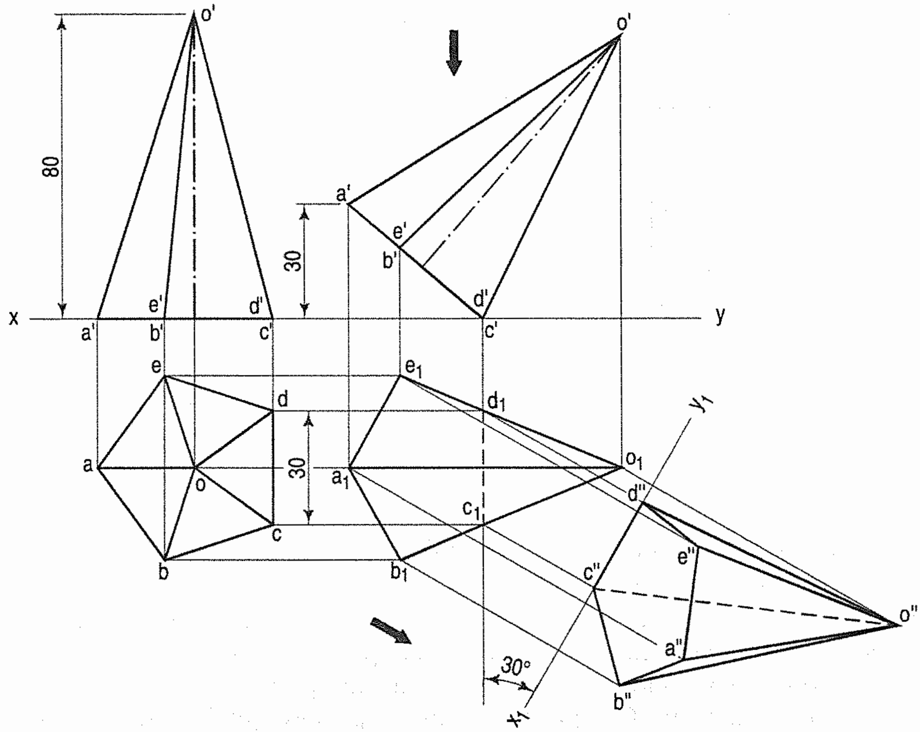


FIG. 13-44

Draw another front view on a reference line inclined at 30° to the edge on which it is resting so that the base is visible.

(i) Draw top view and front view in simple position assuming the axis of the pyramid perpendicular to the H.P.

(ii) Draw a parallel line at a distance of 30 mm from *xy*. Mark the point c' on the line *xy* and reproduce the front view as shown fig. 13-44.

(iii) Project points *a*', *b*', c' etc. and obtain new top view keeping distance of points a_1, b_1, c_1 etc. from *xy* equal to distance of *a*, *b*, *c*, etc. from the line *xy*.

(iv) Draw another reference line x_1y_1 making an angle of 30° with the side of base c_1d_1 and obtain a new front view as shown. Note that the base is visible. Observe from the base *a*, *b*, *c*, *d*, *e* along the projectors.

Problem 13-31. *A regular pentagonal pyramid with the sides of its base 30 mm and height 80 mm rests on an edge of the base. The base is tilted until its apex is 50 mm above the level of the edge of the base on which it rests. Draw the projection of the pyramid when the edge on which it rests, is parallel to the V.P. and the apex of the pyramid points towards V.P.*

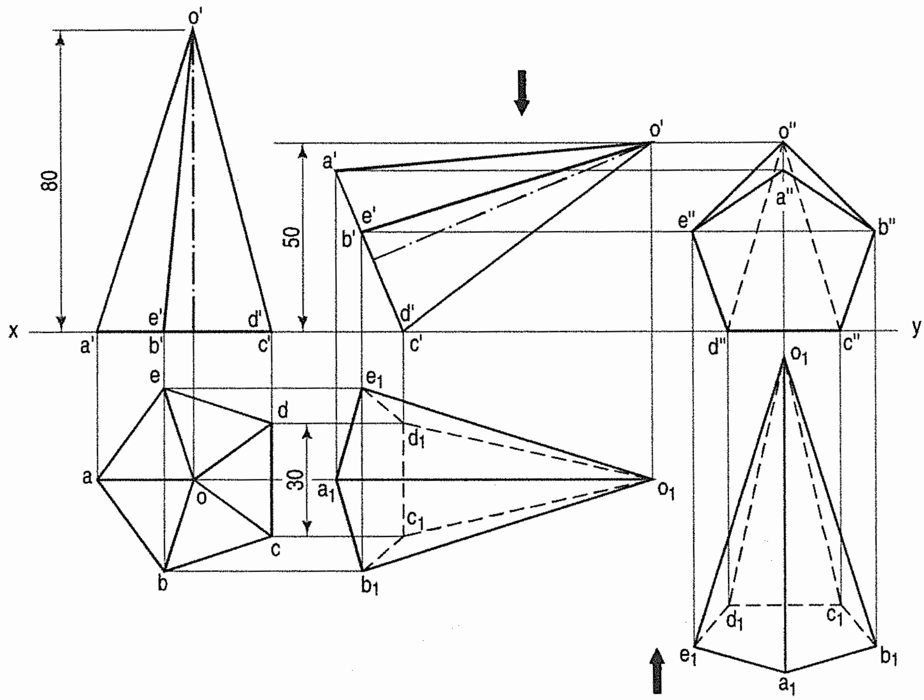


FIG. 13-45

(i) Draw top view and front view assuming the axis of the pyramid perpendicular to the H.P. as shown in fig. 13-45.

(ii) Draw a parallel line at a distance of 50 mm from *xy*. Reproduce the front view as shown. Draw projectors from points *a*', *b*', c' etc. vertically from the front view and horizontally from the points *a*, *b*, *c* etc. from the previous top view. Complete the new top view, joining the intersection of the projectors in the correct sequence as shown.

(iii) Redraw the top view keeping c_1d_1 parallel to xy. Project the points a_1, b_1, c_1 etc. vertically from the new top view and horizontal projectors from the points a', b', c' etc. of the front view. Join the intersection points of both the projectors in the correct sequence as shown.

Problem 13-32. *A right regular pentagonal pyramid, with the sides of the base 30 mm and height 65 mm rests on the edge of its base on the horizontal plane, the base being tilted until the vertex is at 60 mm above the H.P. Draw the projections of the pyramid when the edge on which it rests, is made parallel to FRP. Assuming the pyramid to be resting on its base on the horizontal plane, draw its projections keeping one of the sides of the base perpendicular to xy.*

Method I: Changing position of reference line [fig. 13-46(i)]:

(i) With o' as centre and radius equal to 60 mm, draw an arc. Draw the tangent to the arc passing through c' or d'. This is a new reference line x_1y_1. Project the required top view.

(ii) Draw another reference line x_2y_2 parallel to c_1d_1. Project new front view as shown. Observe auxiliary top view from the base a, b, c, d, e along the projectors.

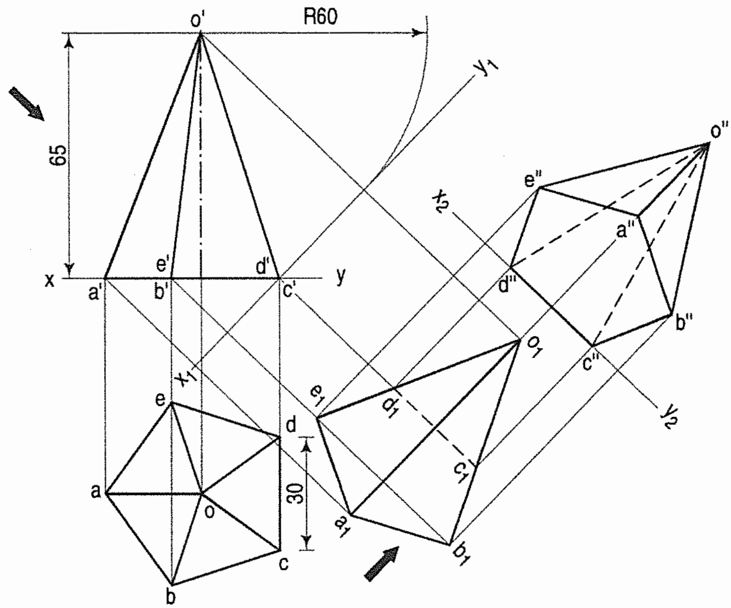


FIG. 13-46(i)

Method II: Changing positions of solid [(fig. 13-46(ii)]:

(i) Draw a line 60 mm parallel to xy. Mark point c' or d' on xy. With c' as centre and the radius equal to o'c', draw an arc cutting the above line at o'. With o' and c' as centre and radius equal to o'c' and a'c' draw an arc cutting each other at the point a'. Join a', o' and c' as shown. Project the required top view as shown.

(ii) Redraw the top view keeping side of base c'd' parallel to xy. Project new front view as shown.

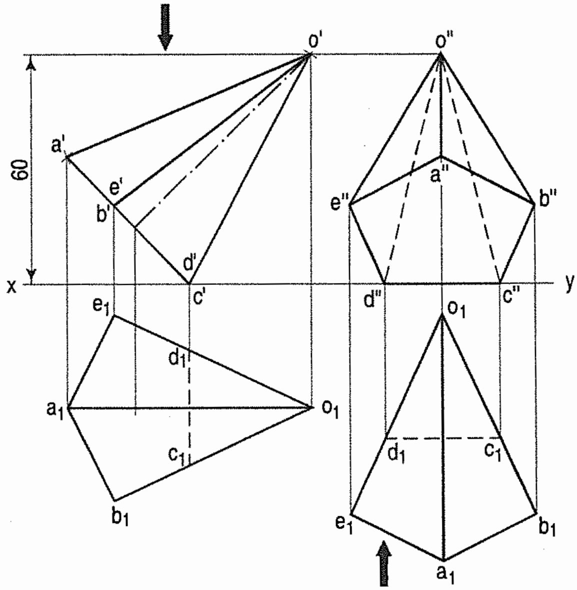


FIG. 13-46(ii)

Problem 13-33. *The front view, part top view and part auxiliary view of a casting are given in fig. 13-47(i). Project its side view.*

See fig. 13-47(ii).

The construction for the ellipse for 38 mm diameter circle has been shown in detail. Horizontal distances are taken from the auxiliary view. Other ellipses are drawn in the same manner.

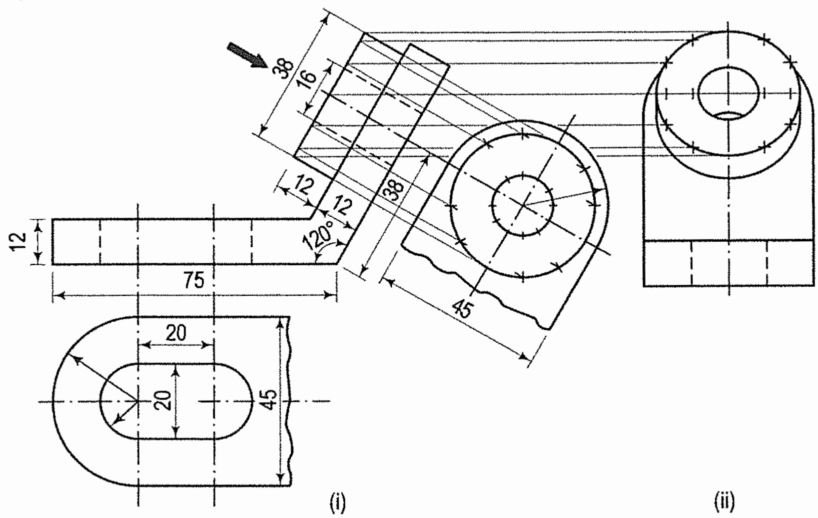


FIG. 13-47

13-5. PROJECTIONS OF SPHERES

The projection of a sphere in any position on any plane is always a circle whose diameter is equal to the diameter of the sphere (fig. 13-48). This circle represents the contour of the sphere.

A flat circular surface is formed when a sphere is cut by a plane. A hemisphere (i.e. a sphere cut by a plane passing through its centre) has a flat circular face of diameter equal to that of the sphere.

When it is placed on the ground on its flat face, its front view is a semi-circle, while its top view is a circle [fig. 13-49(i)].

When the flat face is inclined to the H.P. or the ground and is perpendicular to the V.P. it is seen as an ellipse (partly hidden) in the top view [fig. 13-49(ii)], while the contour of the hemisphere is shown by the arc of the circle drawn with radius equal to that of the sphere.

Fig. 13-50 shows the projections of a sphere, a small portion of which is cut off by a plane. Its flat face is perpendicular to the H.P. and inclined to the V.P. An ellipse is seen in the front view within the circle for the sphere.

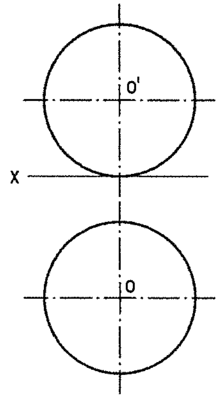


FIG. 13-48

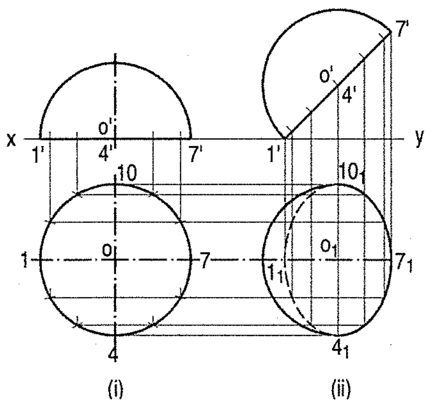


FIG. 13-49

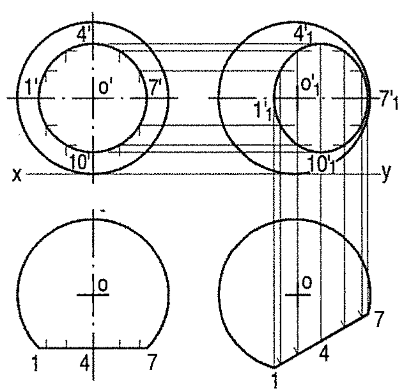


FIG. 13-50

When the flat face of a cut sphere is perpendicular to the V.P. and inclined to the H.P., its projections can be drawn as described in problem 13-34.

Problem 13-34. (fig. 13-51): *A brass flower-vase is spherical in shape with flat, circular top 35 cm diameter and bottom 25 cm diameter and parallel to each other. The greatest diameter is 40 cm. Draw the projections of the vase when its axis is parallel to the V.P. and makes an angle of 60° with the ground.*

(i) Draw the front view of the vase resting on its bottom with its axis vertical. Project the top view.

(ii) Tilt the front view so that the axis makes 60° angle with *xy* and project the top view. Note that a part of the ellipse for the bottom is also visible.

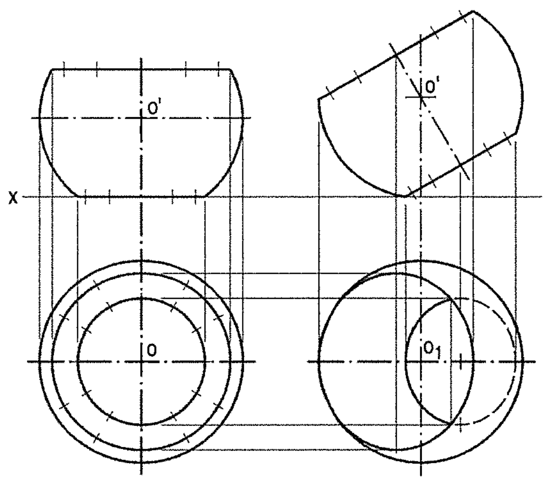


FIG. 13-51

(1) Spheres in contact with each other: Projections of two equal spheres resting on the ground and in contact with each other, with the line joining their centres parallel to the V.P., are shown in fig. 13-52.

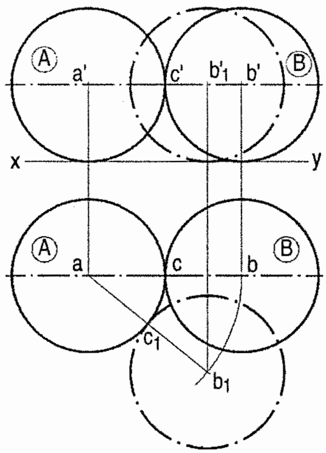


FIG. 13-52

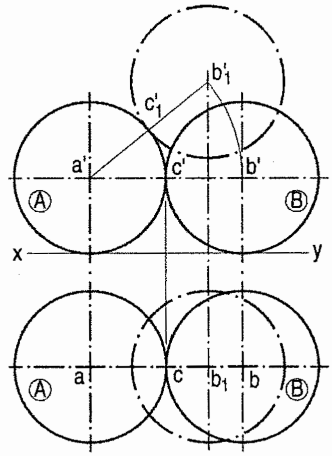


FIG. 13-53

As the spheres are equal in size, the line joining their centres is parallel to the ground also. Hence, both ab and $a'b'$ show the true length of that line (i.e. equal to the sum of the two radii or the diameter of the spheres). The point of contact between the two spheres is also visible in each view.

If the position of one of the spheres, say sphere *B*, is changed so that the line joining their centres is inclined to the V.P., in the front view, the centre b' will move along the line $a'b'$ to b'_1. The true length of the line joining the centres and the point of contact are now seen in the top view only.

When the sphere *B* is so moved that it remains in contact with the sphere *A* and the line joining their centres is parallel to the V.P., but inclined to the ground (fig. 13-53), the true length of that line and the point of contact are visible in the front view only.

Problem 13-35. (fig. 13-54): *Three equal spheres of 38 mm diameter are resting on the ground so that each touches the other two and the line joining the centres of two of them is parallel to the V.P.*

A fourth sphere of 50 mm diameter is placed on top of the three spheres so as to form a pile. Draw three views of the arrangement and find the distance of the centre of the fourth sphere above the ground.

As the spheres are resting on the ground and are equal in size, the lines joining their centres will be parallel to the ground. In the top view, the centres will lie at the corners of an equilateral triangle of sides equal to the sum of the two radii, i.e. 40 mm.

Draw (in the top view) an equilateral triangle *abc* of 40 mm long sides with one side, say *ab*, parallel to *xy*. At its corners, draw three circles of 40 mm diameter. Project the front view. The centres will lie on a line parallel to and 20 mm above *xy*.

When the fourth sphere is placed on top, its centre *d* in the top view will be in the centre of the triangle. In the front view, it will lie on a projector through *d*.

The true distance between the centre of the top sphere and that of any one of the bottom spheres will be equal to the sum of the two radii, viz. 20 mm + 25 mm, i.e. 45 mm.

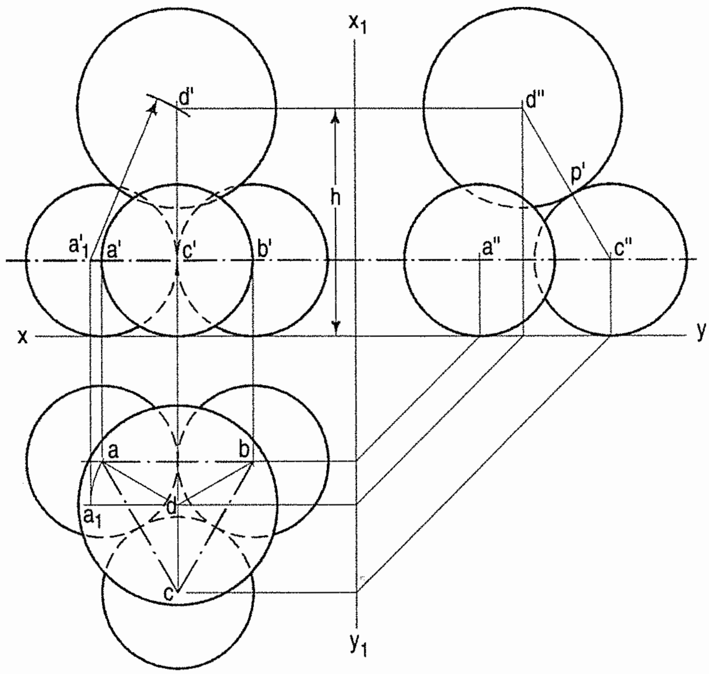


FIG. 13-54

But as none of the lines *da*, *db* or *dc* is parallel to *xy*, their front views will not show their true lengths. Therefore, to locate the position of the centre of the top sphere in the front view,

(i) make one of the lines, say *da*, parallel to *xy*;

(ii) project a_1 to a'_1 on the path of *a'* and

(iii) with a'_1 as centre and radius equal to 45 mm, draw an arc cutting the projector through *d* at the required point *d'*. With *d'* as centre and radius equal to 25 mm, draw the required circle which will be partly hidden as shown. *h* is the distance of the centre of the sphere from the ground.

(iv) Project the side view. As *c'd'* is parallel to the new reference line, *c''d''* will be equal to 45 mm and the point of contact *p'* between the spheres having centres *c* and *d* will be visible.

(2) Unequal spheres: When two unequal spheres are on the ground and are in contact with each other, their point of contact and the true length of the line joining their centres will be seen in the front view if that line is parallel to the V.P. In the top view, the length of the line will be shorter but will remain constant even when it is inclined to the V.P.

Problem 13-36. (fig. 13-55): *Three spheres A, B and C of 75 mm, 50 mm and 30 mm diameters respectively, rest on the ground each touching the other two. Draw their projections and show the three points of contact when the line joining the centres of the spheres A and B is parallel to the V.P.*

(i) With centre *a'* and radius equal to 37.5 mm, draw a circle of sphere *A*, mark *a'* at 37.5 mm above *xy* in front view. With *a'b'* equal to 62.5 mm, mark point *b'* 25 mm above *xy*. With *b'* as centre and radius equal to 25 mm, draw a circle of sphere *B* in front view.

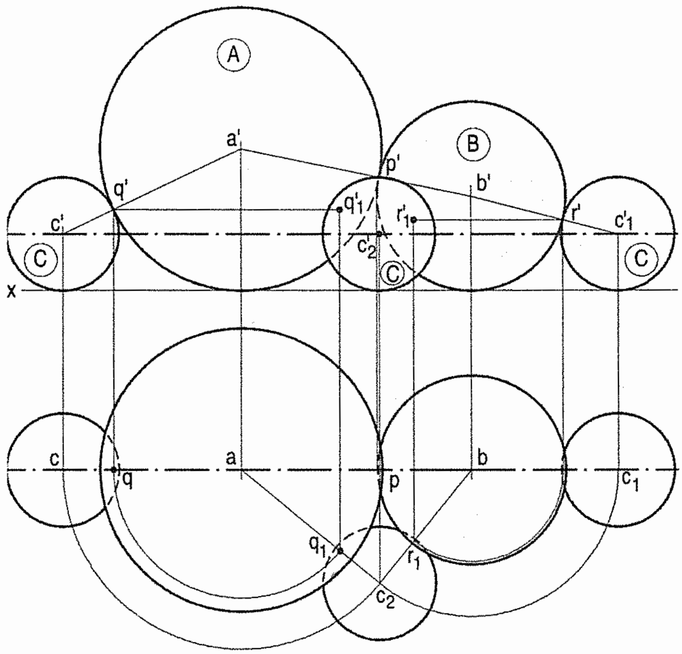


FIG. 13-55

(ii) Project the centres and obtain points *a* and *b* on a line parallel to *xy* in top view. With *a* as centre and radius equal to 37.5 mm of sphere *A*, and with *b* as centre and radius equal to 25 mm of sphere *B*, draw circles in the top view.

(iii) Similarly, draw the views of sphere *C* in contact with spheres *A* and *B*.

(iv) With *a* as centre and radius equal to *ac*, and with *b* as centre and radius equal to bc_1, draw arcs intersecting each other at c_2. With c_2 centre draw top view of the sphere *C*.

(v) Draw the projector through c_2 to cut the path of c' at c'_2. Then c'_2 is the required centre of the sphere *C* in the front view. *p*, q_1 and r_1, and *p'*, q'_1 and r'_1 are the points of contact in the top view and the front view respectively.

Problem 13-37. (fig. 13-56): *A square prism, base 20 mm side and axis 50 mm long, is resting on its base on the ground with two faces perpendicular to the V.P. Determine the radius of four equal spheres resting on the ground, each touching a face of the prism and other two spheres. Draw the projections of the arrangement.*

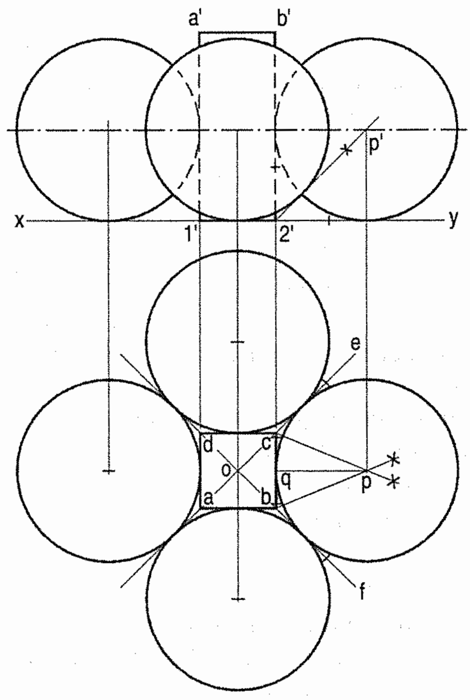


FIG. 13-56

(i) Draw the front and top views of the prism. In the top view, draw diagonals of the square (intersecting each other at *o*) and produce them on both sides.

(ii) Draw the bisectors of angles *bce* and *cbf* intersecting each other at *p*. From *p*, draw a perpendicular *pq* to *bc*. Then *pq* is the required radius of the sphere and *p* is the centre of the circle for the sphere.

(iii) Obtain the other three centres in the same manner. Or, with *o* as centre and radius equal to *op*, draw a circle to cut the centre lines through *o* at the required centres. Draw the four circles.

(iv) Draw a bisector of angle *b*'2'*y* intersecting the projector through *p* at *p*'. Then *p*' is the centre of the sphere in the front view. The centres for the other circles will lie on the horizontal line through *p*'. Project their exact positions from the top view and draw the circles.

Problem 13-38. (fig. 13-57): *Six equal spheres are resting on the ground, each touching other two spheres and a triangular face of a hexagonal pyramid resting on its base on the ground.*

Draw the projections of the solids when a side of the base of the pyramid is perpendicular to the V.P.

Determine the diameter of each sphere. Base of the pyramid 20 mm side; axis 50 mm long.

(i) Draw the projections of the pyramid in the required position. Assuming the solid to be a prism, locate the positions of the centre of one sphere (viz. *p* and *p*') in the two views.

(ii) Draw a line joining *p*' with *a*' (the centre of the base) which coincides with 2'. The centre of the required sphere will lie on this line. Draw a bisector of angle o'3'y cutting *a*'*p*' at *c*'. Draw a line *c*'*q*' perpendicular to *xy*.

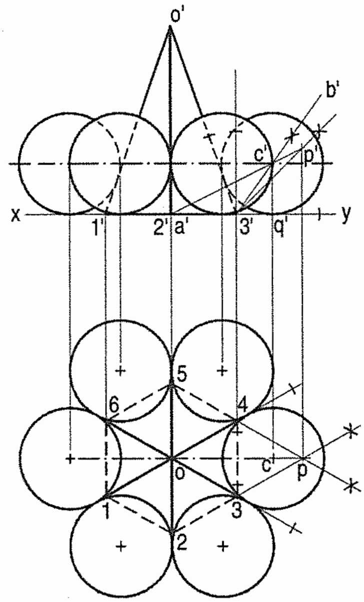


FIG. 13-57

(iii) With c' as centre and radius *c*'*q*', draw one of the required circles. Project *c*' to *c* on *op* in the top view. Then *c* is the centre of the circle in the top view. Other centres may be located in the top view as shown and projected down in the front view.

(iv) Draw the six circles in the top view and four in the front view as shown in the fig. 13-57.

Problem 13-39. *The projections of a paper-weight with a spherical knob are given in fig. 13-58(i). Draw the two views and project another top view when its flat base makes an angle of 60° with the H.P.*

See fig. 13-58(ii) which is self-explanatory.

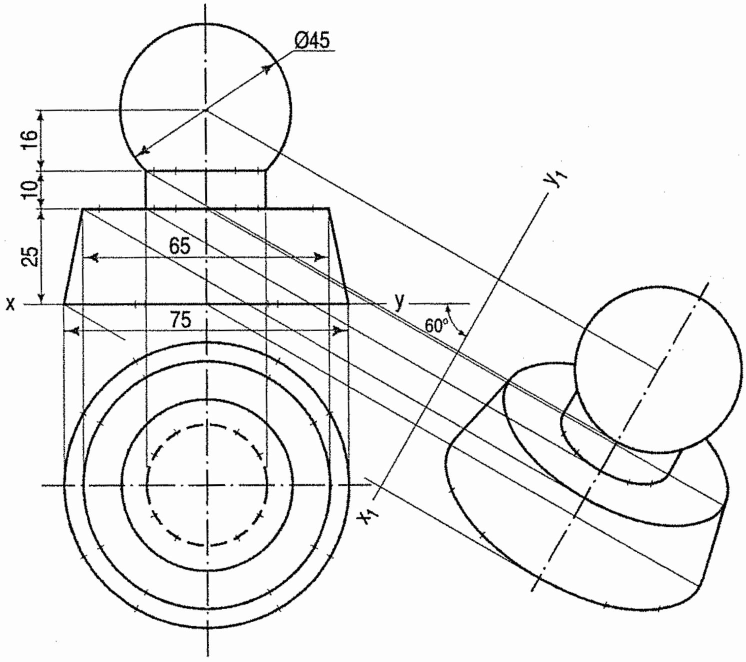


FIG. 13-58

Problem 13-40. (fig. 13-59): *A vertical hexagonal prism of base side 20 mm and thickness 15 mm has one side of hexagon perpendicular to the V.P. A right cone of 34 mm diameter and height 40 mm is placed on the top face of prism such that the base of cone touches top surface of prism while the axes of both coincide. Draw the front view and top view of the combined object. Draw also projections when axes of combined solid is inclined at 35° with auxiliary plane.*

(i) Draw the top view of hexagonal prism keeping one of sides perpendicular to *xy*. (i.e. *ab* or *ed*). Project above *xy* line and draw the front view of prism of height 15 mm.

(ii) Inscribe circle in the top view touching sides of the prism. Project it in the front view and mark the height of the cone as shown.

(iii) Draw auxiliary x_1y_1 inclined at 35° with the axes of the combined solids.

(iv) Draw the projectors from the various points of combined solids in the front view.

(v) Taking distance of various points from the top view of combined solids from *xy* and mark same distances along the respective projectors.

(vi) Complete auxiliary top view as shown.

Problem 13-41. (fig. 13-60): *A vertical cylindrical disc of thickness 10 mm and diameter 50 mm is resting on the ground. A vertical frustum of pentagonal pyramid, having bottom of 20 mm sides, top face of 40 mm sides with 60 mm height is resting on the top surface of the disc so that axes of the both solids coincide. Take one of sides of the base of pentagon is perpendicular to V.P. Draw the projections of combined solid when the axis of combined solids is inclined to 30° with the H.P.*

(i) Draw the top view of frustum of pyramid (pentagon) keeping one of the sides perpendicular to *xy* as shown.

(ii) Project the front view marking height of cylindrical disc and frustum of the pyramid 10 mm and 60 mm respectively.

(iii) Draw a line at angle of 30° with *xy*. (As axis is inclined with H.P., its inclination observed in the front view).

(iv) Reproduce the front view considering inclined line as axes of the combined solids.

(v) Draw the vertical projectors from various points of the front view.

(vi) Draw horizontal projectors to intersect respective vertical projectors. Obtain the auxiliary top view as shown.

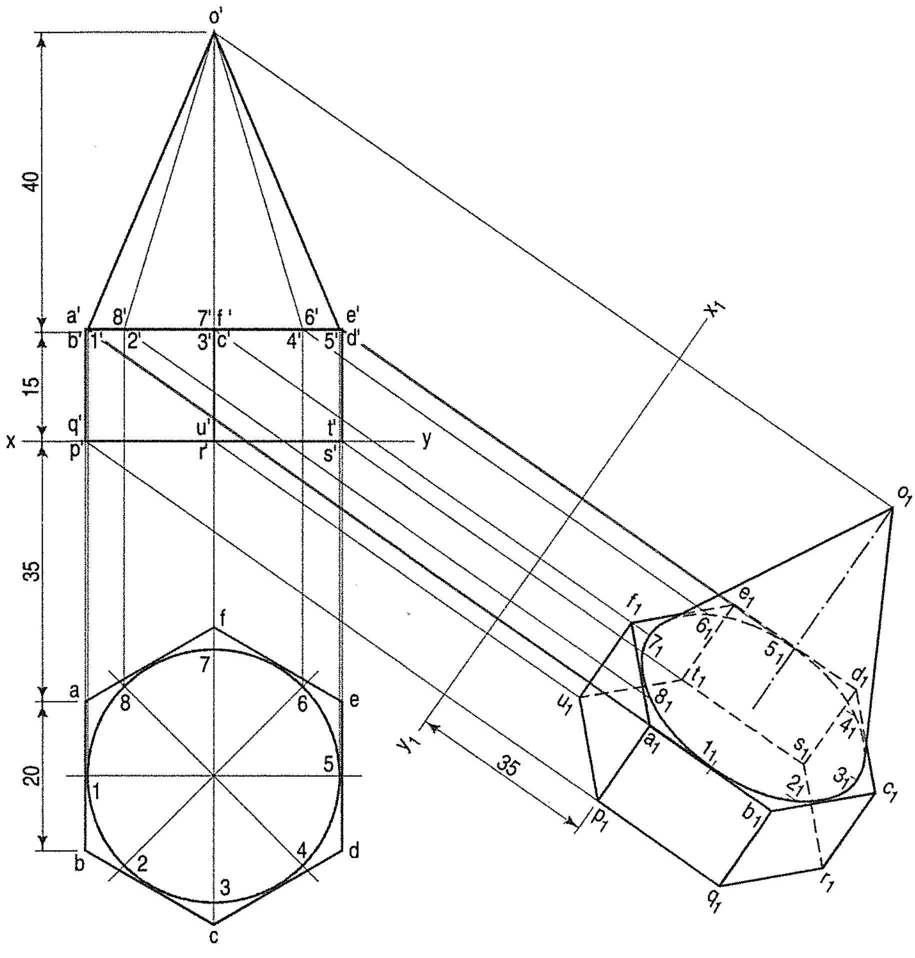


FIG. 13-59

Problem 13-42. (fig. 13-61): *A right hexagonal prism of side 25 mm and 20 mm thick with one side of the base is perpendicular to the V.P. resting on the ground. A vertical frustum of square pyramid of base 20 mm sides and top face side 30 mm and height 50 mm is resting on the prism such that one side of square makes 45° with the V.P. Assume that axes of both solids are coinciding. Draw the projections of the combined solids when top corner of the square pyramid is 70 mm above the ground (H.P.). Determine angle of combined solids with the H.P.*

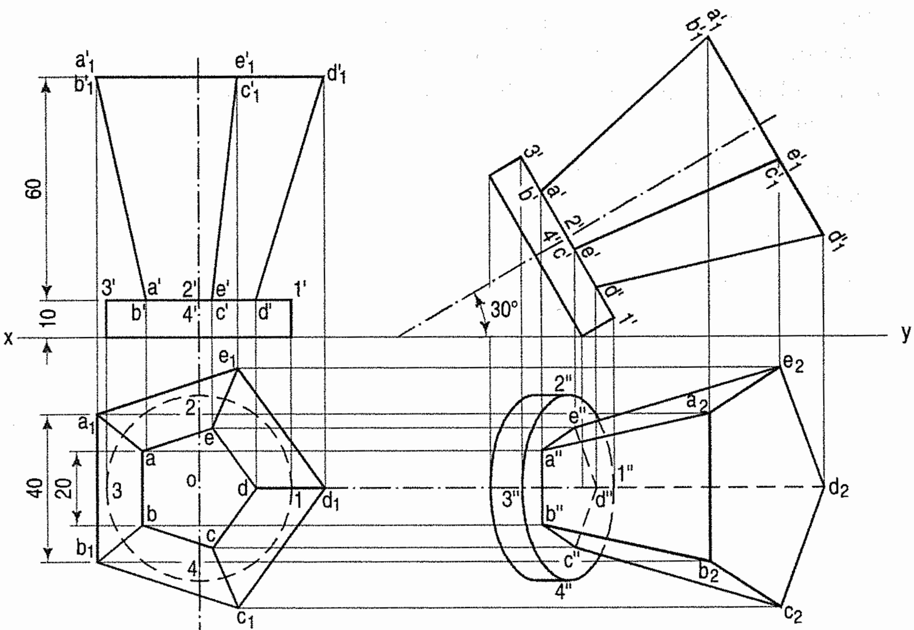


FIG. 13-60

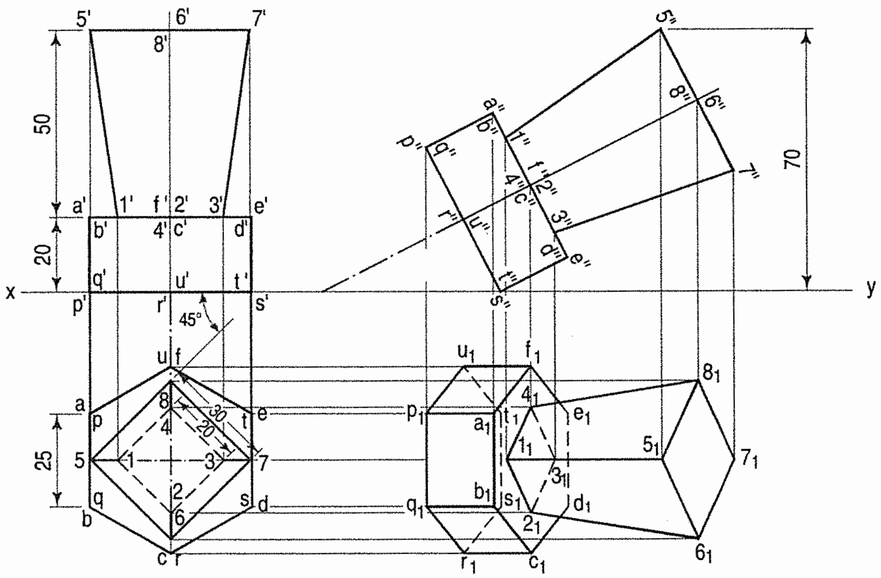


FIG. 13-61

(i) Draw the top view and front view as shown in figure. Keep one side of the hexagonal perpendicular to the *xy*.

(ii) Project the front view as shown.

(iii) Draw a parallel line at distance 70 mm away from *xy*. Reproduce the front view of the combined solids as shown.

(iv) Draw the projectors from the new front view.

(v) Draw from the top view horizontal projectors to intersect respective projectors drawn from the new front view.

(vi) Complete the top view as shown.

EXERCISES 13(b)

1. A rectangular block 75 mm × 50 mm × 25 mm thick has a hole of 30 mm diameter drilled centrally through its largest faces. Draw the projections when the block has its 50 mm long edge parallel to the H.P. and perpendicular to the V.P. and has the axis of the hole inclined at 60° to the H.P.
2. Draw the projections of a square pyramid having one of its triangular faces in the V.P. and the axis parallel to and 40 mm above the H.P. Base 30 mm side; axis 75 mm long.
3. A cylindrical block, 75 mm diameter and 25 mm thick, has a hexagonal hole of 25 mm side, cut centrally through its flat faces. Draw three views of the block when it has its flat faces vertical and inclined at 30° to the V.P. and two faces of the hole parallel to the H.P.
4. Draw three views of an earthen flower pot, 25 cm diameter at the top, 15 cm diameter at the bottom, 30 cm high and 2.5 cm thick, when its axis makes an angle of 30° with the vertical.
5. A tetrahedron of 75 mm long edges has one edge parallel to the H.P. and inclined at 45° to the V.P. while a face containing that edge is vertical. Draw its projections.
6. A hexagonal prism, base 30 mm side and axis 75 mm long, has an edge of the base parallel to the H.P. and inclined at 45° to the V.P. Its axis makes an angle of 60° with the H.P. Draw its projections.
7. A pentagonal prism is resting on a corner of its base on the ground with a longer edge containing that corner inclined at 45° to the H.P. and the vertical plane containing that edge and the axis inclined at 30° to the V.P. Draw its projections. Base 40 mm side; height 65 mm.
8. Draw three views of a cone, base 50 mm diameter and axis 75 mm long, having one of its generators in the V.P. and inclined at 30° to the H.P., the apex being in the H.P.
9. A square pyramid, base 40 mm side and axis 90 mm long, has a triangular face on the ground and the vertical plane containing the axis makes an angle of 45° with the V.P. Draw its projections.
10. A frustum of a pentagonal pyramid, base 50 mm side, top 25 mm side and axis 75 mm long, is placed on its base on the ground with an edge of the base perpendicular to the V.P. Draw its projections. Project another top view on a reference line parallel to the line which shows the true length of the slant edge. From this top view, project a front view on an auxiliary vertical plane inclined at 45° to the top view of the axis.
11. Draw the projections of a cone, base 50 mm diameter and axis 75 mm long, lying on a generator on the ground with the top view of the axis making an angle of 45° with the V.P.
12. The front view, incomplete top view and incomplete auxiliary top view of a casting are given in fig. 13-47. Draw all the three views completely in the third-angle projection.

13. A line sketch (in two views) of a shed with a curved roof is given in fig. 13-62. Draw its front view on an auxiliary vertical plane inclined at 60° to the V.P. All dimensions are in metres. Scale, 10 mm = 0.5 m.

14. The front view of a hexagonal pyramid [base 25 mm side] having one of its triangular faces resting centrally on a triangular face of a square pyramid [base 50 mm side and axis 50 mm long] is given in fig. 13-63. The plane containing the two axes is parallel to the V.P. Draw the top view of the solids. From this top view, project a front view on a reference line x_1y_1 inclined at 30° to xy; (ii) from the given front view, project another top view on a reference line x_2y_2 inclined at 45° to xy.

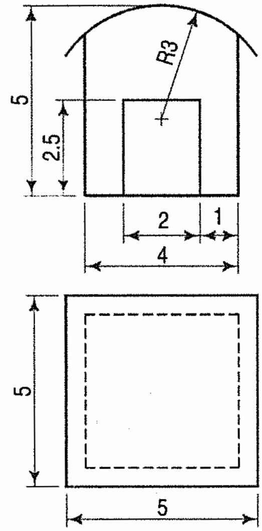


FIG. 13-62

15. A cube of 50 mm long edges is resting on the ground with its vertical faces equally inclined to the V.P. A hexagonal pyramid, base 25 mm side and axis 50 mm long, is placed centrally on top of the cube so that their axes are in a straight line and two edges of its base parallel to the V.P. Draw the front and top views of the solids. Project another top view on an A.I.P. making an angle of 45° with the H.P. From this top view project another front view on an auxiliary vertical plane inclined at 30° to the top view of the combined axis.

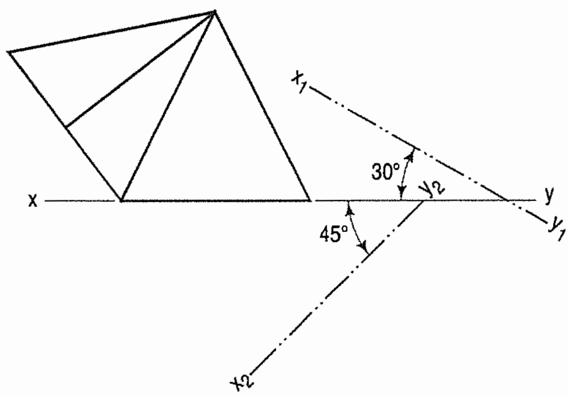


FIG. 13-63

16. Four equal spheres of 25 mm diameter are resting on the ground, each touching the other two spheres, so that a line joining the centres of two touching spheres is inclined at 30° to the V.P. A fifth sphere of 30 mm diameter is placed centrally on top of the four spheres, thus forming a pile. Draw the projections of the spheres and measure the height of the centre of the top sphere above the ground.

17. Three spheres of 25 mm, 50 mm and 75 mm diameter respectively are resting on the ground so that each touches the other two. Draw their projections when the top view of the line joining centres of any two of them is perpendicular to the V.P.

18. Three equal cones, base 50 mm diameter and axis 75 mm long, are placed on the ground on their bases, each touching the other two. A sphere of 40 mm diameter is placed centrally between them. Draw three views of the arrangement and determine the height of the centre of the sphere above the ground.

19. Five equal spheres are resting on the ground each touching the other two spheres and a vertical face of a pentagonal prism of 25 mm side. Determine the diameter of the spheres and draw the projections when a side of the base of the prism is perpendicular to the V.P.

20. Four equal spheres are resting on the ground, each touching the other two spheres and a triangular face of a square pyramid, having base 25 mm side and axis 50 mm long. Draw their projections and find the diameter of the spheres.
21. One of the body diagonals of a cube of 45 mm edge is parallel to the H.P. and inclined at 45° to the V.P. Draw the front view and the top view of the cube.
22. A pentagonal pyramid, base 40 mm side and height 75 mm rests on one edge of its base on the ground so that the highest point in the base is 25 mm above the ground. Draw its projections when the axis is parallel to the V.P. Draw another front view on a reference line inclined at 30° to the edge on which it is resting, and so that the base is visible.
23. A thin lamp shade in the form of a frustum of a cone has its larger end 200 mm diameter, smaller end 75 mm diameter and height 150 mm. Draw its three views when it is lying on its side on the ground and the axis parallel to the V.P.
24. A bucket made of tin sheet has its top 200 mm diameter and bottom 125 mm diameter with a circular ring 40 mm wide attached at the bottom. The total height of the bucket is 250 mm. Draw its projections when its axis makes an angle of 60° with the vertical.
25. A hexagonal pyramid, side of the base 25 mm long and height 70 mm, has one of its triangular faces perpendicular to the H.P. and inclined at 45° to the V.P. The base-side of this triangular face is parallel to the H.P. Draw its projections.
26. A pentagonal pyramid has an edge of the base in the V.P. and inclined at 30° to the H.P., while the triangular face containing that edge makes an angle of 45° with the V.P. Draw three views of the pyramid. Length of the side of the base is 30 mm, while that of the axis is 80 mm.
27. A square pyramid, base 40 mm side and axis 75 mm long is placed on the ground on one of its slant edges, so that the vertical plane passing through that edge and the axis makes an angle of 30° with the V.P. Draw its three views.
28. A hexagonal prism, side of base 40 mm and height 50 mm is lying on the ground on one of its bases with a vertical face perpendicular to the V.P. A tetrahedron is placed on the prism so that the corners of one of its faces coincide with the alternate corners of the top surface of the prism. Draw the projections of the solids. Project another top view on an auxiliary inclined plane making 45° with the H.P.
29. A square duct is in the form of a frustum of a square pyramid. The sides of top and bottom are 150 mm and 100 mm respectively and its length is 150 mm. It is situated in such a way that its axis is parallel to the H.P. and lies in a plane inclined at 60° to the V.P. Draw the projections of the duct, assuming the thickness of the duct-sheet to be negligible.
30. A pentagonal pyramid, base 30 mm edge and axis 75 mm long, stands upon a circular block, 75 mm diameter and 25 mm thick, so that their axes are in a straight line. Draw the projections of the solids when the base of the block is inclined at 30° to the ground, an edge of the base of the pyramid being parallel to the V.P.
31. The body diagonal of a cube is 75 mm long. The cube has a central 25 mm square hole. The faces of the hole make 45° with the side faces of the cube. Draw the projections of the cube when a body diagonal is perpendicular to the H.P.

32. A bucket, 300 mm diameter at the top and 225 mm diameter at the bottom has a circular ring 225 mm diameter and 50 mm wide attached at the bottom. The total height of the bucket is 300 mm. Draw the projections of the bucket when its axis is inclined at 60° to the H.P. and as a vertical plane makes an angle of 45° with the V.P. Assume the thickness of the plate of the bucket to be equal to that of a line.

33. The vertex-angle of the cone just touching the edges of a vertical hexagonal pyramid 125 mm in height is 45°. Draw the projections of the pyramid on a 45° inclined plane when the former is truncated by a plane making 45° with the axis and bisecting the axis.

34. A knob of a machine handle consists of 15 mm diameter × 150 mm long cylindrical portion and 40 mm diameter spherical portion. The centre of the sphere lies on the axis of the cylindrical portion. Draw the projections if its axis is inclined at 45° to the horizontal plane.

35. Six equal spheres rest on the ground in contact with each other and also with the slanting faces of a regular upright hexagonal pyramid, 25 mm edge of base and 125 mm length of axis. Draw the projections and find the diameter of the sphere.

36. A cylinder, 100 mm diameter and 150 mm long, has a rectangular slot 50 mm × 30 mm cut through it. The axis of the slot bisects the axis of the cylinder at right angles and the 50 mm side of the slot makes an angle of 60° with the base of the cylinder. Draw three views of the cylinder.

37. A very thin glass shade for a table lamp is the portion of a sphere 125 mm diameter included between two parallel planes at 15 mm and 55 mm from the centre, making the height 70 mm. If the axis of the shade is inclined at 30° to the vertical, obtain the projections of the shade.

38. A cone frustum, base 75 mm diameter, top 35 mm diameter and height 65 mm has a hole of 30 mm diameter drilled through it so that the axis of the hole coincides with that of the cone. It is resting on its base on the ground and is cut by a section plane perpendicular to the V.P., parallel to an end generator and passing through the top end of the axis. Draw sectional top view and sectional side view of the frustum.

39. Three vertical poles *AB*, *CD* and *EF* are respectively 5, 8 and 12 metres long. Their ends *B*, *D* and *F* are on the ground and lie at the corners of an equilateral triangle of 10 metres long sides. Determine graphically the distance between the top ends of the poles, viz. *AC*, *CE* and *EA*.

40. Two cylinders of 80 mm diameter each meet each other at right angles. The axis of one of the cylinders is parallel to both the reference planes and is 40 mm in front of the axis of the other cylinder. Draw three views of the cylinders showing lines of intersection in them. Take any suitable lengths of the cylinders.

41. A tetrahedron of side 40 mm rests on the top face of a hexagonal prism of base and height 25 mm such that their apex coincide. Draw the projections when the combination rests with one of the sides of the prism on the H.P., is perpendicular to the V.P., and the axis is inclined at 30° to the H.P.

42. A pentagonal pyramid, base 30 mm side and axis 70 mm long, has one of its slant edges in the H.P. and inclined at 30° to the V.P. Draw the projections of the solid when the apex is towards the observer.

Chapter 17

ISOMETRIC PROJECTION

17-1. INTRODUCTION

Isometric projection is a type of pictorial projection in which the three dimensions of a solid are not only shown in one view, but their actual sizes can be measured directly from it.

If a cube is placed on one of its corners on the ground with a solid diagonal perpendicular to the V.P., the front view is the *isometric projection* of the cube. The step-by-step construction is shown in fig. 17-1.

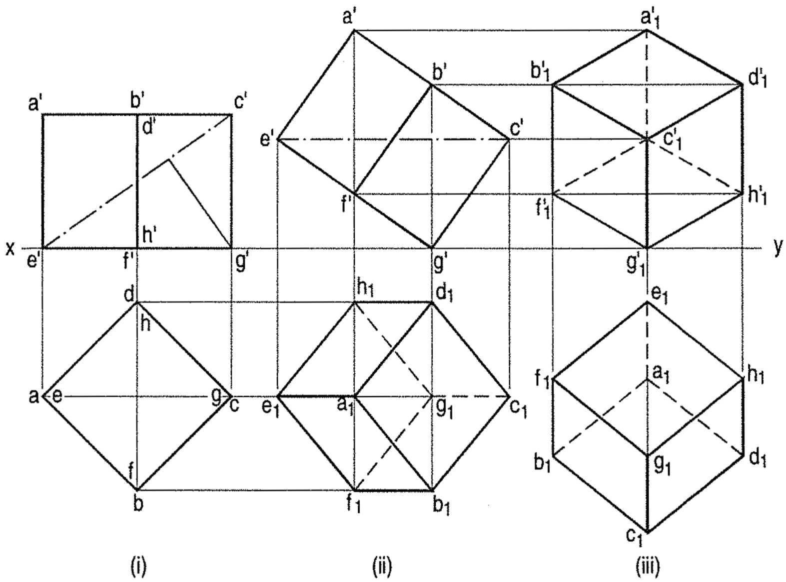


FIG. 17-1

To draw the projections of a cube of 25 mm long edges resting on the ground on one of its corners with a solid diagonal perpendicular to the V.P., assume the cube to be resting on one of its faces on the ground with a solid diagonal parallel to the V.P.

(i) Draw a square *abcd* in the top view with its sides inclined at 45° to *xy*. The line *ac* representing the solid diagonals *AG* and *CE* is parallel to *xy*. Project the front view.

(ii) Tilt the front view about the corner g' so that the line $e'c'$ becomes parallel to *xy*. Project the second top view. The solid diagonal *CE* is now parallel to both the H.P. and the V.P.

(iii) Reproduce the second top view so that the top view of the solid diagonal, viz. $e_1 c_1$ is perpendicular to *xy*. Project the required front view.

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 39 for the introduction.

Fig. 17-2 shows the front view of the cube in the above position, with the corners named in capital letters. Its careful study will show that:

(a) All the faces of the cube are equally inclined to the V.P. and hence, they are seen as similar and equal rhombuses instead of squares.

(b) The three lines *CB*, *CD* and *CG* meeting at *C* and representing the three edges of the solid right-angle are also equally inclined to the V.P. and are therefore, equally foreshortened. They make equal angles of 120° with each other. The line *CG* being vertical, the other two lines *CB* and *CD* make 30° angle each, with the horizontal.

(c) All the other lines representing the edges of the cube are parallel to one or the other of the above three lines and are also equally foreshortened.

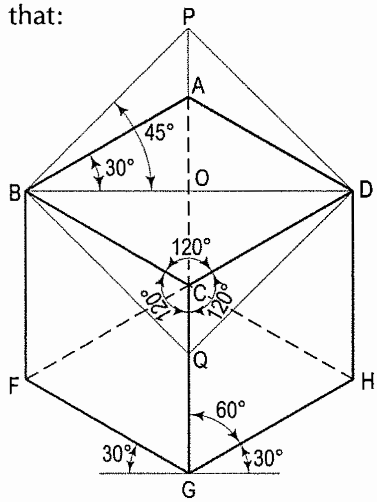


Fig. 17-2

(d) The diagonal *BD* of the top face is parallel to the V.P. and hence, retains its true length.

This chapter deals with various topics of isometric projection as shown below:

1. Isometric axes, lines and planes
2. Isometric scale
3. Isometric drawing or isometric view
4. Isometric graph.

17-2. ISOMETRIC AXES, LINES AND PLANES

The three lines *CB*, *CD* and *CG* meeting at the point *C* and making 120° angles with each other are termed *isometric axes*. The lines parallel to these axes are called *isometric lines*. The planes representing the faces of the cube as well as other planes parallel to these planes are called *isometric planes*.

17-3. ISOMETRIC SCALE

As all the edges of the cube are equally foreshortened, the square faces are seen as rhombuses. The rhombus *ABCD* (fig. 17-2) shows the isometric projection of the top square face of the cube in which *BD* is the true length of the diagonal.

Construct a square *BQDP* around *BD* as a diagonal. Then *BP* shows the true length of *BA*.

In triangle ABO, $\dfrac{BA}{BO} = \dfrac{1}{\cos 30^\circ} = \dfrac{2}{\sqrt{3}}$

In triangle PBO, $\dfrac{BP}{BO} = \dfrac{1}{\cos 45^\circ} = \dfrac{\sqrt{2}}{1}$

$$\frac{BA}{BP} = \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{3}} = 0.815$$

The ratio, $\dfrac{\text{isometric length}}{\text{true length}} = \dfrac{BA}{BP} = \dfrac{\sqrt{2}}{\sqrt{3}} = 0.815$ or $\dfrac{9}{11}$ (approx.).

Thus, the isometric projection is reduced in the ratio $\sqrt{2} : \sqrt{3}$, i.e. *the isometric lengths are 0.815 of the true lengths.*

Therefore, while drawing an *isometric projection*, it is necessary to convert true lengths into isometric lengths for measuring and marking the sizes. This is conveniently done by constructing and making use of an isometric scale as shown below.

(a) Draw a horizontal line *BD* of any length (fig. 17-3). At the end *B*, draw lines *BA* and *BP*, such that $\angle DBA = 30^\circ$ and $\angle DBP = 45^\circ$. Mark divisions of true length on the line *BP* and from each division-point, draw verticals to *BD* meeting *BA* at respective points. The divisions thus obtained on *BA* give lengths on isometric scale.

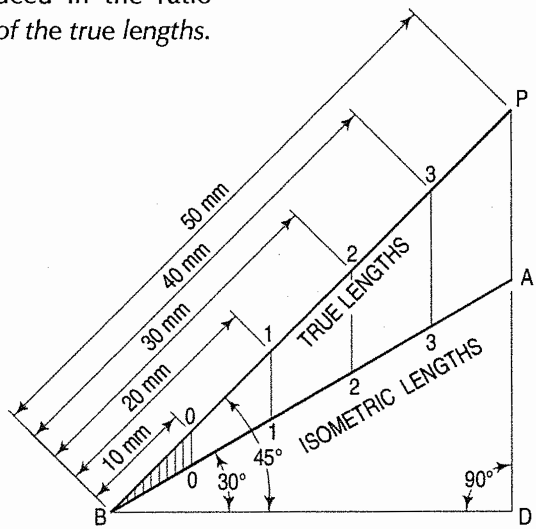


FIG. 17-3

(b) The same scale may also be drawn with divisions of natural scale on a horizontal line *AB* (fig. 17-4). At the ends *A* and *B*, draw lines *AC* and *BC* making 15° and 45° angles with *AB* respectively, and intersecting each other at *C*.

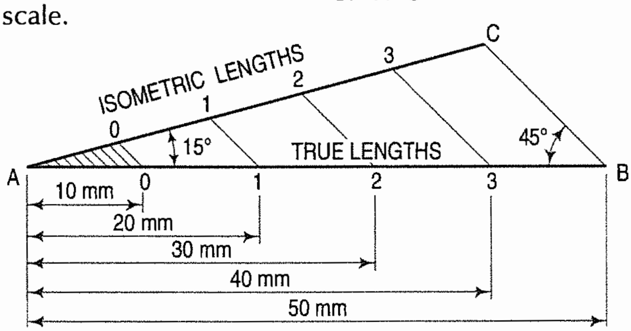


FIG. 17-4

From division-points of true lengths on *AB*, draw lines parallel to *BC* and meeting *AC* at respective points. The divisions along *AC* give lengths to isometric scale.

The lines *BD* and *AC* (fig. 17-2) represent equal diagonals of a square face of the cube, but are not equally shortened in isometric projection. *BD* retains its true length, while *AC* is considerably shortened. Thus, it is seen that lines which are not parallel to the isometric axes are not reduced according to any fixed ratio. Such lines are called non-isometric lines. The measurements should, therefore, be made on *isometric axes and isometric lines only*. The non-isometric lines are drawn by locating positions of their ends on isometric planes and then joining them.

17-4. ISOMETRIC DRAWING OR ISOMETRIC VIEW

If the foreshortening of the isometric lines in an isometric projection is disregarded and instead, the true lengths are marked, the view obtained [fig. 17-5(iii)] will be exactly of the same shape but larger in proportion (about 22.5%) than that obtained by the use of the isometric scale [fig. 17-5(ii)]. Due to the ease in construction and the advantage of measuring the dimensions directly from the drawing, it has become a general practice to use the true scale instead of the isometric scale.

To avoid confusion, the view drawn with the true scale is called *isometric drawing* or *isometric view,* while that drawn with the use of isometric scale is called *isometric projection.*

Referring again to fig. 17-2, the axes *BC* and *CD* represent the sides of a right angle in horizontal position. Each of them together with the vertical axis *CG*, represents the right angle in vertical position. Hence, in isometric view of any rectangular solid resting on a face on the ground, each horizontal face will have its sides parallel to the two sloping axes; each vertical face will have its vertical sides parallel to the vertical axis and the other sides parallel to one of the sloping axes.

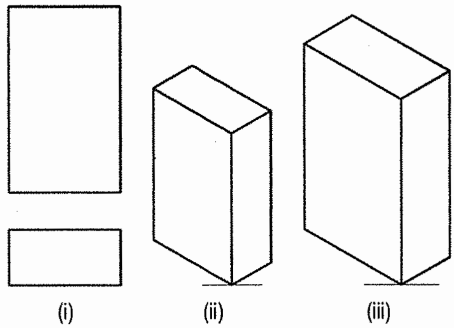


FIG. 17-5

In other words, the vertical edges are shown by vertical lines, while the horizontal edges are represented by lines, making 30° angles with the horizontal. These lines are very conveniently drawn with the T-square and a 30°-60° set-square or drafter.

17-5. ISOMETRIC GRAPH

An isometric graph as shown in fig. 17-6 facilitates the drawing of isometric view of an object. Students are advised to make practice for drawing of isometric view using such graphs. See fig. 17-55 and fig. 17-56 of problem 17-33.

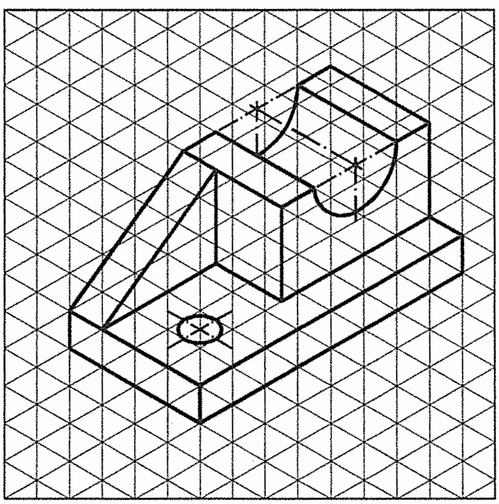


FIG. 17-6

17-6. ILLUSTRATIVE PROBLEMS

The procedure for drawing isometric views of planes, solids and objects of various shapes is explained in stages by means of illustrative problems.

In order that the construction of the view may be clearly understood, construction lines have not been erased. They are, however, drawn fainter than the outlines.

In an isometric view, lines for the hidden edges are generally not shown. In the solutions accompanying the problems, one or two arrows have been shown. They indicate the directions from which if the drawing is viewed, the given orthographic views would be obtained. Students need not show these arrows in their solutions.

17-6-1. ISOMETRIC DRAWING OF PLANES OR PLANE FIGURES

CHAROTAR COGNIFRONT This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 40 for the following problem.

Problem 17-1. *The front view of a square is given in fig. 17-7(i). Draw its isometric view.*

As the front view is a square, the surface of the square is vertical. In isometric view, vertical lines will be drawn vertical, while horizontal lines will be drawn inclined at 30° to the horizontal.

(i) Through any point *d*, draw a vertical line *da* = *DA* [fig. 17-7(ii)].

(ii) Again through *d*, draw a line *dc* = *DC* inclined at 30° to the horizontal and at 60° to *da*.

(iii) Complete the rhombus *abcd* which is the required isometric view. The view can also be drawn in direction of the other sloping axis as shown in fig. 17-7(iii).

Problem 17-2. *If fig. 17-7(i) is the top view of a square, draw its isometric view.*

As the top view is a square, the surface of the square is horizontal. In isometric view, all the sides will be drawn inclined at 30° to the horizontal.

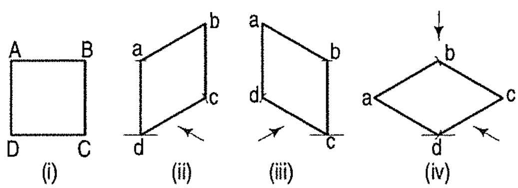


FIG. 17-7

(i) From any point *d* [fig. 17-7(iv)], draw two lines *da* and *dc* inclined at 30° to the horizontal and making 120° angle between themselves.

(ii) Complete the rhombus *abcd* which is the required isometric view.

Problem 17-3. *The top view of a rectangle, the surface of which is horizontal is shown in fig. 17-8(i). Draw its isometric view.*

Draw the required view as explained in problem 17-2 and as shown in either fig. 17-8(ii) or fig. 17-8(iii).

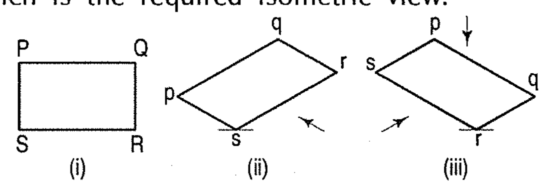


FIG. 17-8

Problem 17-4. *The front view of a triangle having its surface parallel to the V.P. is shown in fig. 17-9(i). Draw its isometric view.*

The surface of the triangle is vertical and the base *ab* is horizontal. *ab* will be drawn parallel to a sloping axis. The two sides of the triangle are inclined.

Hence they will not be drawn parallel to any isometric axis. In an isometric view, angles do not increase or decrease in any fixed proportion. They are drawn after determining the positions of the ends of the arms on isometric lines.

Therefore, enclose the triangle in the rectangle *ABQP*. Draw the isometric view *abqp* of the rectangle [fig. 17-9(ii)].

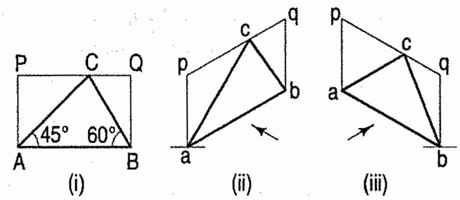


FIG. 17-9

Mark a point *c* in *pq* such that *pc* = *PC*. Draw the triangle *abc* which is the required isometric view. It can also be drawn in the other direction as shown in fig. 17-9(iii).

Problem 17-5. *The front view of a quadrilateral whose surface is parallel to the V.P. is shown in fig. 17-10(i). Draw its isometric view.*

Enclose the quadrilateral in a rectangle *ABEF*.

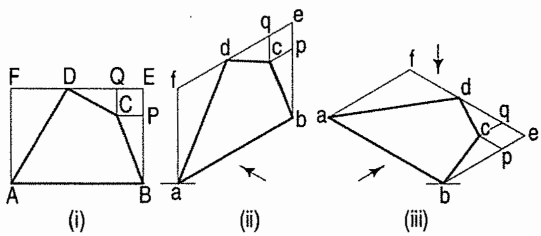


FIG. 17-10

(i) Draw lines *CP* and *CQ* parallel to the sides *FE* and *BE* respectively.

(ii) Draw the isometric view of the rectangle [fig. 17-10(ii)] and obtain the point *d* in *fe* as explained in problem 17-4. Draw the isometric view *cpeq* of the rectangle *CPEQ*.

(iii) Draw the quadrilateral *abcd* which is the required isometric view.

If the given view is the top view of a quadrilateral whose surface is horizontal, i.e. parallel to the H.P., its isometric view will be as shown in fig. 17-10(iii).

Problem 17-6. *If the view given in fig. 17-11(i) is*

(a) *the front view of a hexagon whose surface is parallel to the V.P. or*

(b) *the top view of the hexagon whose surface is horizontal, draw its isometric views.*

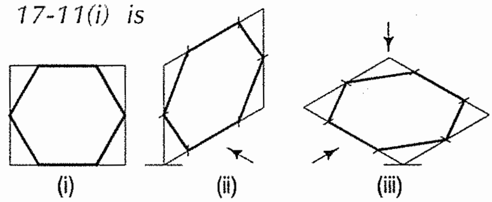


FIG. 17-11

(a) fig 17-11(ii). (b) fig 17-11(iii).

In both cases, the views can be drawn in the other direction also.

Problem 17-7. *Fig. 17-12(i) shows the front view of a circle whose surface is parallel to the V.P. Draw the isometric view of the circle.*

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 41 for the following problem.

I. Method of points:

(i) Enclose the circle in a square, touching it in points 1, 2, 3 and 4. Draw the diagonals of the square cutting the circle in points 5, 6, 7 and 8.

(ii) Draw the isometric view of the square [fig. 17-12(ii) and [fig. 17-12(iii)] and on it mark the mid-points 1, 2, 3 and 4 of its sides. Obtain points 5, 6, 7 and 8 on the diagonals as explained in problem 17-5.

Or, after determining the position of one point, draw through it, lines parallel to the sides of the rhombus and obtain the other three points. Draw a neat and smooth curve passing through the eight points viz. 1, 6, 2, 7 etc. The curve is the required isometric view. It is an ellipse.

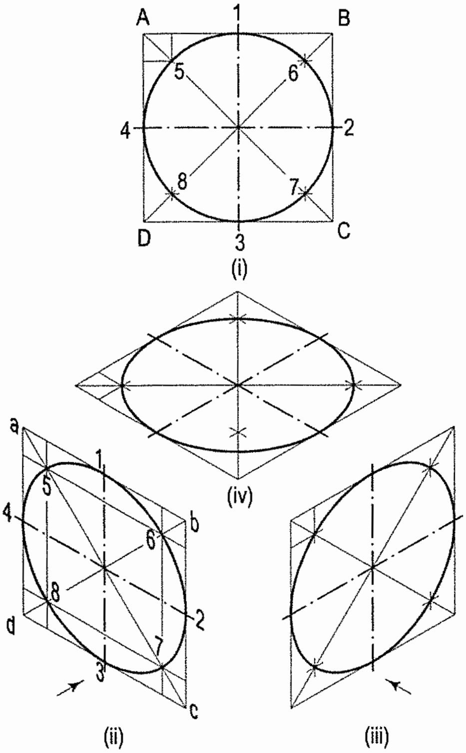


FIG. 17-12

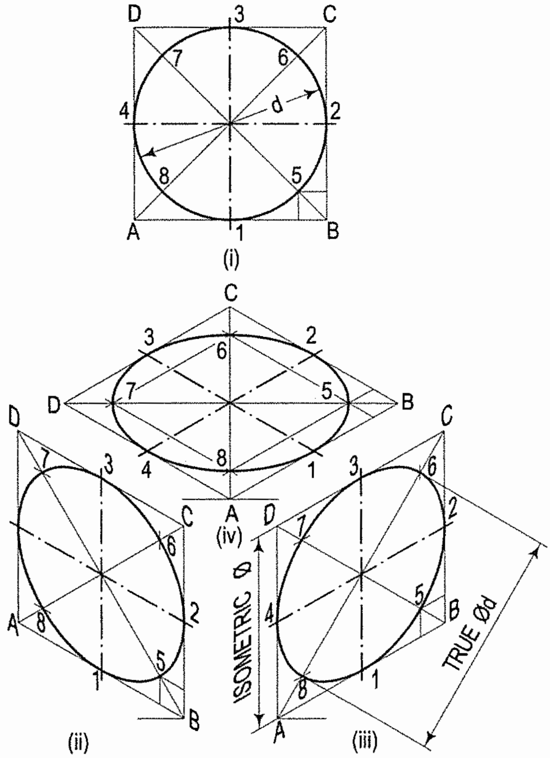


FIG. 17-13

If the view given in fig. 17-12(i) is the top view of a circle whose surface is horizontal, its isometric view will be as shown in fig. 17-12(iv).

As the isometric views have been drawn with the true scale, the major axis of the ellipse is longer than the diameter of the circle.

Fig. 17-13(ii), fig. 17-13(iii) and fig. 17-13(iv) show the isometric projection of the circle drawn with isometric scale. Note that when the length of the side of the rhombus is equal to the isometric diameter of the circle, the length of the major axis of the ellipse is equal to the true diameter of the circle.

II. Four-centre method:

Draw the isometric view of the square [fig. 17-14(i)]. Draw perpendicular bisectors of the sides of the rhombus, intersecting each other on the longer diagonal at points *p* and *q*, and which meet at the 120°-angles *b* and *d*.

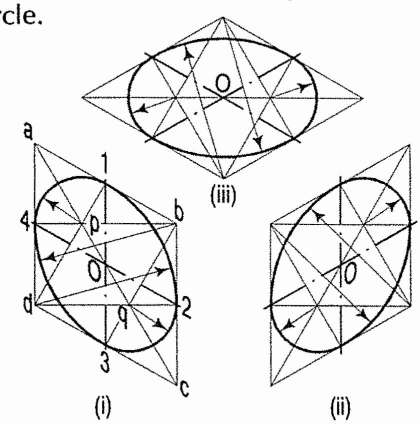


FIG. 17-14

Or, draw lines joining the 120°-angles *b* and *d* with the mid-points of the opposite sides and intersecting each other on the longer diagonal at points *p* and *q*. Two of these lines will be drawn horizontal, while the other two will make 60°-angles with the horizontal. With centres *b* and *d*, draw arcs 3-4 and 1-2 respectively. With centres *p* and *q*, draw arcs 1-4 and 2-3 respectively and complete the required ellipse. Fig. 17-14(ii) shows the ellipse obtained in the rhombus drawn in the direction of the other sloping axis. Fig. 17-14(iii) shows the isometric view of the circle when its surface is horizontal.

The ellipse obtained by the four-centre method is not a true ellipse and differs considerably in size and shape from the ellipse plotted through points. But owing to the ease in construction and to avoid the labour of drawing freehand neat curves, this method is generally employed.

Problem 17-8. *To draw the isometric view of a circle of a given diameter, around a given point.*

Let *O* be the given point and *D* the diameter of the circle.

(a) When the surface of the circle is vertical [fig. 17-14(i)].

(i) Through *O*, draw a vertical centre line and another centre line inclined at 30° to the horizontal, i.e. parallel to a sloping isometric axis. On these lines, mark points 1, 2, 3 and 4 at a distance equal to 0.5*D* from *O*.

(ii) Through these points, draw lines parallel to the centre lines and obtain the rhombus *abcd* of sides equal to *D*.

(iii) Draw the required ellipse in this rhombus by the four-centre method.

By drawing the second centre line parallel to the other sloping axis, the isometric view is obtained in another position as shown in fig. 17-14(ii).

(b) When the surface of the circle is horizontal [fig. 17-14(iii)].

Through *O*, draw the two centre lines parallel to the two sloping isometric axes, i.e. inclined at 30° to the horizontal. Draw the required ellipse as explained in (a) above.

Note: This construction is very useful in drawing isometric views of circular holes in solids.

Problem 17-9. *Fig. 17-15(i) shows the front view of a semi-circle whose surface is parallel to the V.P. Draw its isometric view.*

(i) Enclose the semi-circle in a rectangle. Draw the isometric view of the rectangle [fig. 17-15(ii) and [fig. 17-15(iii)].

(ii) Using the four-centre method, draw the half-ellipse in it which is the required view. The centre for the longer arc may be obtained as shown or by completing the rhombus.

If the view given in fig. 17-15(i) is the top view of a horizontal semi-circle, its isometric view would be drawn as shown in fig. 17-16(i) and fig. 17-16(ii).

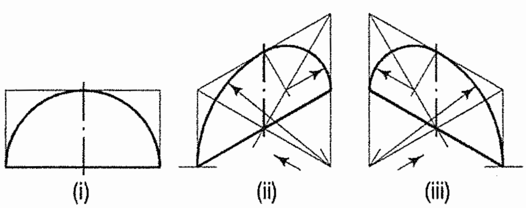


FIG. 17-15

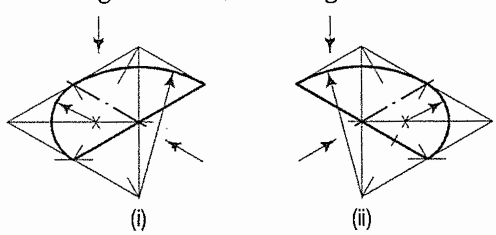


FIG. 17-16

Problem 17-10. *Fig. 17-17(i) shows the front view of a semi-circle whose surface is parallel to the V.P. Draw the isometric view of the semi-circle.*

See fig. 17-17(ii) and fig. 17-17(iii).

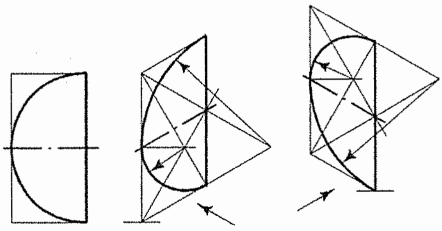


FIG. 17-17

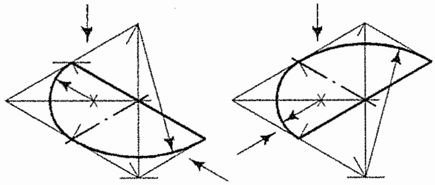


FIG. 17-18

If the view shown in fig. 17-17(i) is the top view of a semi-circle whose surface is horizontal, its isometric view will be as shown in fig. 17-18(i) or fig. 17-18(ii).

Problem 17-11. *Fig. 17-19(i) shows the front view of a plane parallel to the V.P. Draw its isometric view.*

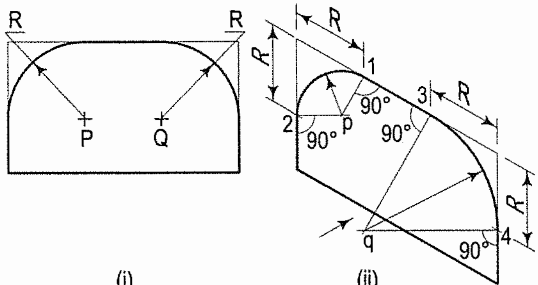


FIG. 17-19

(i) The upper two corners of the plane are rounded with quarter circles. Enclose the plane in a rectangle.

(ii) Draw the isometric view of the rectangle. From the upper two corners of the parallelogram, mark points on the sides at a distance equal to *R*, the radius of the arcs. At these points erect perpendiculars to the respective sides to intersect each other at points *p* and *q*. With *p* and *q* as centres, and radii *p*1 and *q*3, draw the arcs and complete the required view.

It is interesting to note that although the arcs are of the same radius, they are drawn with different radii in their isometric views.

17-6-2. ISOMETRIC DRAWING OF PRISMS AND PYRAMIDS

We have seen that the isometric view of a cube is determined from its orthographic view in a particular position. The three edges of the solid right-angle of the cube are shown by lines parallel to the three isometric axes. A square prism or a rectangular prism also has solid right-angles. Hence, lines for its edges are also drawn parallel to the three isometric axes.

While drawing the isometric view of any solid, the following important points should be carefully noted:

(i) The isometric view should be drawn according to the given views and in such a way that maximum possible details are visible.

(ii) At every point for the corner of a solid, at least three lines for the edges must converge. Of these, at least two must be for visible edges.

Lines for the hidden edges need not be shown, but it is advisable to check up every corner so that no line for a visible edge is left out.

(iii) Two lines (for visible edges) will never cross each other.

Problem 17-12. *Draw the isometric view of a square prism, side of the base 20 mm long and the axis 40 mm long, when its axis is (i) vertical and (ii) horizontal.*

(i) When the axis is vertical, the ends of the prism will be horizontal. Draw the isometric view (the rhombus 1-2-3-4) of the top end [fig. 17-20(i)]. Its sides will make 30°-angles with the horizontal. The length of the prism will be drawn in the third direction, i.e. vertical. Hence, from the corners of the rhombus, draw vertical lines 1-5, 2-6 and 3-7 of length equal to the length of the axis. The line 4-8 should not be drawn, as that edge will not be visible. Draw lines 5-6 and 6-7, thus completing the required isometric view. Lines 7-8 and 8-5 also should not be drawn. Beginning may also be made by drawing lines from the point 6 on the horizontal line and then proceeding upwards.

(ii) When the axis is horizontal, the ends will be vertical. The ends can be drawn in two ways as shown in fig. 17-20(ii) and fig. 17-20(iii). In each case, the length is shown in the direction of the third isometric axis.

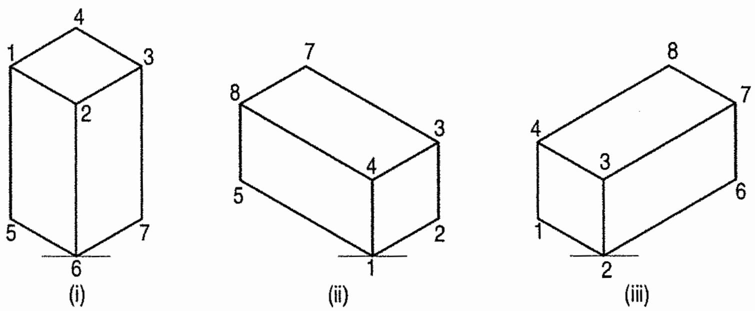


FIG. 17-20

Problem 17-13. *Three views of a block are given in fig. 17-21(i). Draw its isometric view.*

The block is in the form of a rectangular prism. Its shortest edges are vertical. Lines for these edges will be drawn vertical. Lines for all other edges which are horizontal, will be drawn inclined at 30° to the horizontal in direction of the two sloping axes as shown in fig. 17-21(ii).

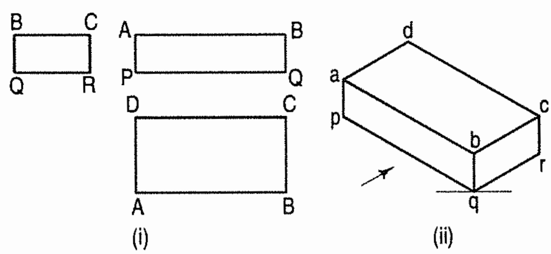


FIG. 17-21

Methods of drawing non-isometric lines.

When an object contains inclined edges which in the isometric view would be shown by non-isometric lines, the view may be drawn by using any one of the following methods:

(i) box method or

(ii) co-ordinate or offset method.

(i) Box method: This method is used when the non-isometric lines or their ends lie in isometric planes. The object is assumed to be enclosed in a rectangular box. Initially, the box is drawn in isometric. The ends of the lines for the inclined edges are then located by measuring on or from the outlines of the box.

Problem 17-14. *Three views of a block are given in fig. 17-22(i). Draw its isometric view.*

(i) Enclose the block in a rectangular box.

(ii) Draw the isometric view of the box [fig. 17-22(ii)].

(iii) Mark points e and *f* on the line *bc* such that be = *BE* and *fc* = *FC*.

(iv) Complete the required view as shown.

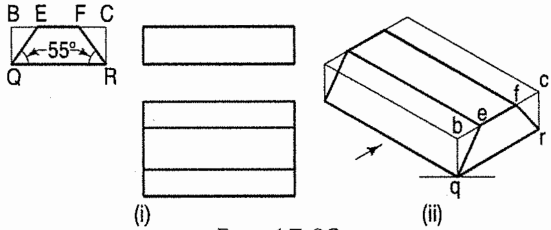


FIG. 17-22

Problem 17-15. *Draw the isometric view of the plate shown in three views in fig. 17-23(i).*

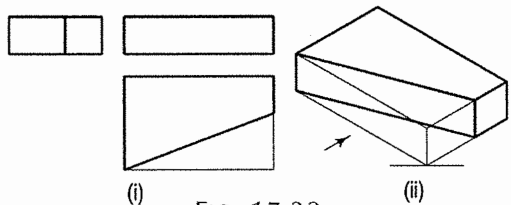


FIG. 17-23

Obtain the required view as explained in problem 17-14 and as shown in fig. 17-23(ii).

CHAROTAR COGNIFRONT This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 42 for the following problem.

Problem 17-16. *Draw the isometric view of the frustum of the hexagonal pyramid shown in fig. 17-24(i).*

(i) Enclose the front view and the top view in rectangles.

(ii) Draw the isometric view of the rectangular box [fig. 17-24(ii)]. Locate the six points of the base of the frustum on the sides of the bottom of the box. The upper six points on the top surface of the box are located by drawing isometric lines, e.g. *P*1 and *Q*1 intersecting at a point 1.

(iii) Join the corners and complete the isometric view as shown.

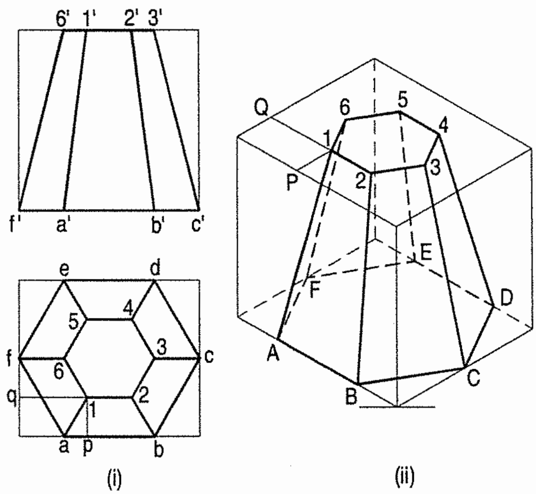


FIG. 17-24

(ii) Co-ordinate or Offset method: This method is adopted for objects in which neither non-isometric lines nor their ends lie in isometric planes.

Perpendiculars are dropped from each end of the edge to a horizontal or a vertical reference plane. The points at which the perpendiculars meet the plane, are located by drawing co-ordinates or offsets to the edges of the plane.

CHAROTAR COGNIFRONT This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 43 for the following problem.

Problem 17-17. *Draw the isometric view of the pentagonal pyramid, the projections of which are given in fig. 17-25(i).*

(i) Enclose the base (in the top view) in an oblong.

(ii) Draw an offset *oq* (i.e. *pq*) on the line *ab*.

(iii) Draw the isometric view of the oblong and locate the corners of the base in it [fig. 17-25(ii)].

(iv) Mark a point *Q* on the line *AB* such that *AQ* = *aq*. From *Q*, draw a line *QP* equal to *qo* and parallel to 2*C*. At *P*, draw a vertical *OP* equal to *o'p'*.

(v) Join *O* with the corners of the base, thus completing the isometric view of the pyramid.

Fig. 17-25(iii) shows the isometric view of the same pyramid with its axis in horizontal position.

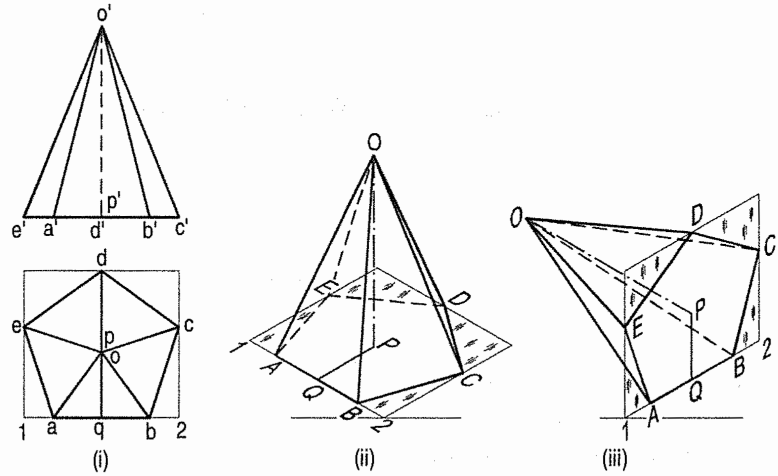


FIG. 17-25

Problem 17-18. *Draw the isometric view of the truncated triangular pyramid shown in fig. 17-26(i).*

(i) Draw the perpendiculars *d' x'*, *e' y'* and *f' z'* the front view and the offsets *dq*, *er* and *fc* in the top view.

(ii) Draw the isometric view of the whole pyramid [fig. 17-26(ii)].

(iii) Transfer the offsets and the verticals to this view and obtain points *D*, *E* and *F* on the lines *OA*, *OB* and *OC* respectively.

(iv) Draw lines *DE*, *EF* and *FD* and complete the required isometric view.

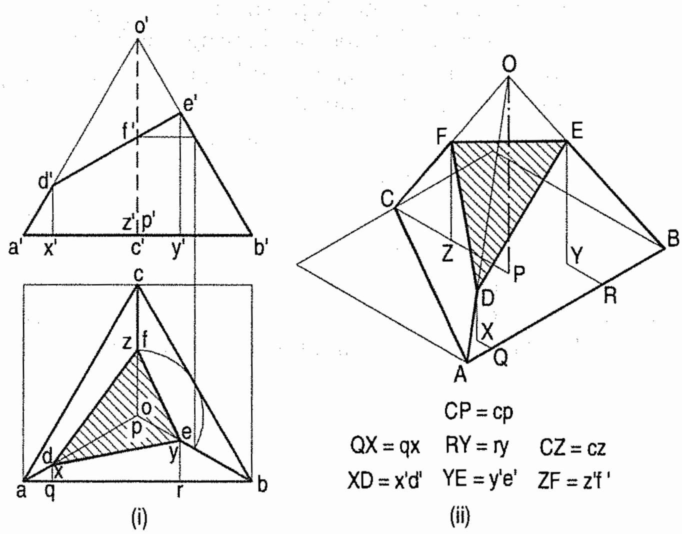


FIG. 17-26

17-6-3. ISOMETRIC DRAWING OF CYLINDERS

Problem 17-19. *Draw the isometric view of the cylinder shown in fig. 17-27(i).*

The axis of the cylinder is vertical, hence its ends are horizontal. Enclose the cylinder in a square prism.

Method I:

Draw the isometric view of the prism [fig. 17-27(ii)]. In the two rhombuses, draw the ellipses by the four-centre method. Draw two common tangents to the two ellipses. Erase the inner half of the lower ellipse and complete the required view.

Method II:

Draw the rhombus for the upper end of the prism [fig. 17-27(iii)] and in it, draw the ellipse by the four-centre method. From the centres for the arcs, draw vertical lines of length equal to the length of the axis, thus determining the centres for the lower ellipse. Draw the arcs for the half ellipse. Draw common tangents, thus completing the required view.

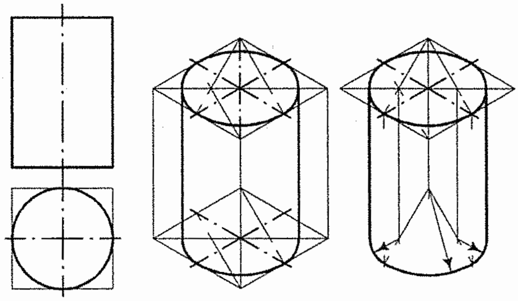


FIG. 17-27

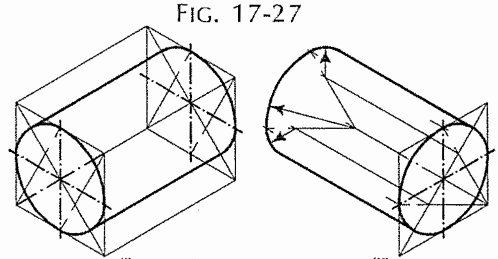


FIG. 17-28

When the axis of the cylinder is horizontal, its isometric view is drawn by method I as shown in fig. 17-28(i).

Fig. 17-28(ii) shows the view drawn by method II, but the axis is shown sloping in the other direction.

Fig. 17-29 and fig. 17-30 respectively show the isometric views (drawn by method II) of a half-cylindrical disc with its axis in vertical and horizontal positions.

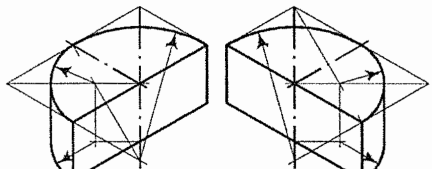


FIG. 17-29

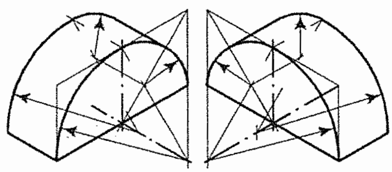


FIG. 17-30

17-6-4. ISOMETRIC DRAWING OF CONES

Problem 17-20. *Draw the isometric view of a cone, base 40 mm diameter and axis 55 mm long (i) when its axis is vertical and (ii) when its axis is horizontal.*

(i) Draw the ellipse for the base [fig. 17-31(i)]. Determine the position of the apex by the offset method.

(ii) Draw tangents to the ellipse from the apex. Erase the part of the ellipse between the tangents and complete the view as shown.

(iii) See fig. 17-31(ii) which is self-explanatory.

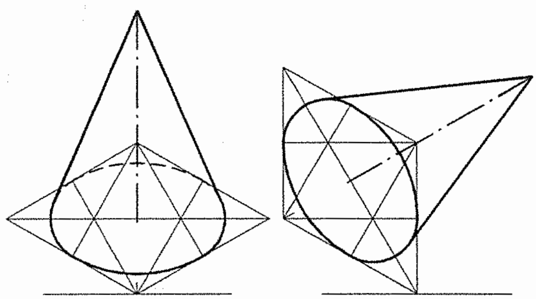


FIG. 17-31

Problem 17-21. *Draw the isometric view of the frustum of a cone shown in fig. 17-32(i).*

(i) Draw the ellipse for the base [fig. 17-32(ii)]. Draw the axis.

(ii) Around the top end of the axis, draw the ellipse for the top.

(iii) Draw common tangents, erase the unwanted part of the ellipse and complete the view as shown.

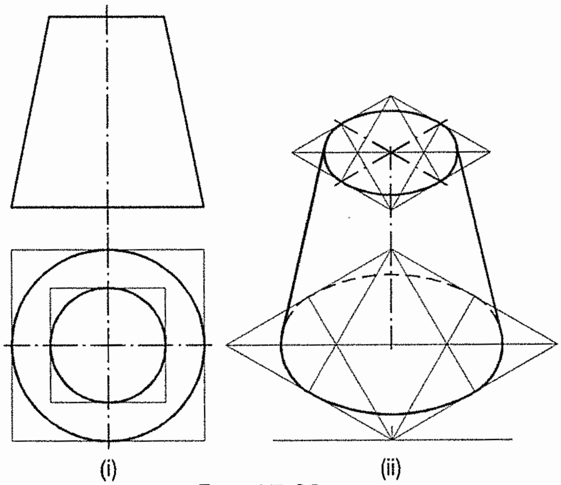


FIG. 17-32

17-6-5. ISOMETRIC DRAWING OF SPHERE

The orthographic view of a sphere seen from any direction is a circle of diameter equal to the diameter of the sphere. Hence, the isometric projection of a sphere is also a circle of the same diameter as explained below.

The front view and the top view of a sphere resting on the ground are shown in fig. 17-33(i). *C* is its centre, *D* is the diameter and *P* is the point of its contact with the ground.

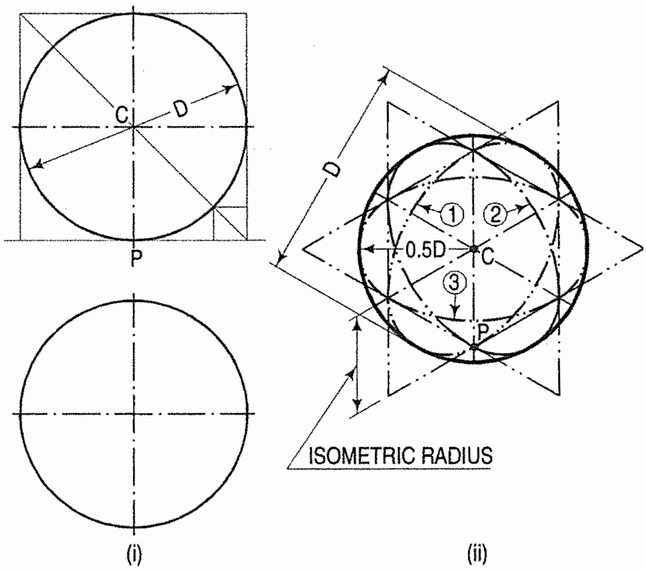


FIG. 17-33

Assume a vertical section through the centre of the sphere. Its shape will be a circle of diameter *D*. The isometric projection of this circle is shown in fig. 17-33(ii) by ellipses 1 and 2, drawn in two different vertical positions around the same centre *C*. The length of the major axis in each case is equal to *D*. The distance of the point *P* from the centre *C* is equal to the isometric radius of the sphere.

Again, assume a horizontal section through the centre of the sphere. The isometric projection of this circle is shown by the ellipse 3, drawn in a horizontal position around the same centre *C*. In this case also, the distance of the outermost points on the ellipse from the centre *C* is equal to 0.5*D*.

Thus, it can be seen that in an isometric projection, the distances of all the points on the surface of a sphere from its centre, are equal to the radius of the sphere.

Hence, the isometric projection of a sphere is a circle whose diameter is equal to the true diameter of the sphere.

Also, the distance of the centre of the sphere from its point of contact with the ground is equal to the isometric radius of the sphere, viz. *CP*.

It is, therefore, of the utmost importance to note that, *isometric scale must invariably be used, while drawing isometric projections of solids in conjunction with spheres or having spherical parts.*

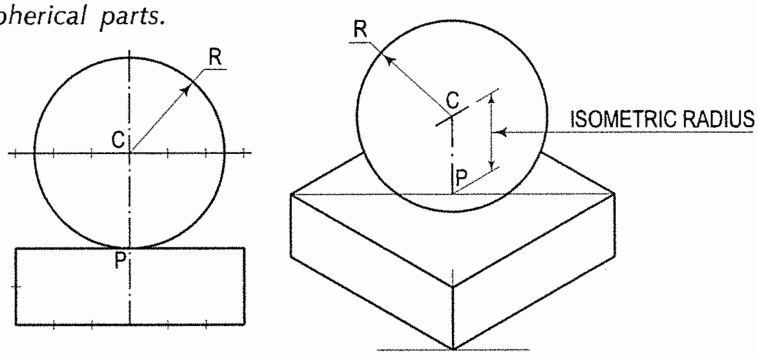


FIG. 17-34

Problem 17-22. *Draw the isometric projection of a sphere resting centrally on the top of a square prism, the front view of which is shown in fig. 17-34(i).*

(i) Draw the isometric projection (using isometric scale) of the square prism and locate the centre *P* of its top surface [fig. 17-34(ii)].

(ii) Draw a vertical at *P* and mark a point *C* on it, such that *PC* = the *isometric radius* of the sphere.

(iii) With *C* as centre and *radius equal to the radius of the sphere*, draw a circle which will be the isometric projection of the sphere.

17-7. TYPICAL PROBLEMS OF ISOMETRIC DRAWING

The solutions given in the following typical problems are mostly self-explanatory. Explanations are however given where deemed necessary. Construction lines are left intact for guidance. Dotted lines for hidden edges have been shown in some views to make the construction more clear. Unless otherwise stated, all dimensions are given in millimetres.

Problem 17-23. *A hexagonal prism having the side of base 26 mm and the height of 60 mm is resting on one of the corner of the base and its axis is inclined to 30° to the H.P. Draw its projections and also prepare the isometric view of the prism in the above stated condition.*

(i) Draw the projections of the prism as shown in figure 17-35.

(ii) Construct the isometric view as shown in fig. 17-36.

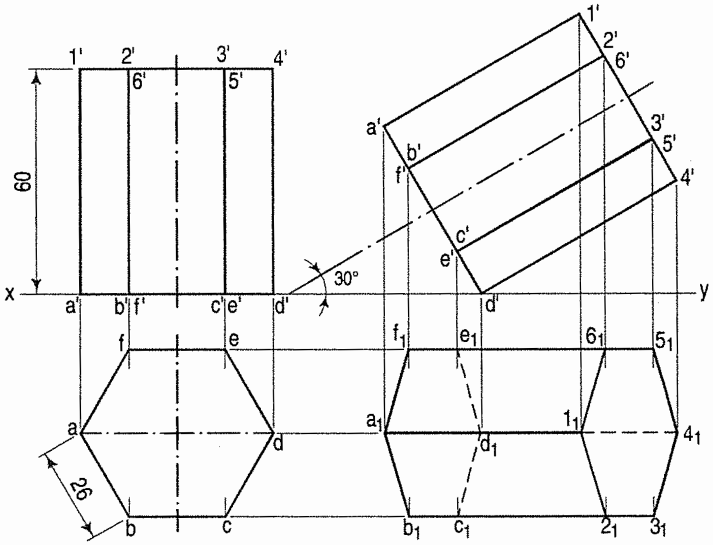


FIG. 17-35

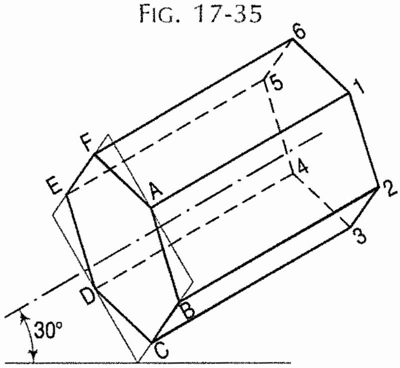


FIG. 17-36

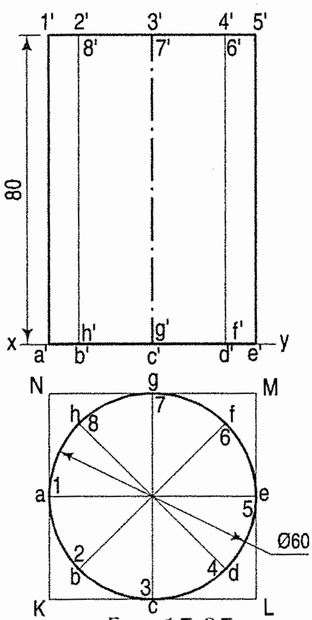


FIG. 17-37

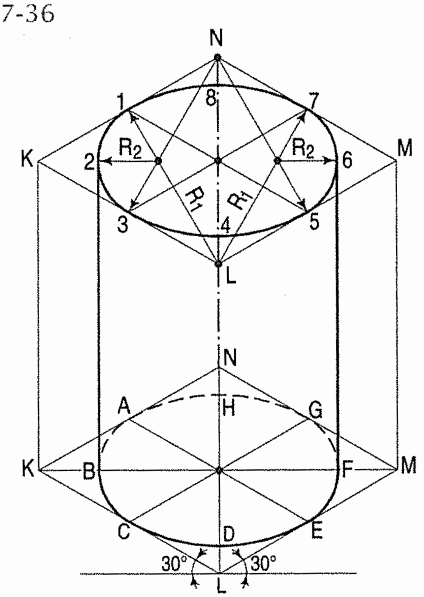


FIG. 17-38

Problem 17-24. (fig. 17-37)**:** *A cylindrical block of base, 60 mm diameter and height 80 mm, standing on the H.P. with its axis perpendicular to the H.P. Draw its isometric view.*

The method shown in fig. 17-38 is self-explanatony.

Problem 17-25. *The projection of pentagonal pyramid is shown in fig. 17-39. Draw its isometric view.*

See fig. 17-40.

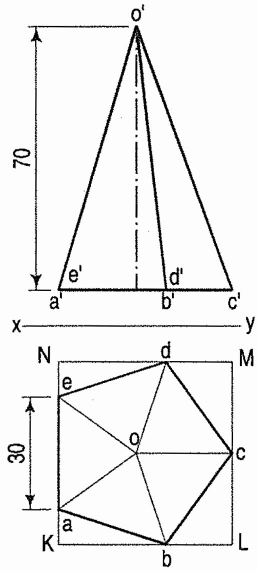


FIG. 17-39

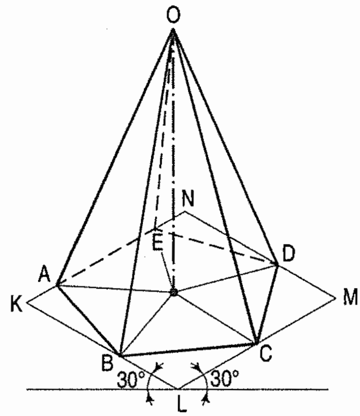


FIG. 17-40

Problem 17-26. *The projection of the frustum of the cone is shown in fig. 17-41. Draw its isometric view.*

See fig. 17-42.

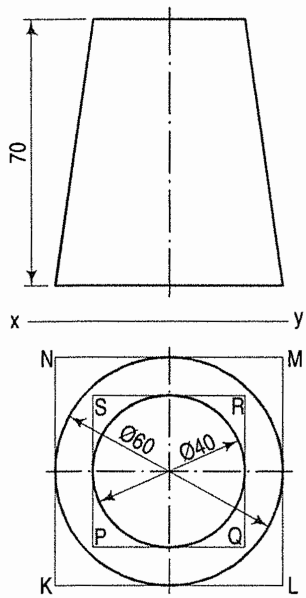


FIG. 17-41

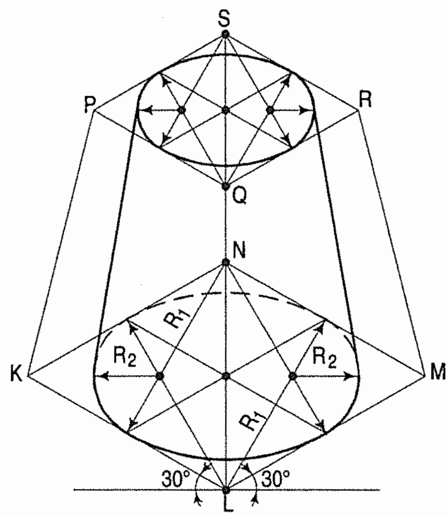


FIG. 17-42

Problem 17-27. *The orthographic projections of the object is shown in fig. 17-43. Draw the isometric view of the object.*

See fig. 17-44.

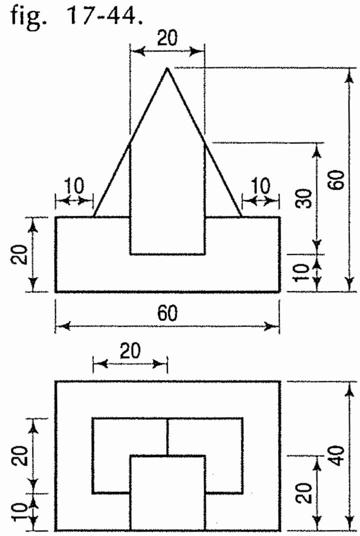


FIG. 17-43

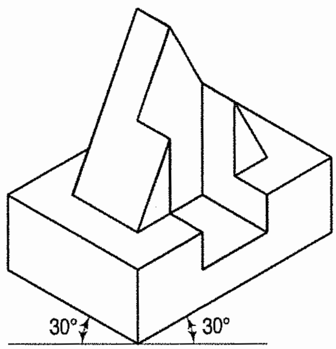


FIG. 17-44

Problem 17-28. *Draw the isometric view of the casting shown in two views in fig. 17-45.*

See fig. 17-46. Lines for the visible lower edges of the rectangular hole should be shown.

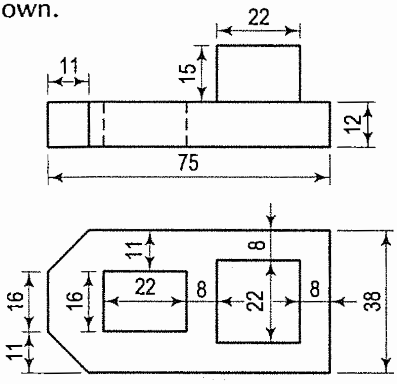


FIG. 17-45

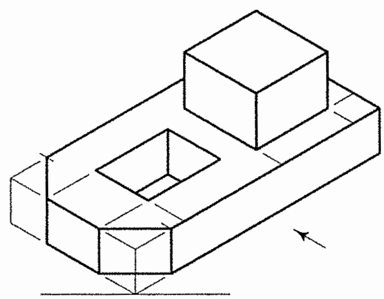


FIG. 17-46

Problem 17-29. *Draw the isometric view of the block, two views of which are shown in fig. 17-47.*

See fig. 17-48. Centres for the arcs for lower circular edges are obtained by drawing vertical lines from the centres for the upper arcs.

Problem 17-30. *Draw the isometric view of the casting, shown in three views in fig. 17-49.*

See fig. 17-50.

Problem 17-31. *Draw the isometric view of the casting, two views of which are shown in fig. 17-51.*

See fig. 17-52.

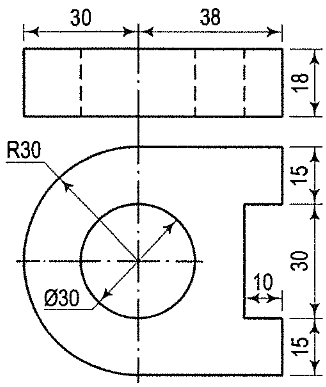


FIG. 17-47

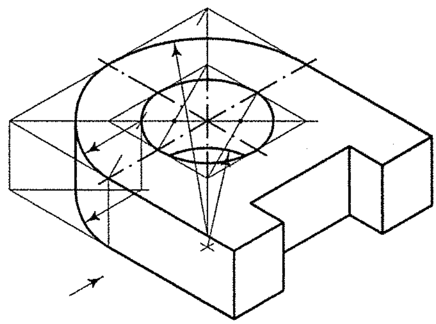
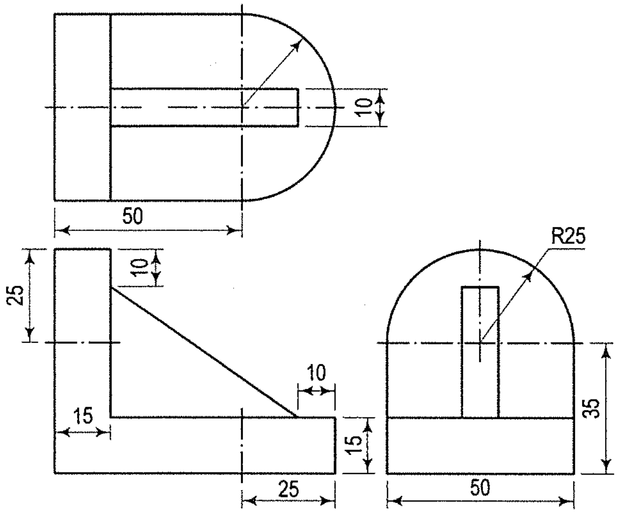


FIG. 17-48



(Third-angle projection)
FIG. 17-49

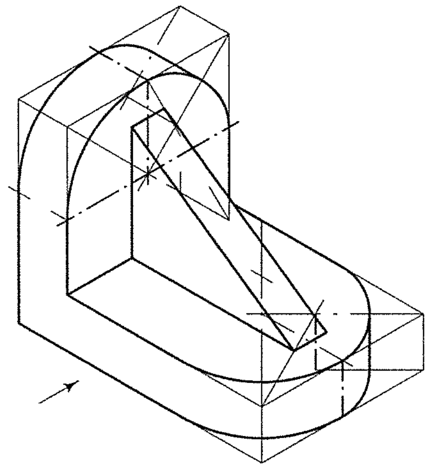
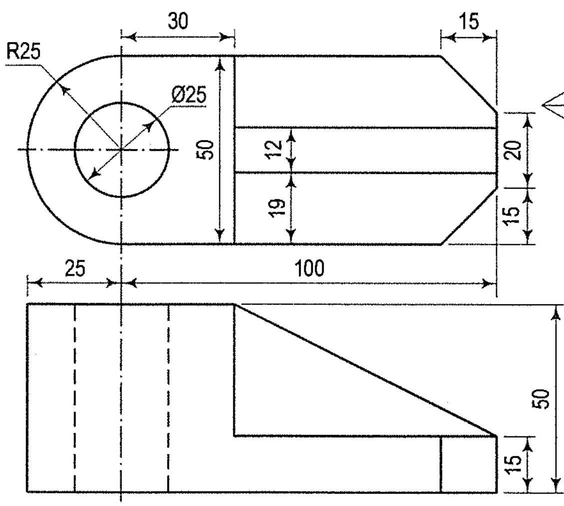


FIG. 17-50



(Third-angle projection)
FIG. 17-51

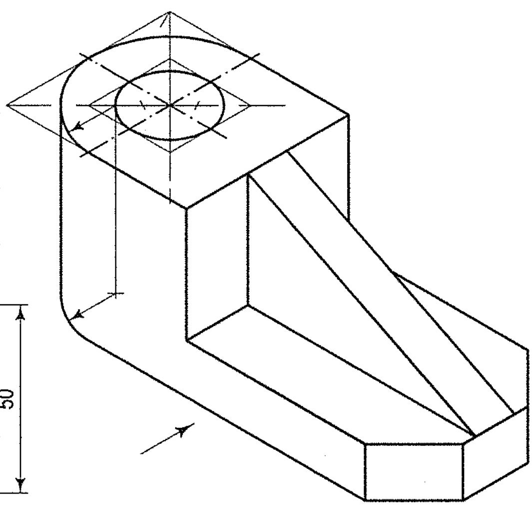


FIG. 17-52

Problem 17-32. *The front view of a board fitted with a letter H and mounted on a wooden post is given in fig. 17-53. Draw its isometric view, assuming the thickness of the board and of the letter to be equal to 3 cm. Scale, half full size. (All dimensions are given in centimeters.)*

See fig. 17-54.

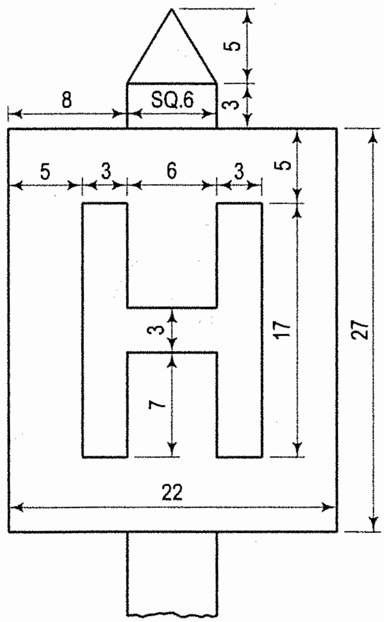


FIG. 17-53

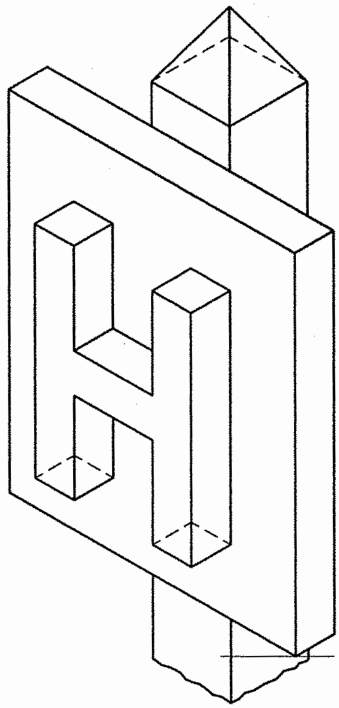


FIG. 17-54

Problem 17-33. *Draw the isometric view of the casting shown in two views in fig. 17-55.*

See fig. 17-56.

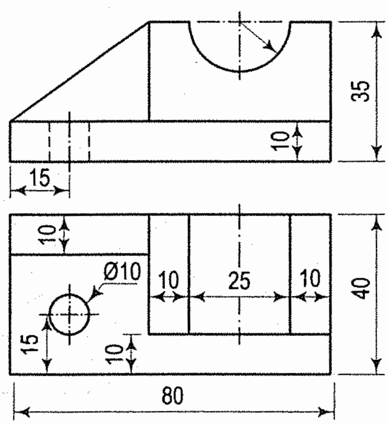


FIG. 17-55

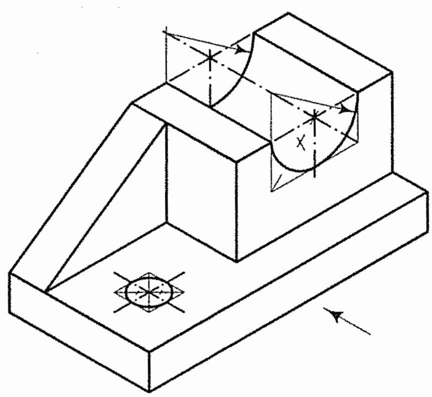


FIG. 17-56

Problem 17-34. *Draw the isometric view of the model of steps, two views of which are shown in fig. 17-57.*

See fig. 17-58.

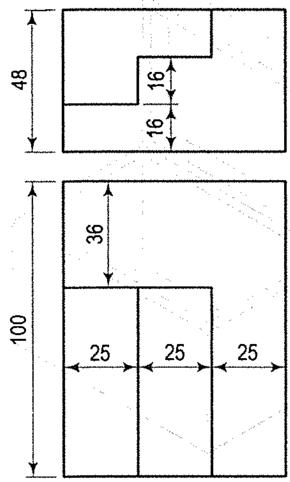


FIG. 17-57

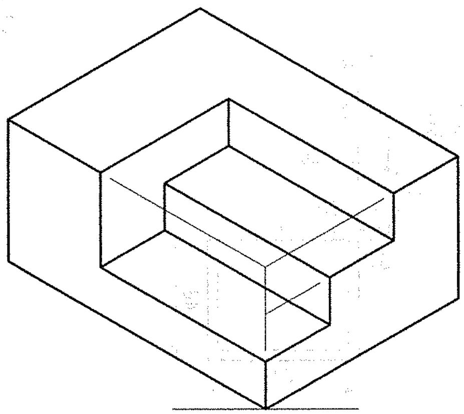


FIG. 17-58

Problem 17-35. *Two pieces of wood joined together by a dovetail joint are shown in two views in fig. 17-59. Draw the isometric view of the two pieces separated but in a position ready for fitting.*

See fig. 17-60.

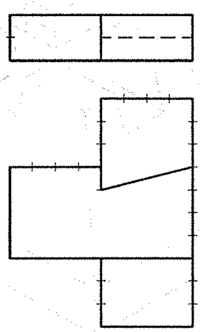


FIG. 17-59

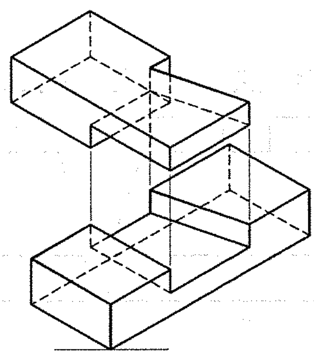


FIG. 17-60

Problem 17-36. *The outside dimensions of a box made of 4 cm thick planks are 90 cm × 60 cm × 60 cm height. The depth of the lid on the outside is 12 cm. Draw the isometric view of the box when the lid is (a) 90° open and (b) 120° open.*

Draw the orthographic view of the box with the lid in required positions as shown in fig. 17-61.

(a) This position is simple to draw in isometric view. Care must, however, be taken to deduct the thickness of the wood for the bottom and the top, when showing in the lines for the inside of the box and the lid (fig. 17-62).

(b) In this position, points *P*, *Q*, *R* etc. for the lid are located by enclosing the lid in the oblong and transferring the same on the isometric view as shown in fig. 17-62. The view is left incomplete to avoid congestion.

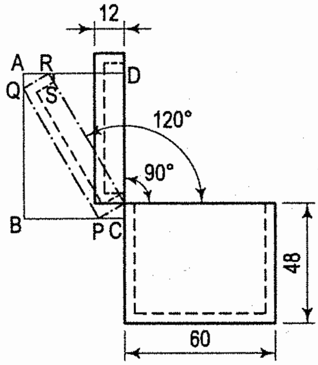


FIG. 17-61

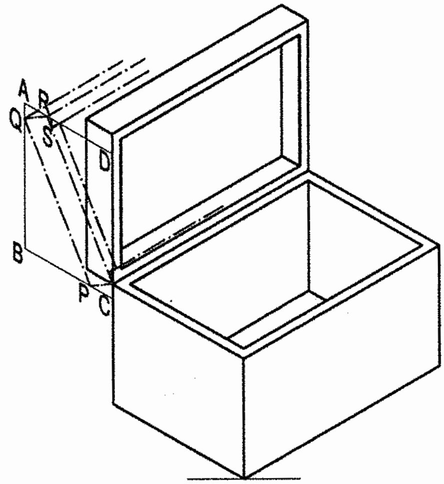


FIG. 17-62

Problem 17-37. *Two views of a cast-iron block are shown in fig. 17-63. Draw its isometric view.*

See fig. 17-64.

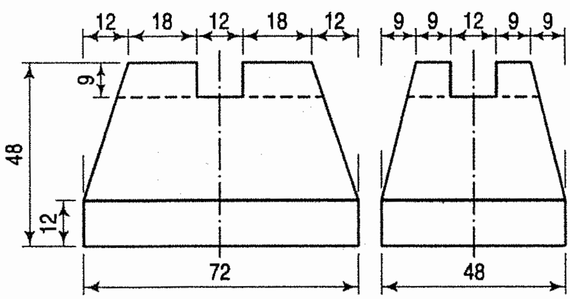


FIG. 17-63

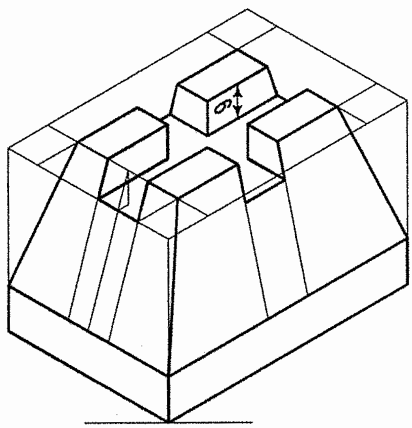


FIG. 17-64

The slope of the lines for the grooves on the outer surface on all the four sides is different and is obtained as shown by construction lines. The depth is measured along vertical lines.

Problem 17-38. *Draw the isometric view of the casting shown in two views in fig. 17-65.*

See fig. 17-66.

Problem 17-39. *Draw the isometric view of the simple moulding shown in fig. 17-67.*

See fig. 17-68.

The points on the curve are located by co-ordinate method. The parallel curve is obtained by drawing lines in the third direction and equal to the thickness of the moulding.

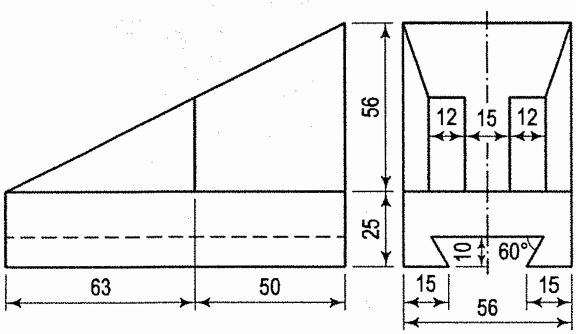


Fig. 17-65

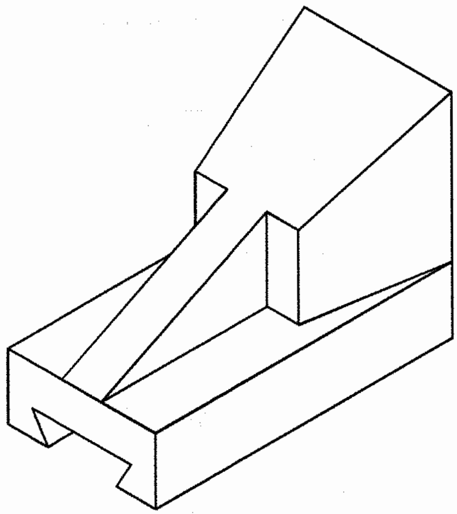


Fig. 17-66

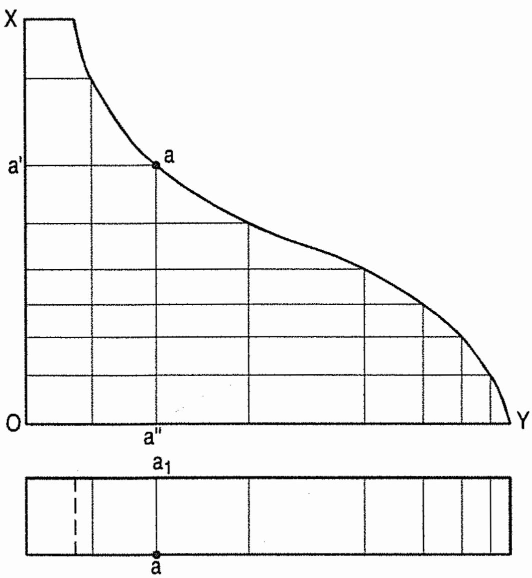


Fig. 17-67

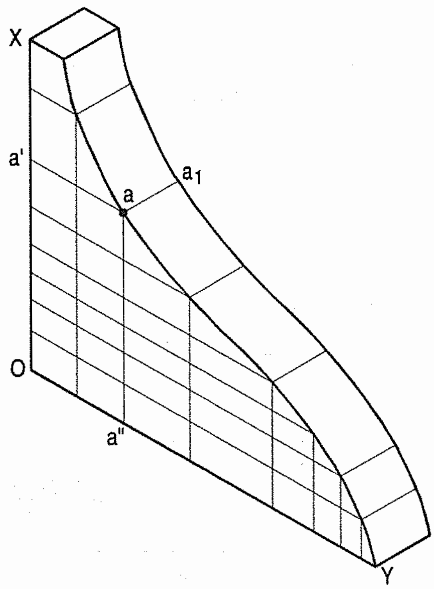


Fig. 17-68

Problem 17-40. *The front view of three solids placed one above the other, with their axes in a straight line is shown in fig. 17-69. Draw the isometric view of the arrangement.*

See fig. 17-70.

In this problem, isometric lengths must be taken for all dimensions except for the radius of the circle for the sphere.

The centre C of the sphere is at a distance equal to the isometric radius from the centre P of the top face of the cone frustum. *The circle for the sphere is drawn with the true radius.*

The ellipse for the section of the sphere is drawn within the rhombus constructed around the point Q on the axis.

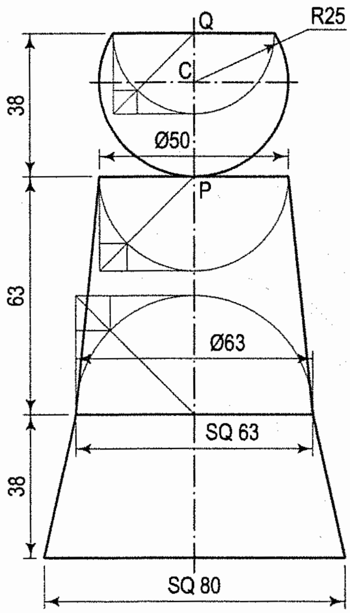


FIG. 17-69

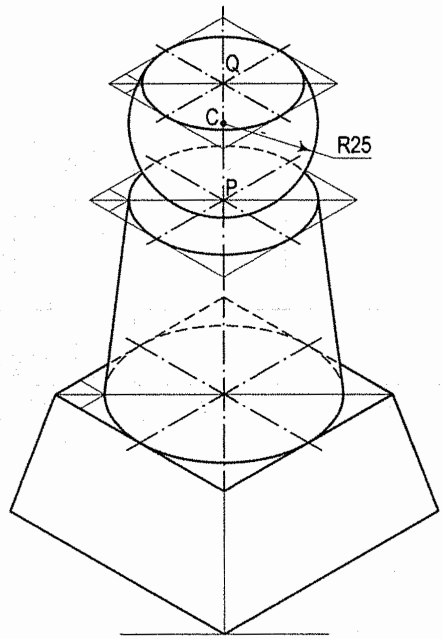


FIG. 17-70

Problem 17-41. *Draw the isometric view of the clamping piece shown in fig. 17-71.*

See fig. 17-72.

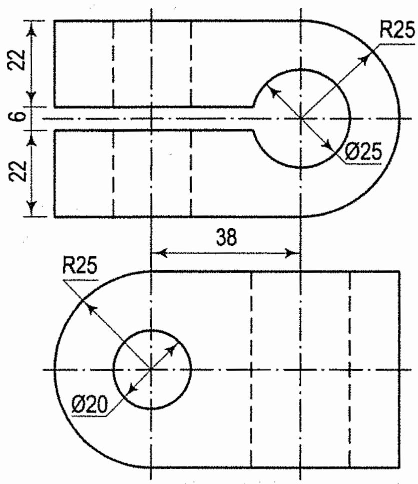


FIG. 17-71

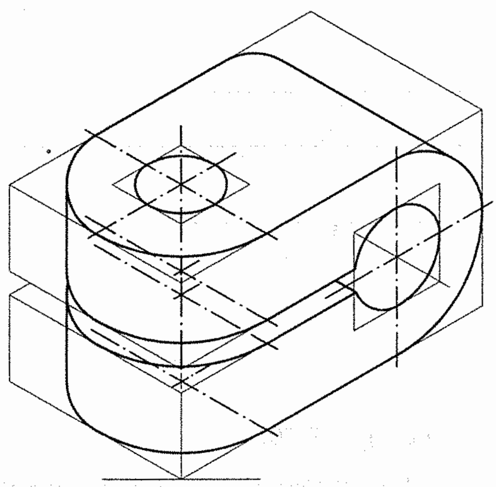


FIG. 17-72

Problem 17-42. (fig. 17-73): *Draw the isometric view of a hexagonal nut for a 24 mm diameter bolt, assuming approximate dimensions. The threads may be neglected but chamfer must be shown.*

See fig. 17-74.

Problem 17-43. *Draw the isometric view of the paper-weight with spherical knob shown in fig. 17-75.*

See fig. 17-76. Isometric scale should be used for all dimensions except for the radius of the circle for the sphere.

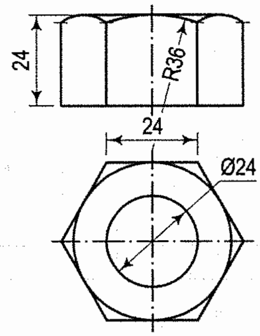


FIG. 17-73

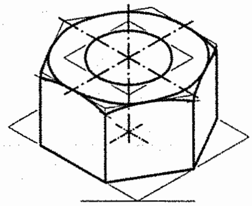


FIG. 17-74

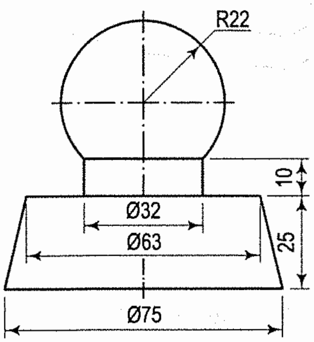


FIG. 17-75

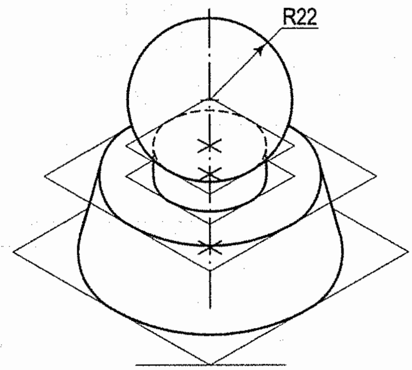


FIG. 17-76

Problem 17-44. (fig. 17-77): *Draw the isometric view of a square-headed bolt 24 mm diameter and 70 mm long, with a square neck 18 mm thick and a head, 40 mm square and 18 mm thick.*

See fig. 17-78.

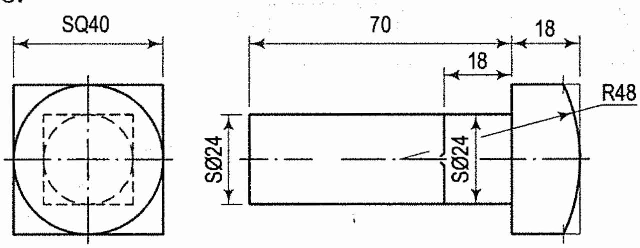


FIG. 17-77

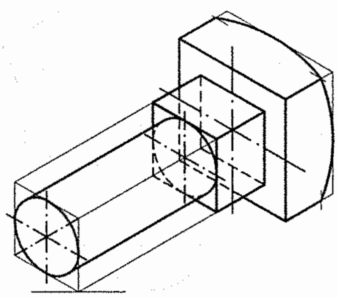


FIG. 17-78

Problem 17-45. *Draw the isometric view of the casting shown in fig. 17-79.* See fig. 17-80.

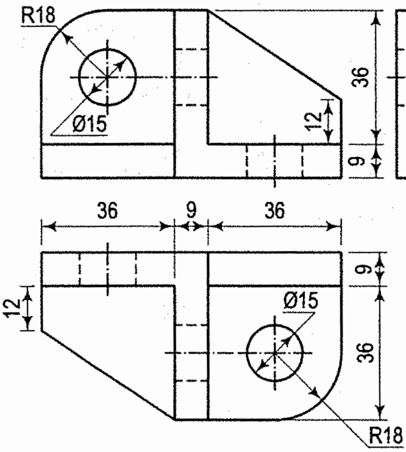


FIG. 17-79

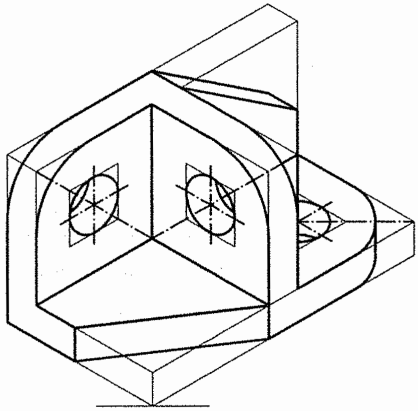


FIG. 17-80

Problem 17-46. *The projections of a casting are shown in fig. 17-81. Draw its isometric view.*

See fig. 17-82.

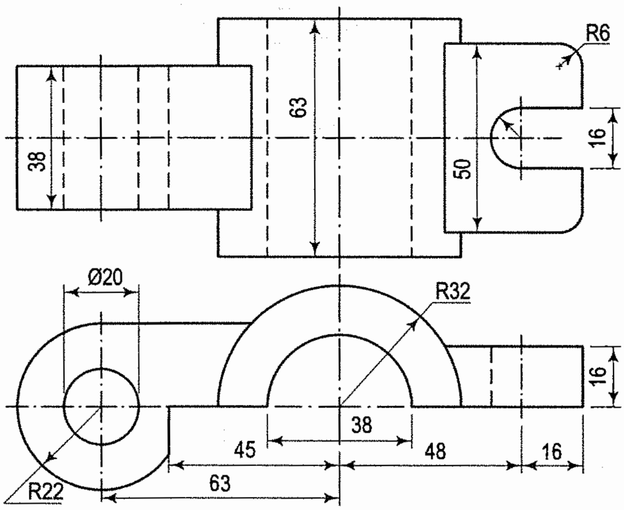


FIG. 17-81

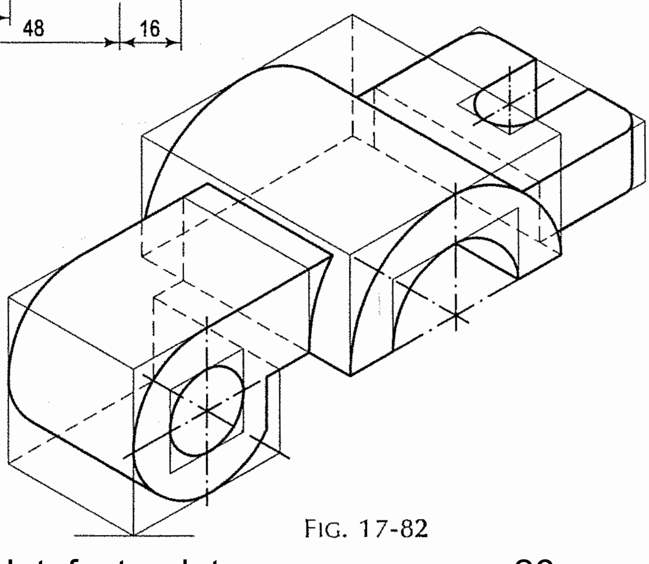


FIG. 17-82

Problem 17-47. *Draw the isometric view of the bracket shown in two views in fig. 17-83.*

See fig. 17-84.

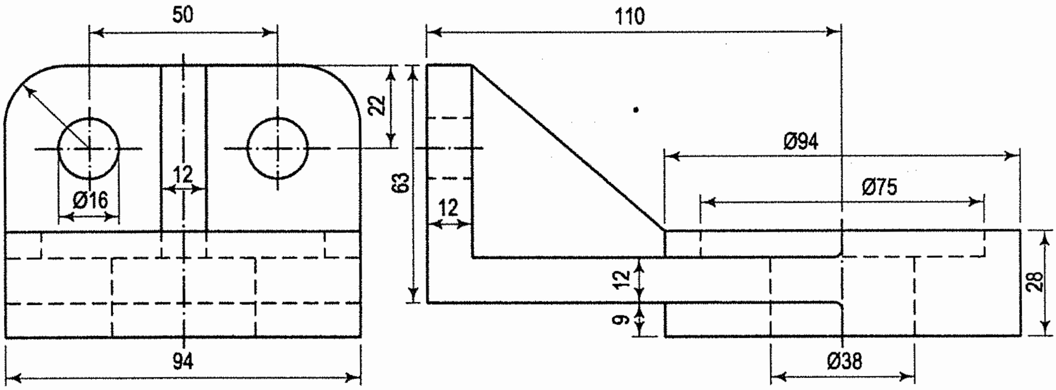


FIG. 17-83

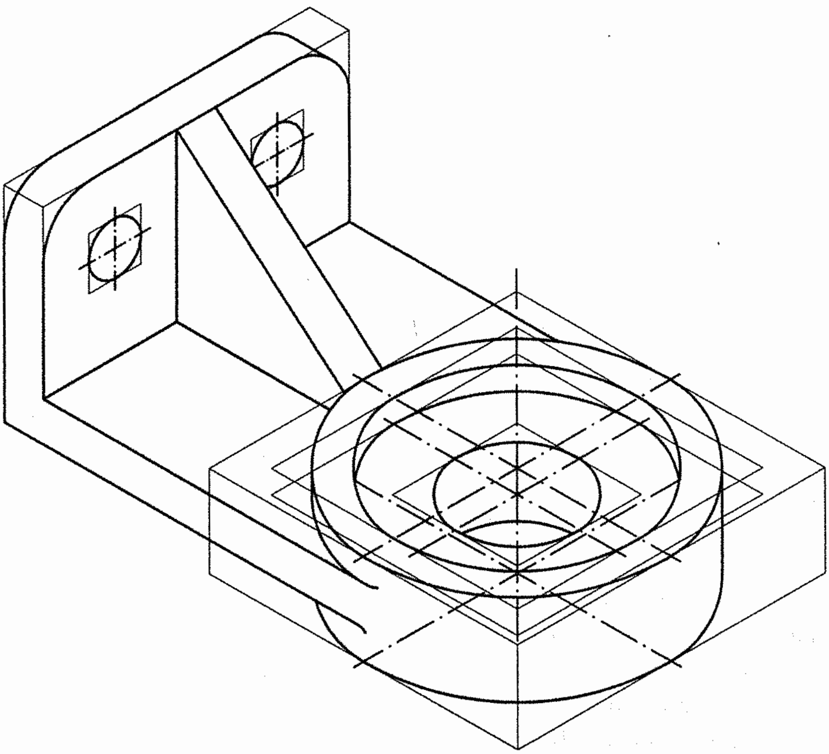


FIG. 17-84

Problem 17-48. *Draw the isometric view of the machine-handle shown in fig. 17-85.*

See fig. 17-86.

All measurements must be in isometric lengths except those for the diameters of spherical parts.

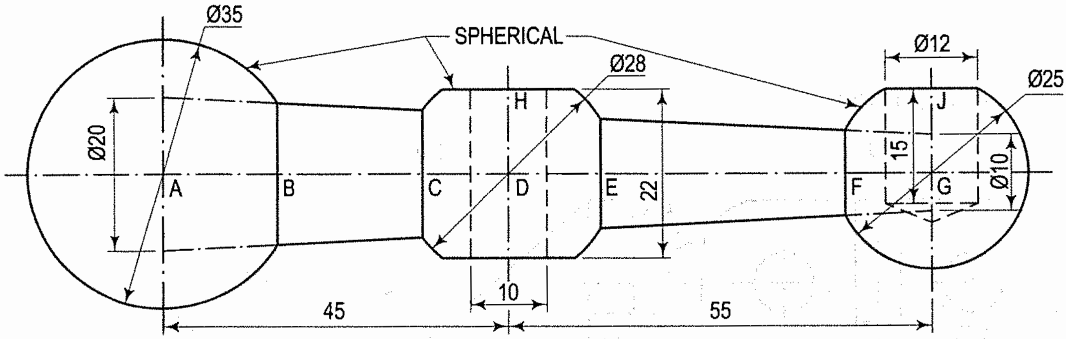


FIG. 17-85

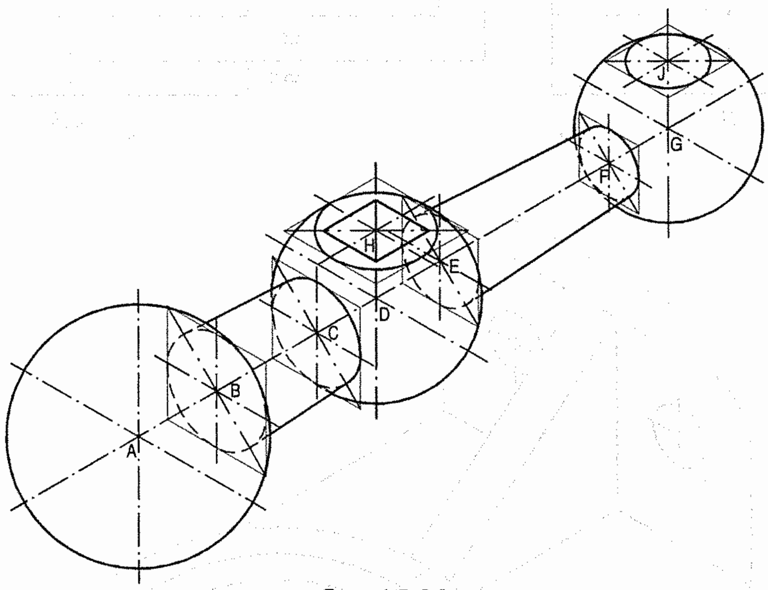


FIG. 17-86

(i) Draw an axis and mark on it the positions of points *A*, *B* etc.

(ii) At points *B*, *C*, *E* and *F*, draw ellipses for circular sections of the conical handle. Ellipses at *B* and *E* will be completely hidden.

(iii) With points *A*, *D* and *G* as centres, draw circles for the spheres, with their respective true radii.

(iv) Mark points *H* and *J* on the vertical axes through *D* and *G* respectively and draw ellipses for the respective sections of the spheres.

(v) Around *H*, draw a rhombus for the square hole.

(vi) The dotted lines for the depth of the holes are omitted.

Problem 17-49. *The front view of a stool having a square top and four legs is shown in fig. 17-87. Draw its isometric view.*

The legs lie along the slant edges of a frustum of a square pyramid (fig. 17-88).

Positions of the connecting horizontal strips between the legs at the top and at the bottom are determined by marking the heights along the axis and then drawing isometric lines upto the line *AB*, which shows the slope of the face of the frustum.

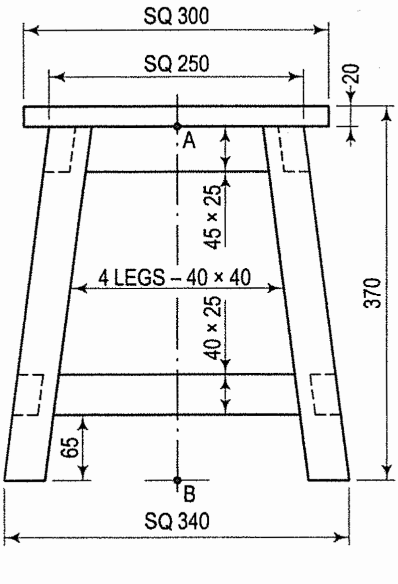


FIG. 17-87

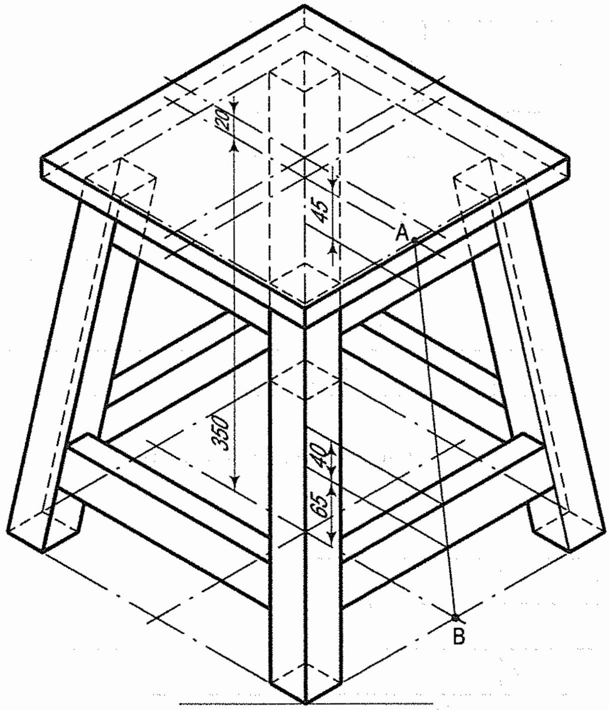


FIG. 17-88

EXERCISES 17

1. Projections of castings of various shapes are given in figs. 17-89 to 17-115. Draw the isometric view of each casting.

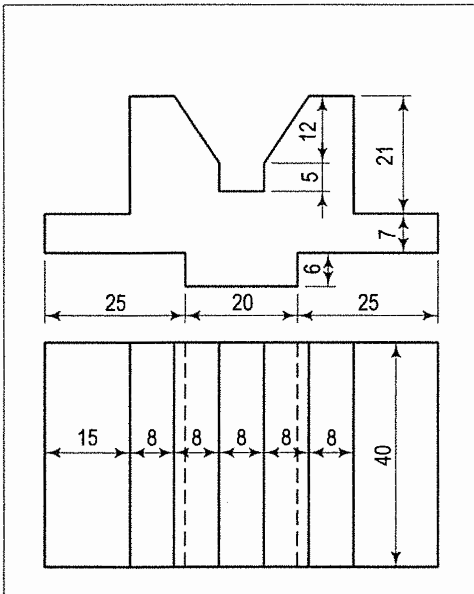


FIG. 17-89

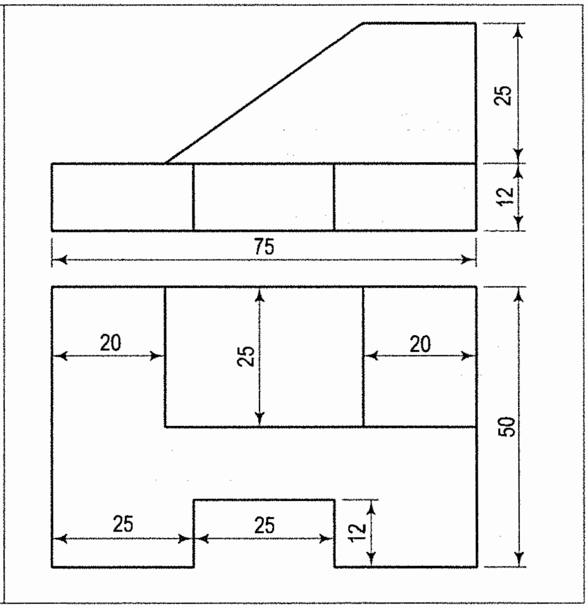
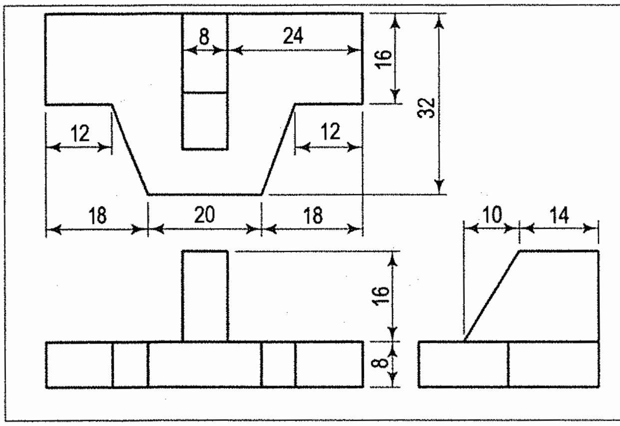


FIG. 17-90



(Third-angle projection)
FIG. 17-91

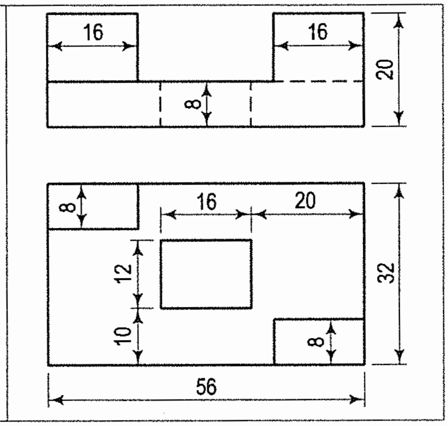


FIG. 17-92

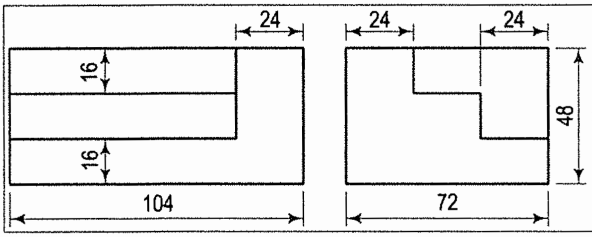


FIG. 17-93

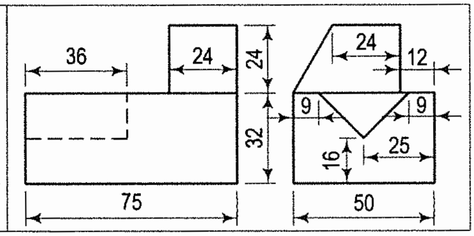
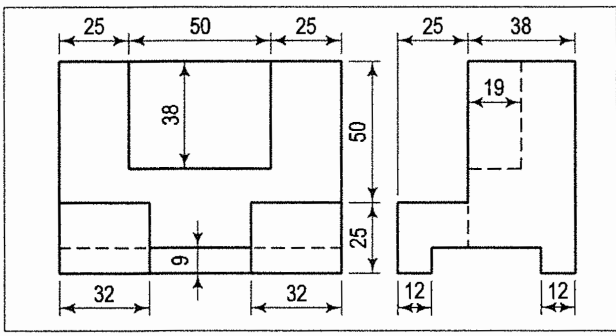
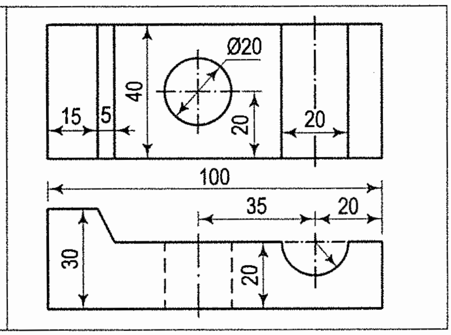


FIG. 17-94



(Third-angle projection)
FIG. 17-95



(Third-angle projection)
FIG. 17-96

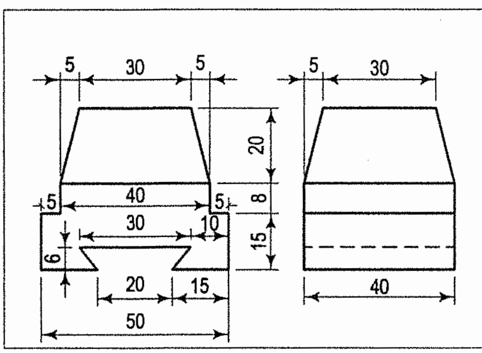


FIG. 17-97

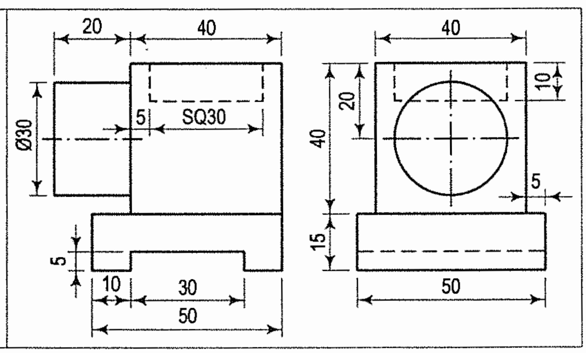


FIG. 17-98

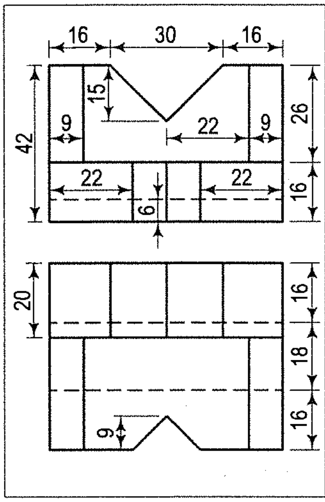


FIG. 17-99

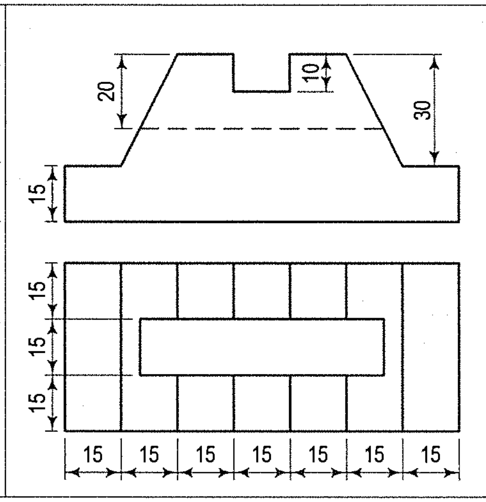


FIG. 17-100

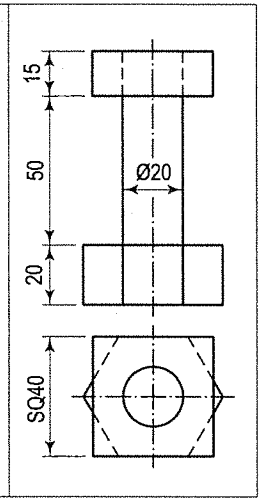


FIG. 17-101

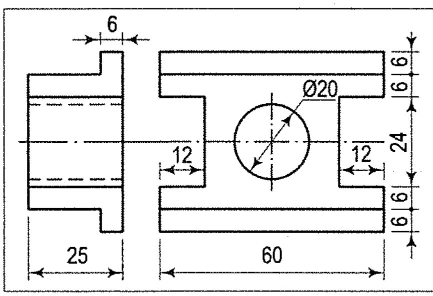


FIG. 17-102

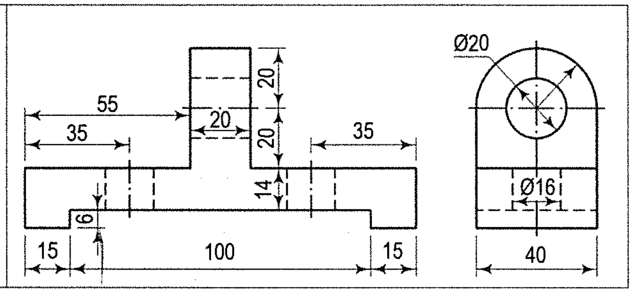


FIG. 17-103

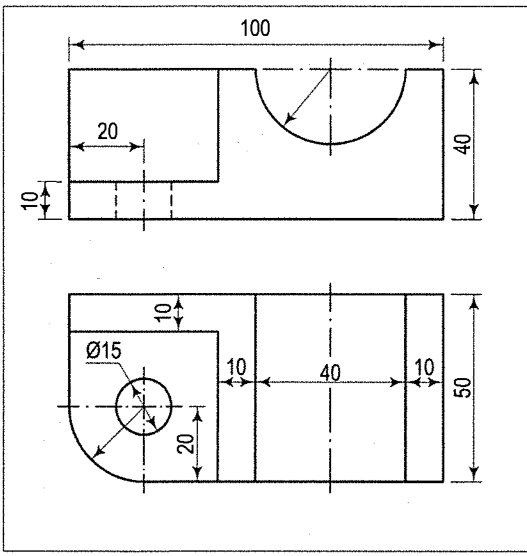
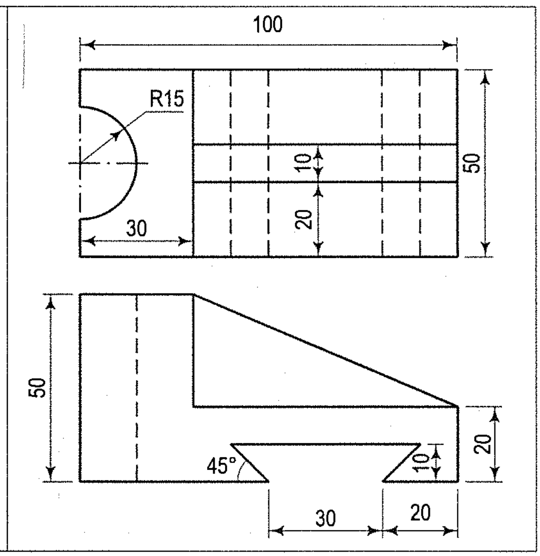


FIG. 17-104



(Third-angle projection)
FIG. 17-105

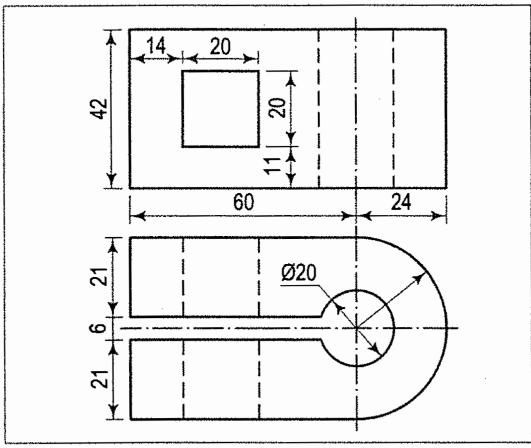
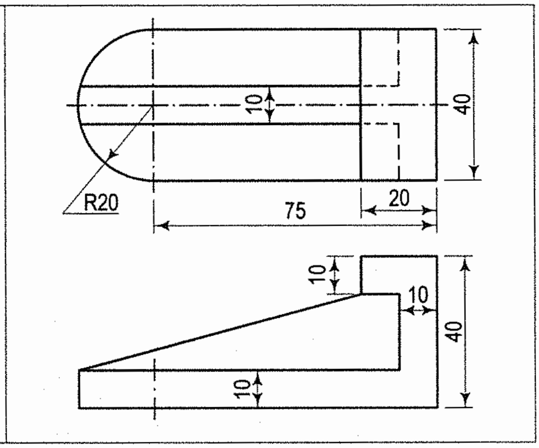


FIG. 17-106



(Third-angle projection)
FIG. 17-107

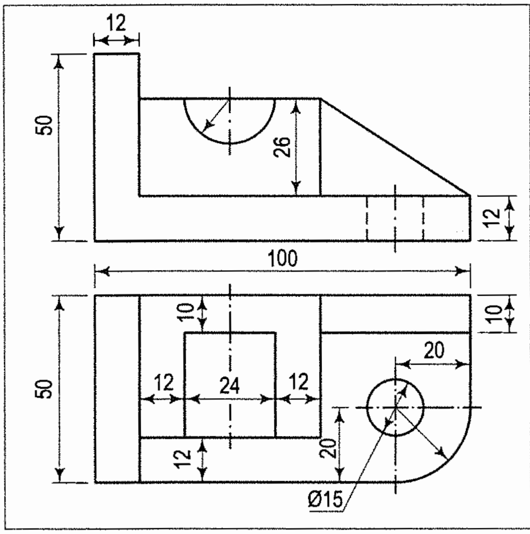
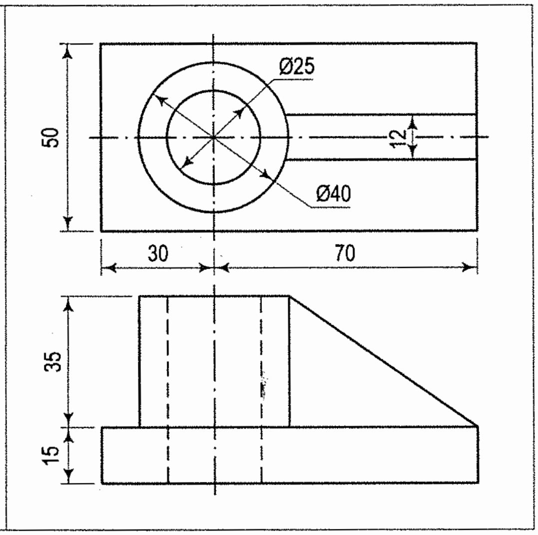


FIG. 17-108



(Third-angle projection)
FIG. 17-109

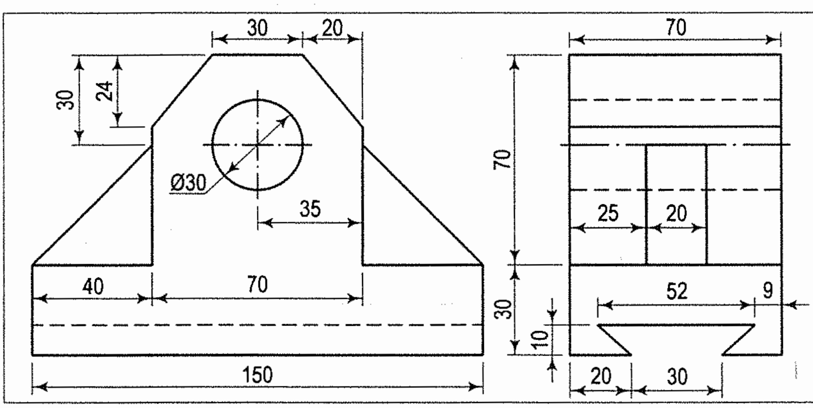


FIG. 17-110

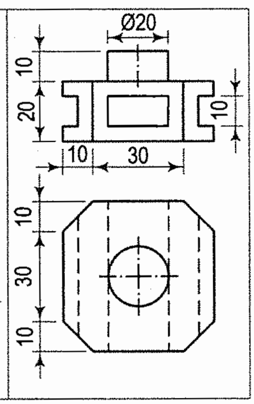
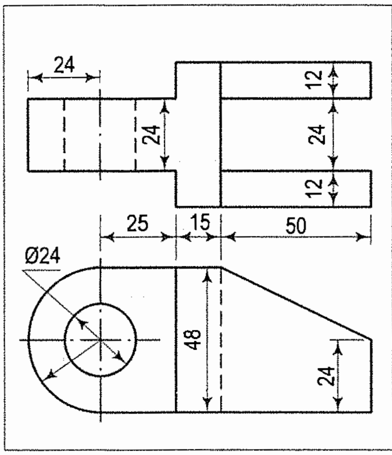


FIG. 17-111



(Third-angle projection)
FIG. 17-112

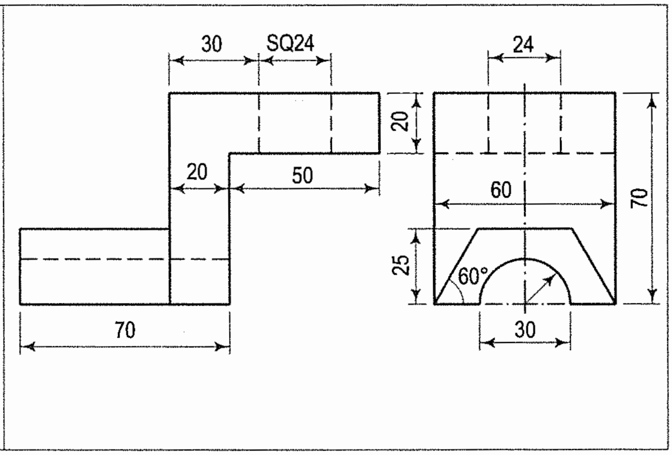


FIG. 17-113

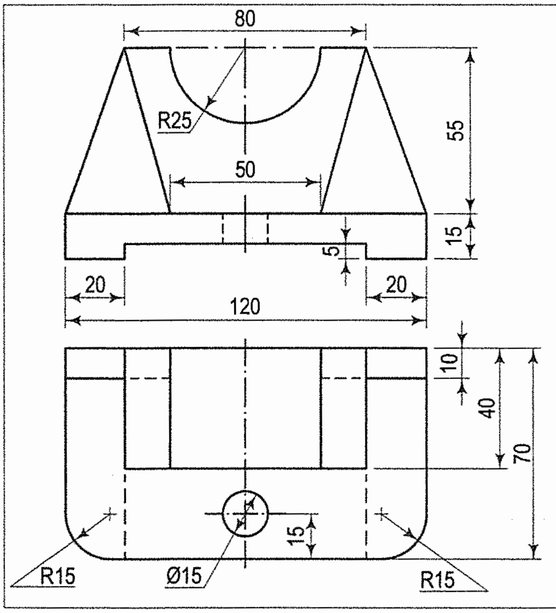


FIG. 17-114

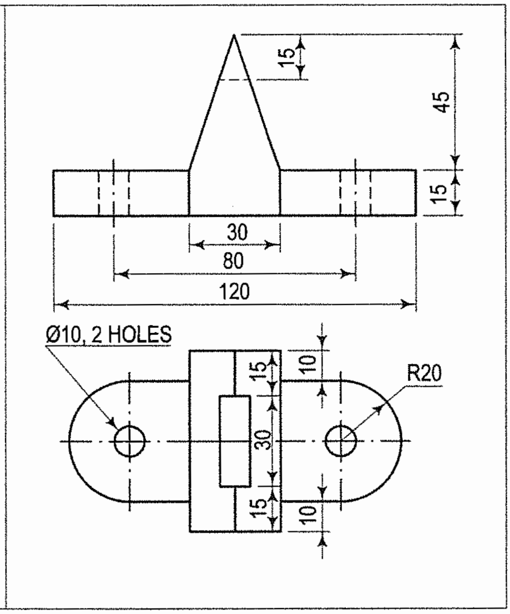
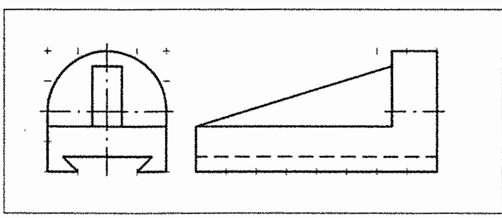


FIG. 17-115

2. Assuming unit length to be equal to 10 mm, draw the isometric views of objects shown in figs. 17-116 to 17-125.



(Third-angle projection)
FIG. 17-116

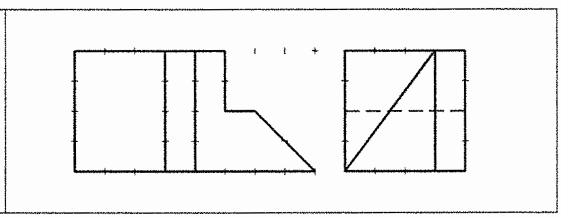


FIG. 17-117

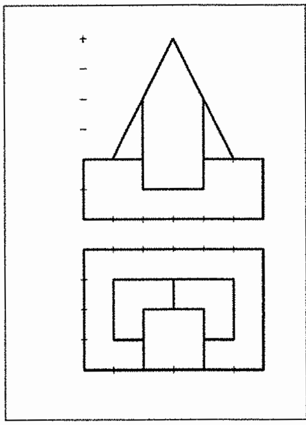


FIG. 17-118

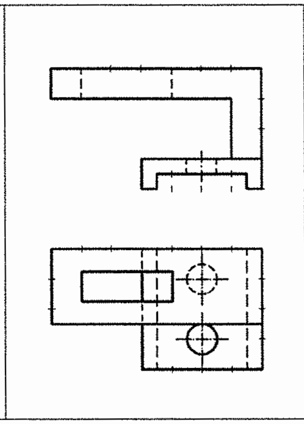


FIG. 17-119

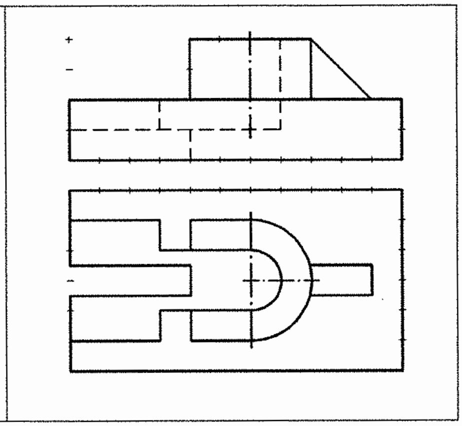
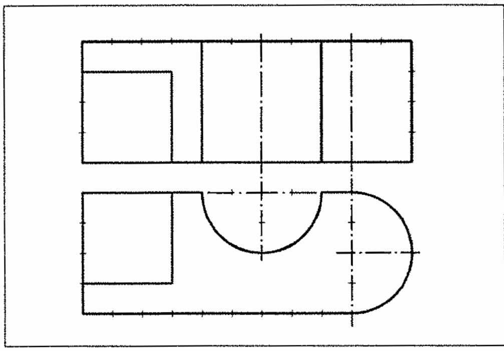


FIG. 17-120



(Third-angle projection)
FIG. 17-121

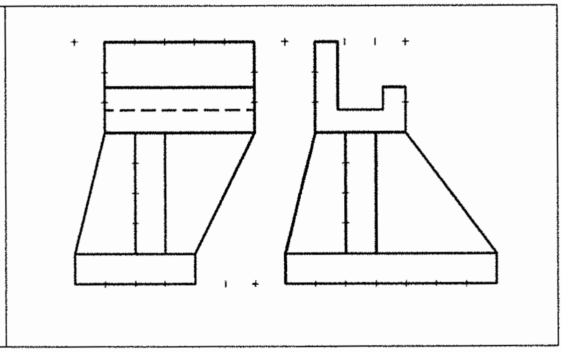


FIG. 17-122

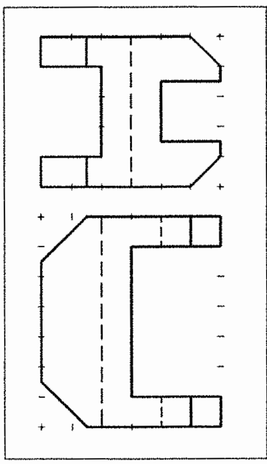
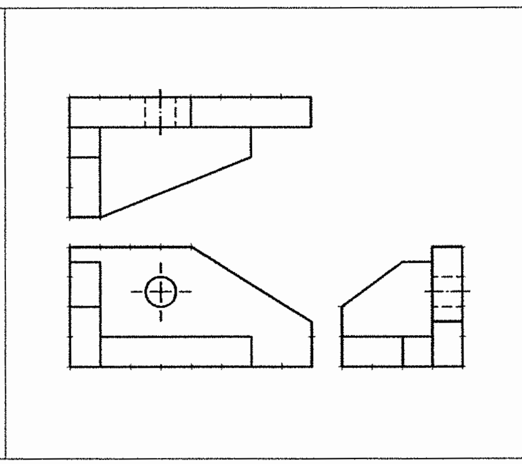


FIG. 17-123



(Third-angle projection)
FIG. 17-124

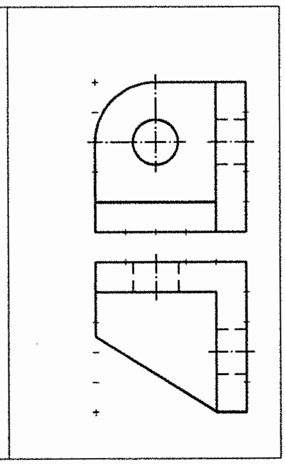


FIG. 17-125

3. Assuming simple graph of 10 mm grid, draw the isometric views of objects shown in fig. 17-126 and fig. 17-127.

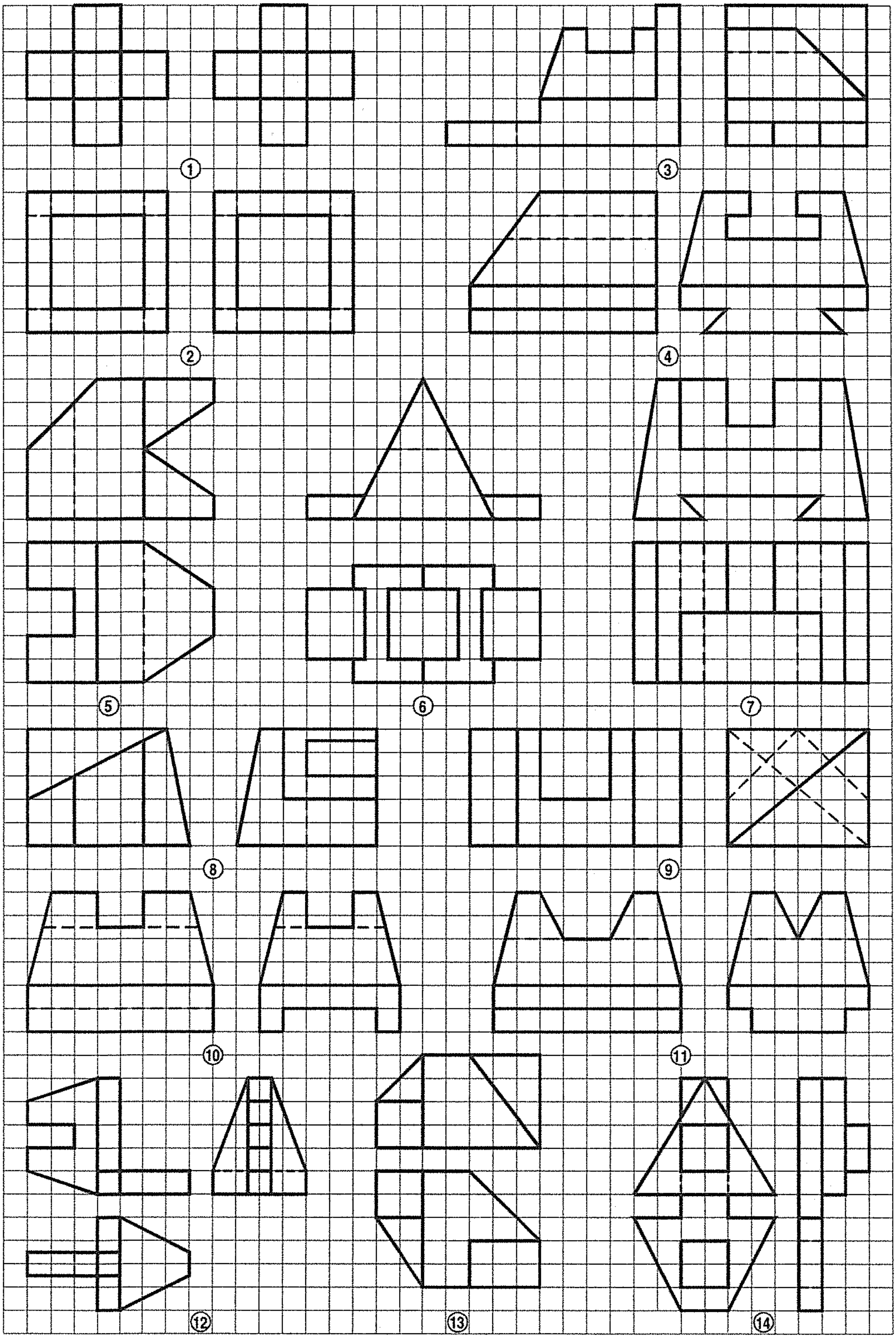


FIG. 17-126

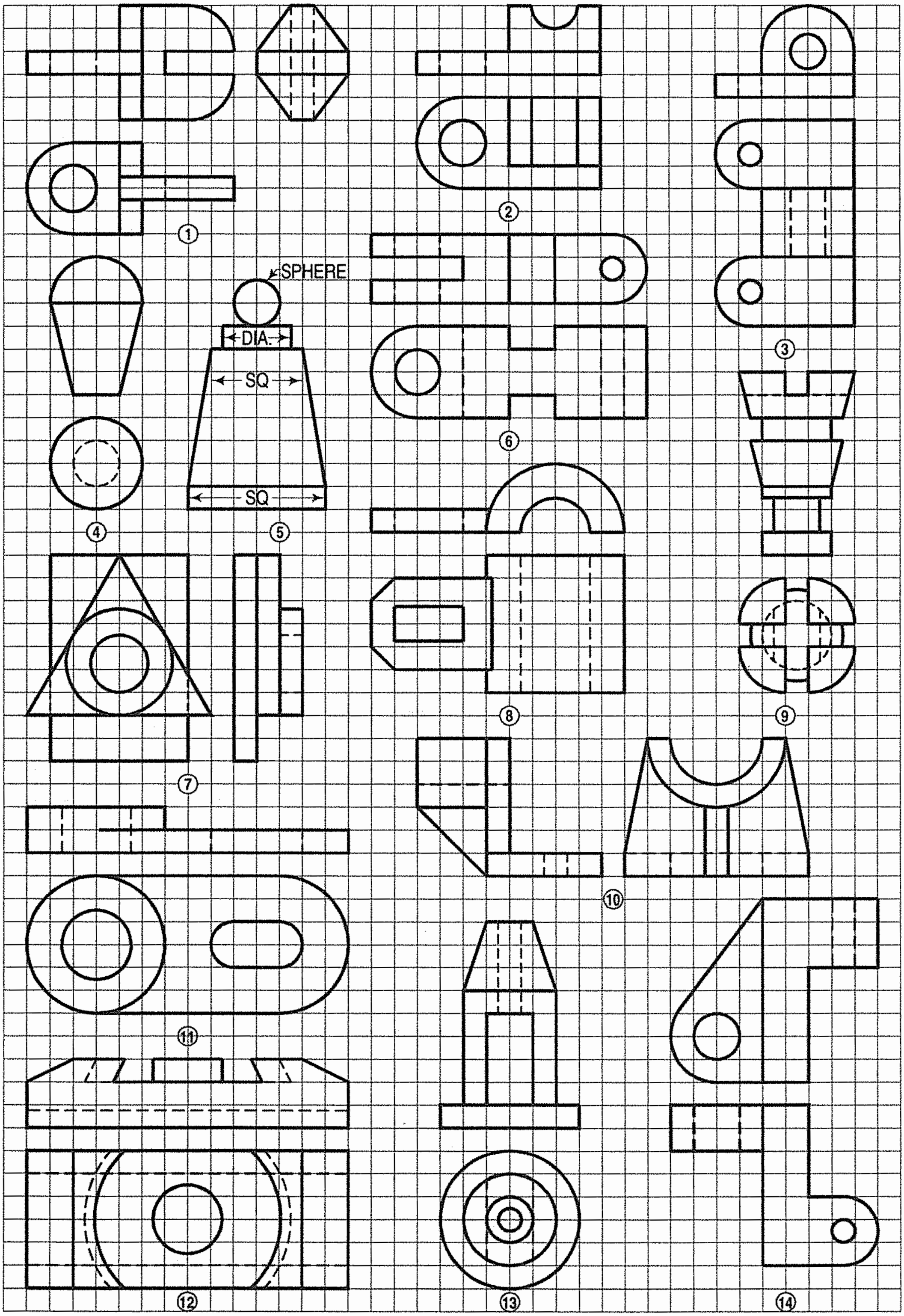


FIG. 17-127

4. Orthographic views of 35 objects with either (i) a line or (ii) lines or (iii) a view missing are given in fig. 17-128. Complete the given views. Also draw freehand, the isometric view of each object.

[*For answer see fig. 17-194.*]

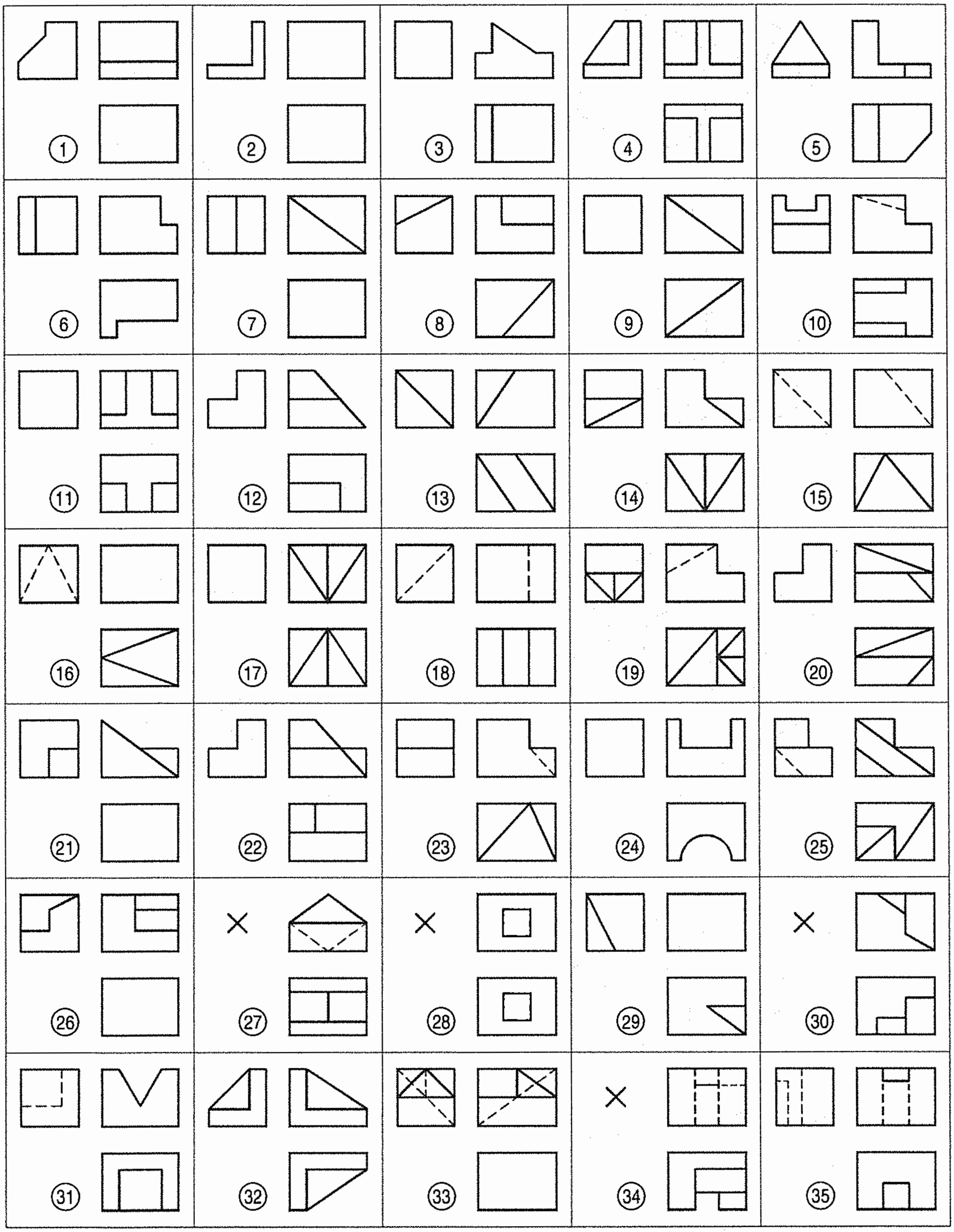
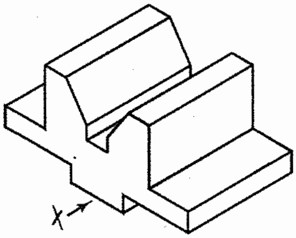
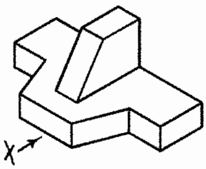


FIG. 17-128

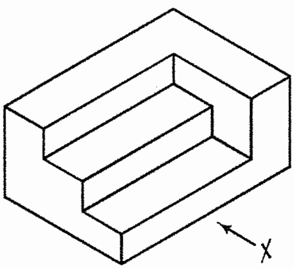
SOLUTIONS TO EXERCISES 17



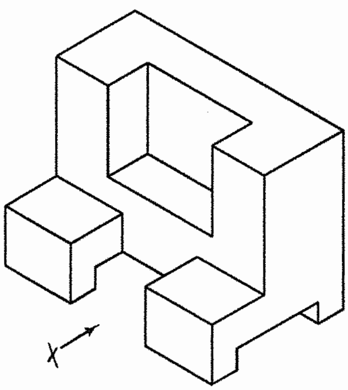
(Ex. 1. Fig. 17-89)
Fig. 17-129



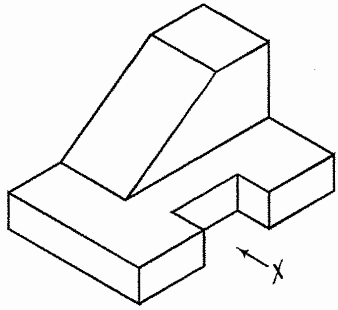
(Ex. 1. Fig. 17-91)
Fig. 17-131



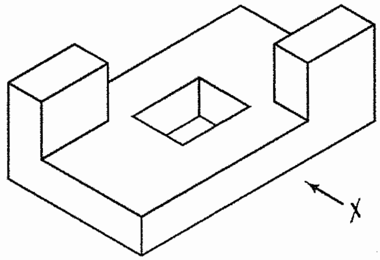
(Ex. 1. Fig. 17-93)
Fig. 17-133



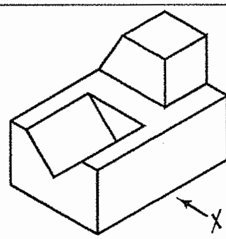
(Ex. 1. Fig. 17-95)
Fig. 17-135



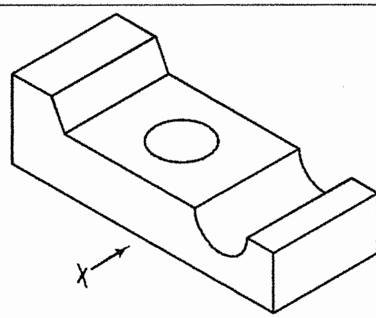
(Ex. 1. Fig. 17-90)
Fig. 17-130



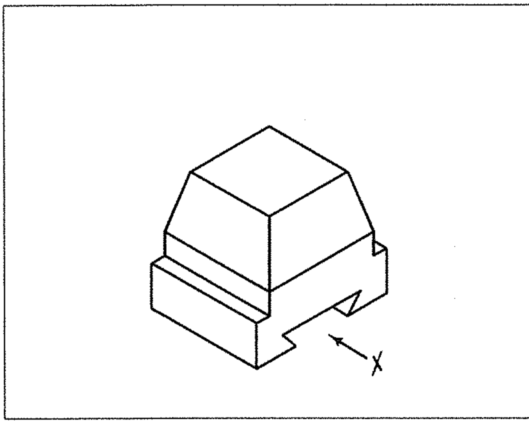
(Ex. 1. Fig. 17-92)
Fig. 17-132



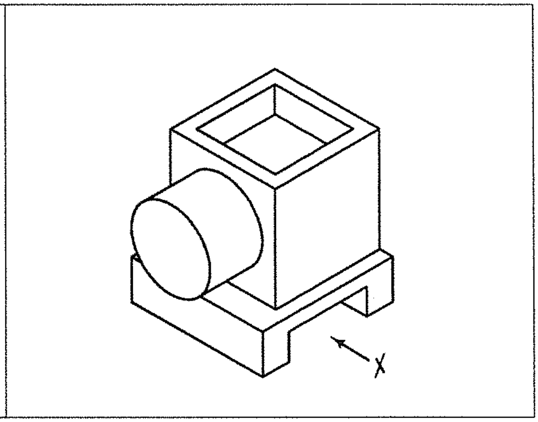
(Ex. 1. Fig. 17-94)
Fig. 17-134



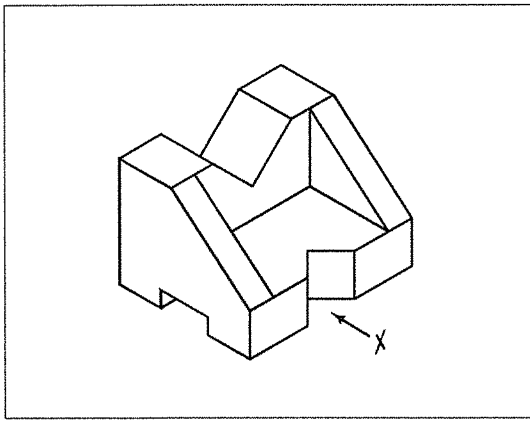
(Ex. 1. Fig. 17-96)
Fig. 17-136



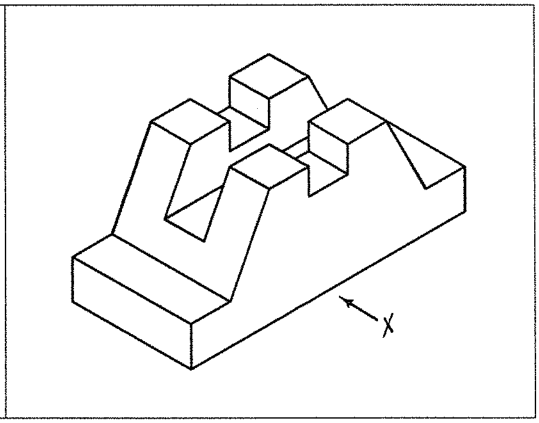
(Ex. 1. Fig. 17-97)
Fig. 17-137



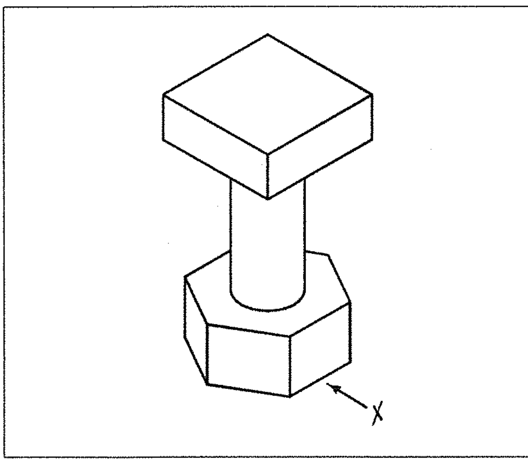
(Ex. 1. Fig. 17-98)
Fig. 17-138



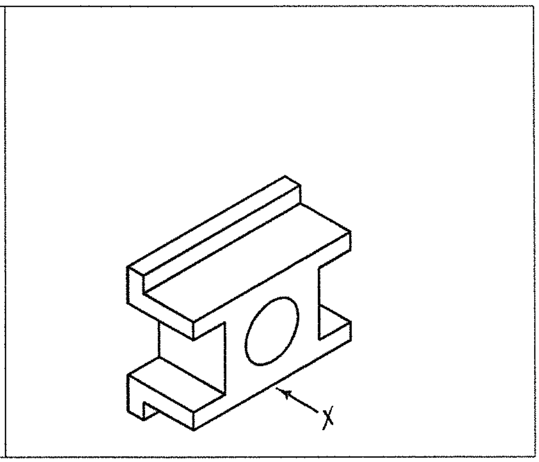
(Ex. 1. Fig. 17-99)
Fig. 17-139



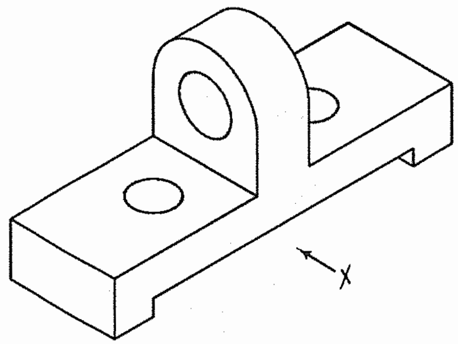
(Ex. 1. Fig. 17-100)
Fig. 17-140



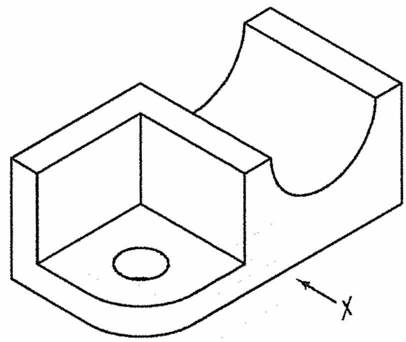
(Ex. 1. Fig. 17-101)
Fig. 17-141



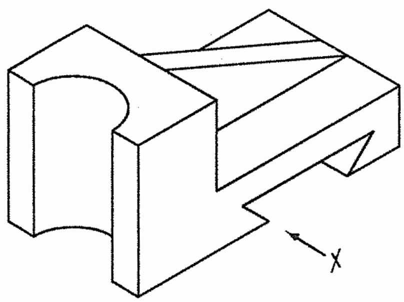
(Ex. 1. Fig. 17-102)
Fig. 17-142



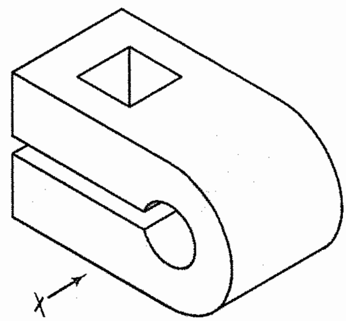
(Ex. 1. Fig. 17-103)
Fig. 17-143



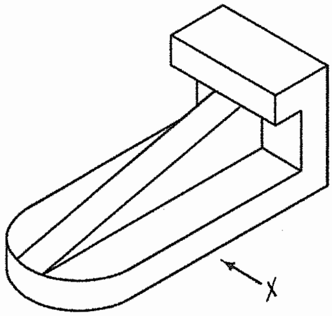
(Ex. 1. Fig. 17-104)
Fig. 17-144



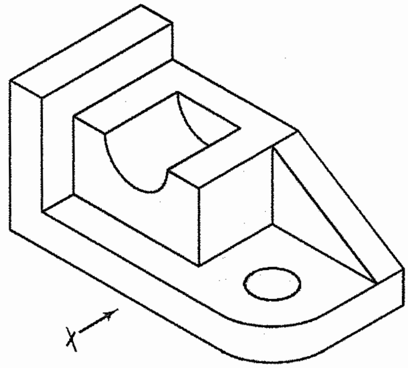
(Ex. 1. Fig. 17-105)
Fig. 17-145



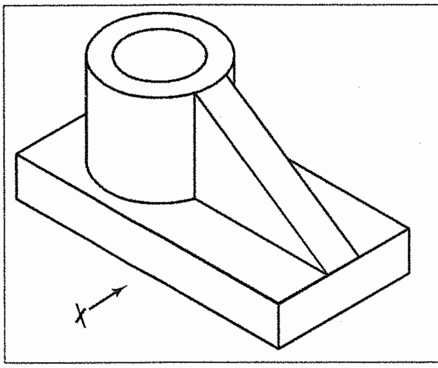
(Ex. 1. Fig. 17-106)
Fig. 17-146



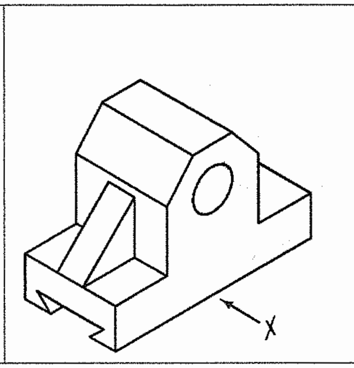
(Ex. 1. Fig. 17-107)
Fig. 17-147



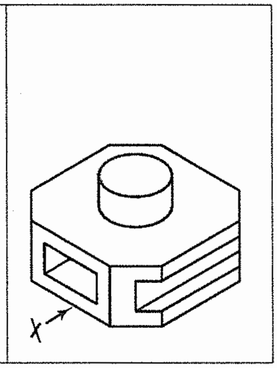
(Ex. 1. Fig. 17-108)
Fig. 17-148



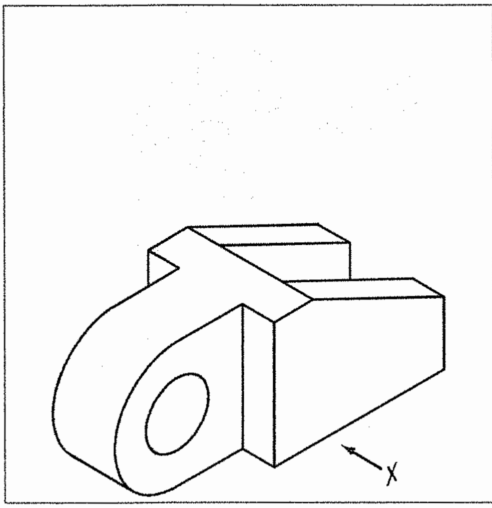
(Ex. 1. Fig. 17-109)
Fig. 17-149



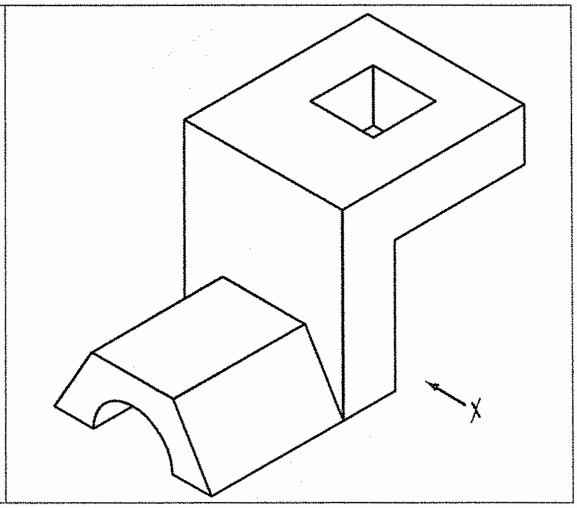
(Ex. 1. Fig. 17-110)
Fig. 17-150



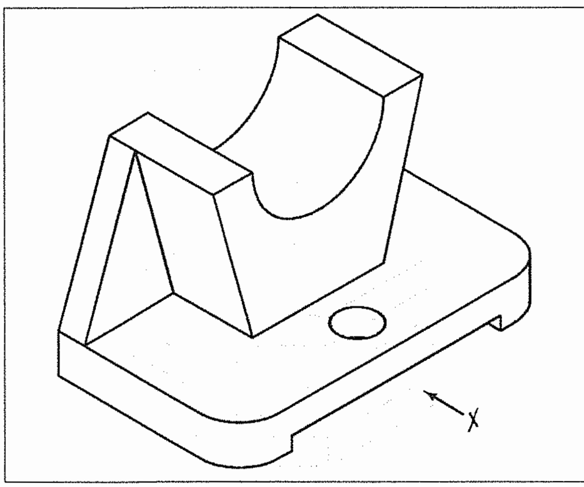
(Ex. 1. Fig. 17-111)
Fig. 17-151



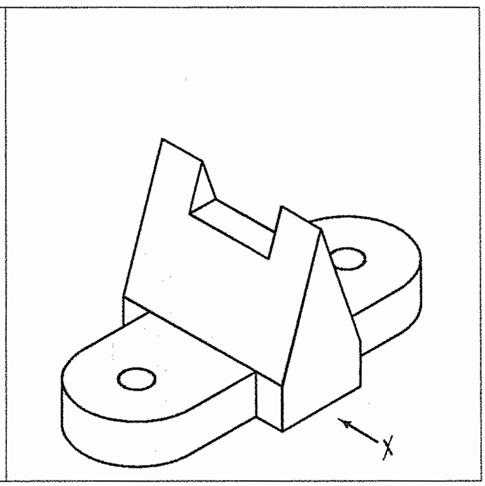
(Ex. 1. Fig. 17-112)
Fig. 17-152



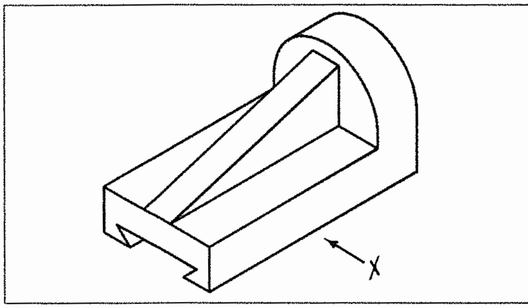
(Ex. 1. Fig. 17-113)
Fig. 17-153



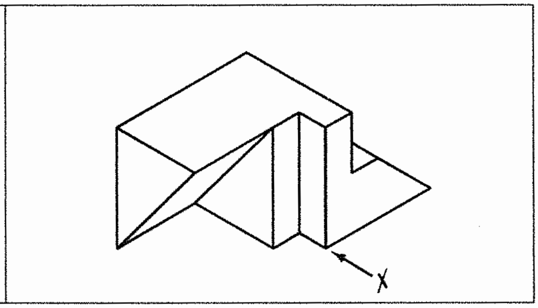
(Ex. 1. Fig. 17-114)
Fig. 17-154



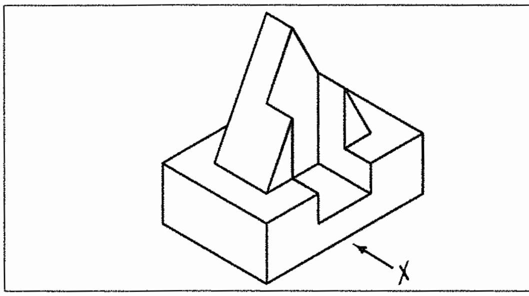
(Ex. 1. Fig. 17-115)
Fig. 17-155



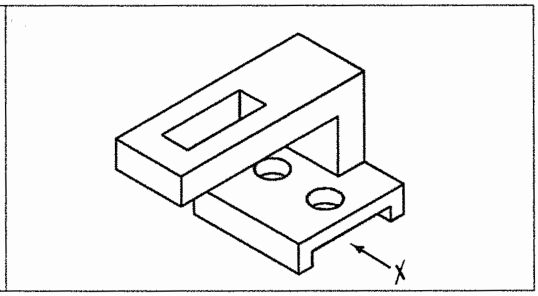
(EX. 2. FIG. 17-116)
FIG. 17-156



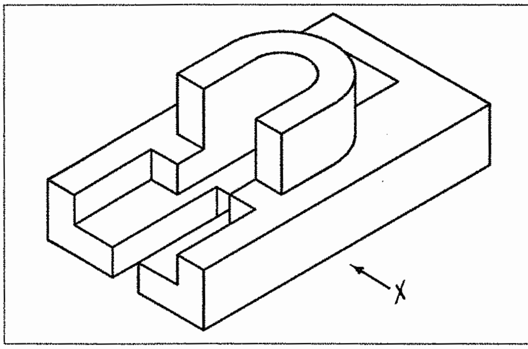
(EX. 2. FIG. 17-117)
FIG. 17-157



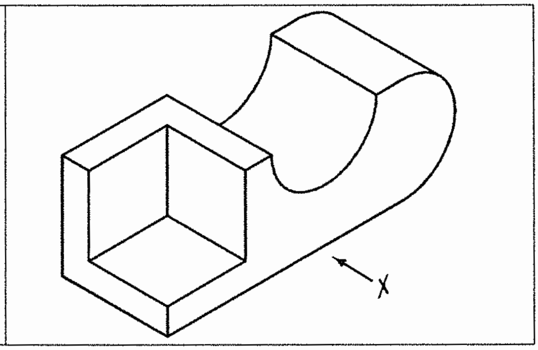
(EX. 2. FIG. 17-118)
FIG. 17-158



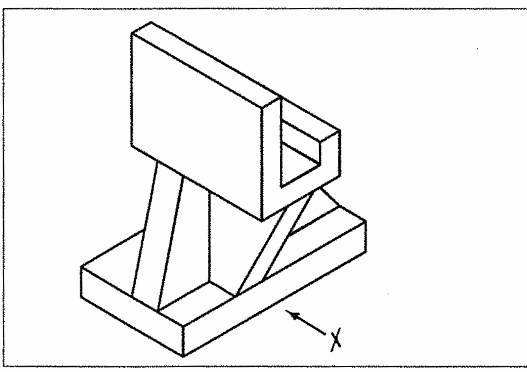
(EX. 2. FIG. 17-119)
FIG. 17-159



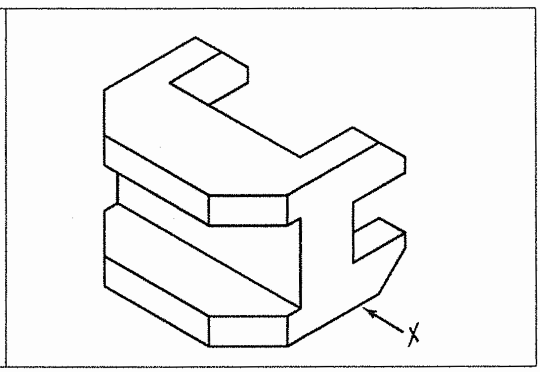
(EX. 2. FIG. 17-120)
FIG. 17-160



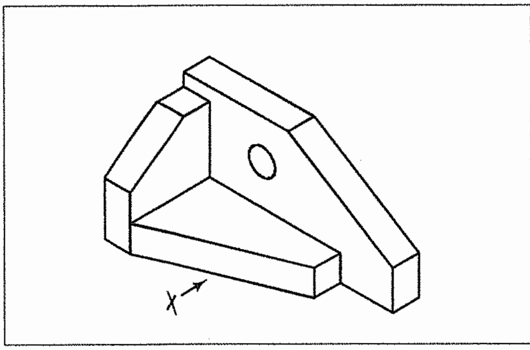
(EX. 2. FIG. 17-121)
FIG. 17-161



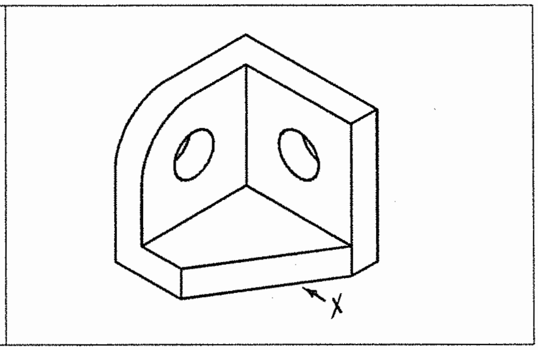
(EX. 2. FIG. 17-122)
FIG. 17-162



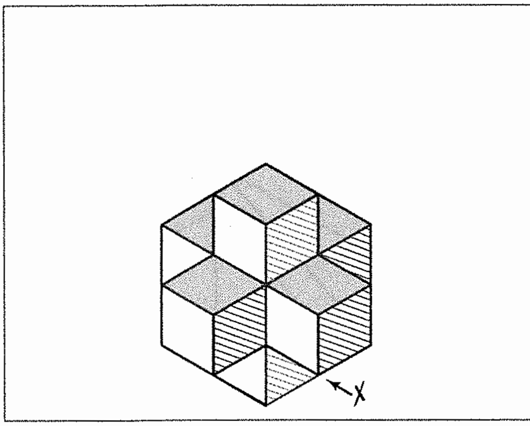
(EX. 2. FIG. 17-123)
FIG. 17-163



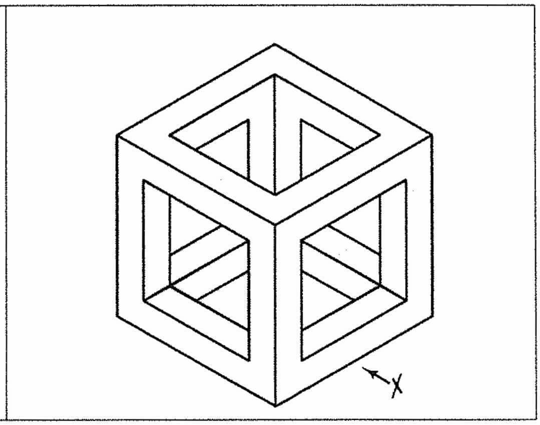
(Ex. 2. Fig. 17-124)
Fig. 17-164



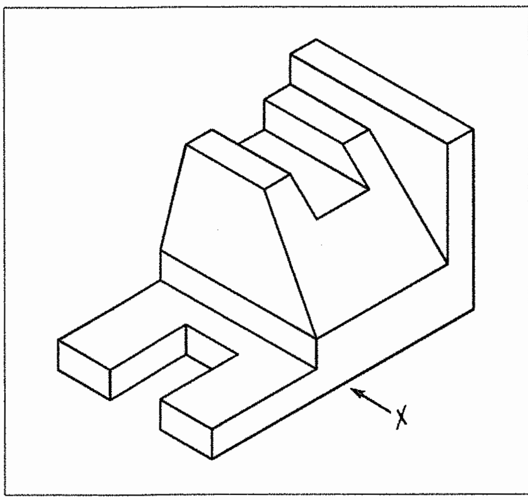
(Ex. 2. Fig. 17-125)
Fig. 17-165



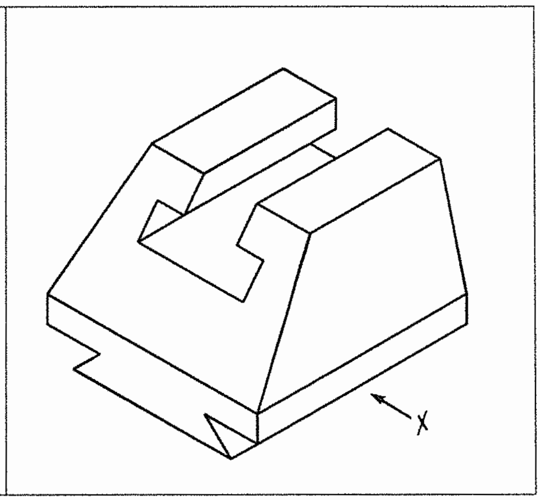
[Ex. 3. Fig. 17-126(1)]
Fig. 17-166



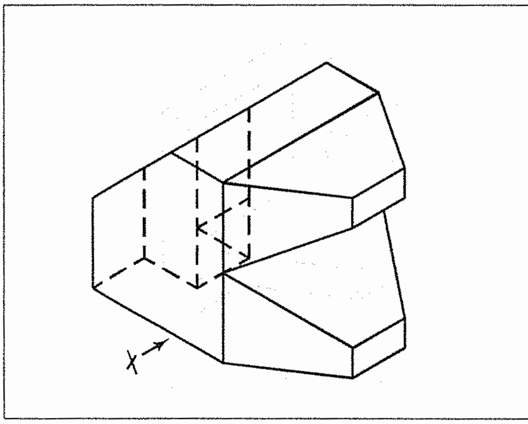
[Ex. 3. Fig. 17-126(2)]
Fig. 17-167



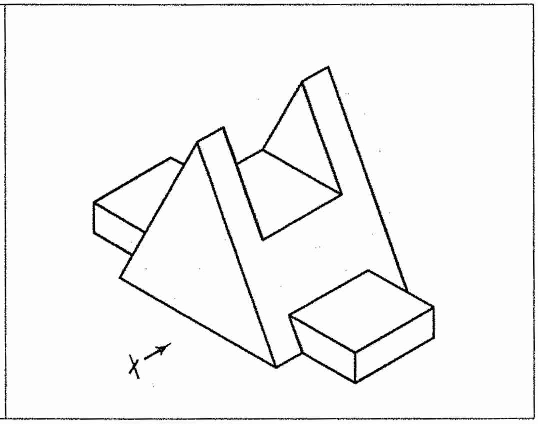
[Ex. 3. Fig. 17-126(3)]
Fig. 17-168



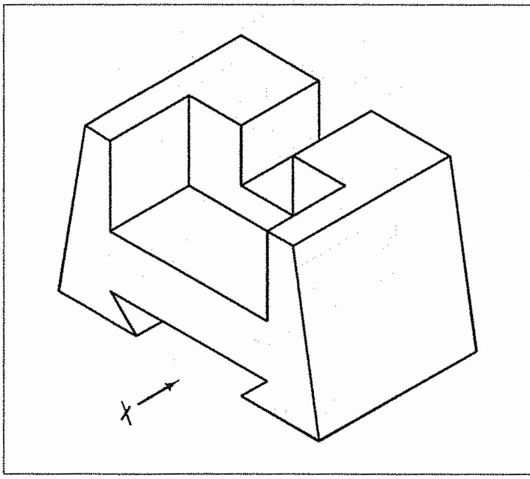
[Ex. 3. Fig. 17-126(4)]
Fig. 17-169



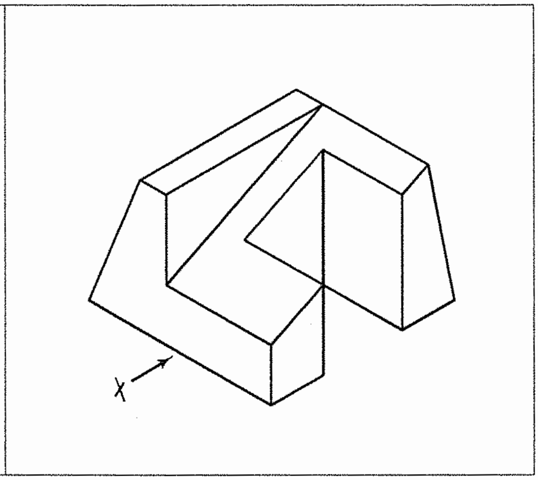
[Ex. 3. Fig. 17-126(5)]
Fig. 17-170



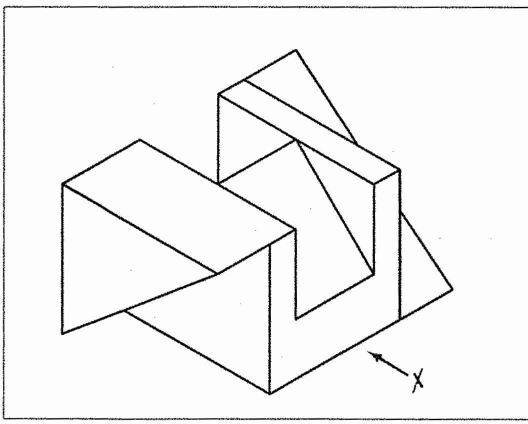
[Ex. 3. Fig. 17-126(6)]
Fig. 17-171



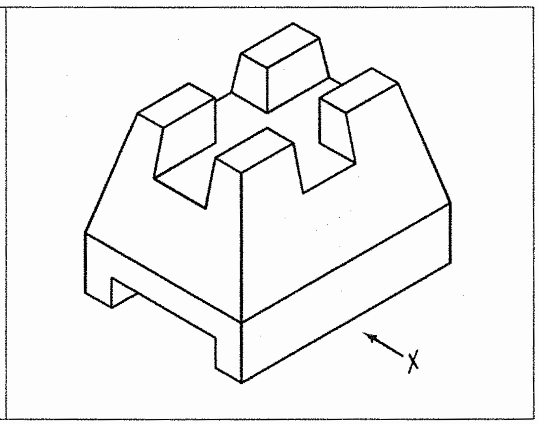
[Ex. 3. Fig. 17-126(7)]
Fig. 17-172



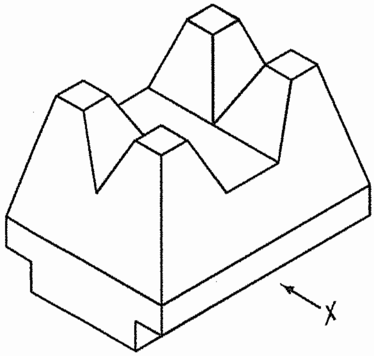
[Ex. 3. Fig. 17-126(8)]
Fig. 17-173



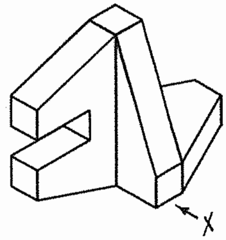
[Ex. 3. Fig. 17-126(9)]
Fig. 17-174



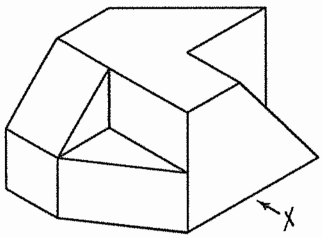
[Ex. 3. Fig. 17-126(10)]
Fig. 17-175



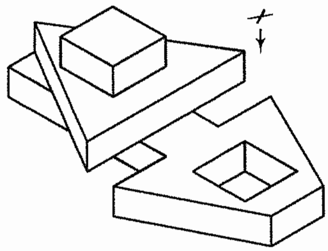
[Ex. 3. Fig. 17-126(11)]
Fig. 17-176



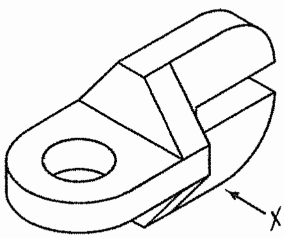
[Ex. 3. Fig. 17-126(12)]
Fig. 17-177



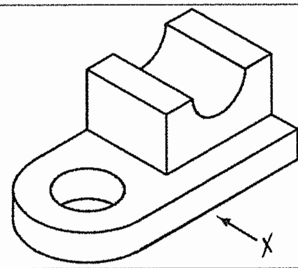
[Ex. 3. Fig. 17-126(13)]
Fig. 17-178



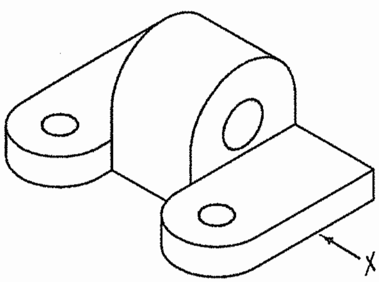
[Ex. 3. Fig. 17-126(14)]
Fig. 17-179



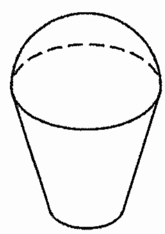
[Ex. 3. Fig. 17-127(1)]
Fig. 17-180



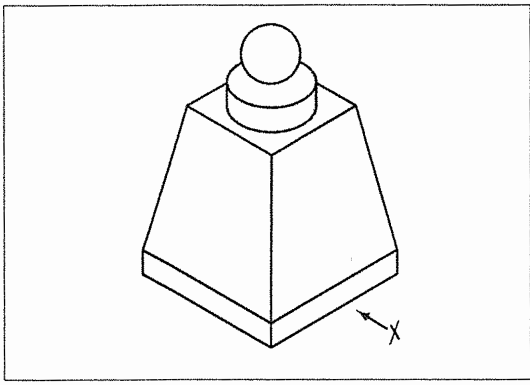
[Ex. 3. Fig. 17-127(2)]
Fig. 17-181



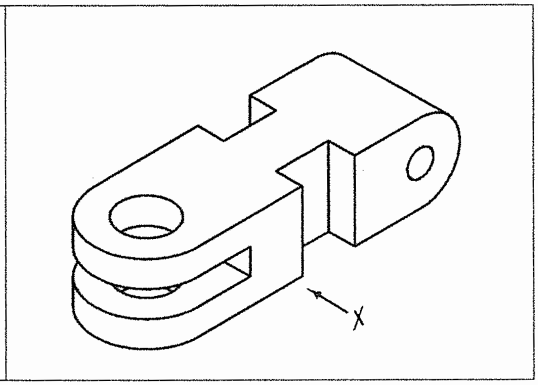
[Ex. 3. Fig. 17-127(3)]
Fig. 17-182



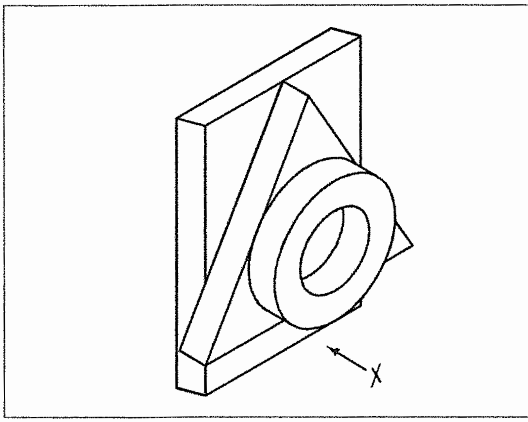
[Ex. 3. Fig. 17-127(4)]
Fig. 17-183



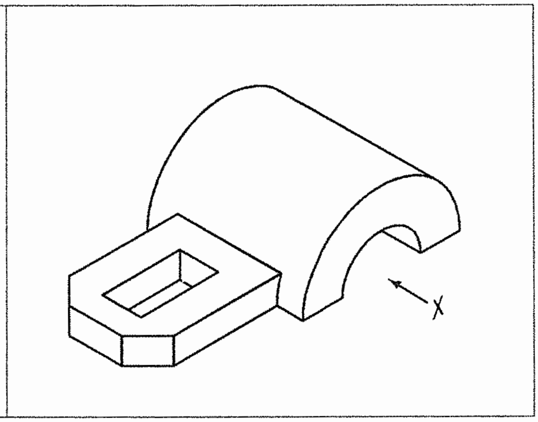
[Ex. 3. Fig. 17-127(5)]
Fig. 17-184



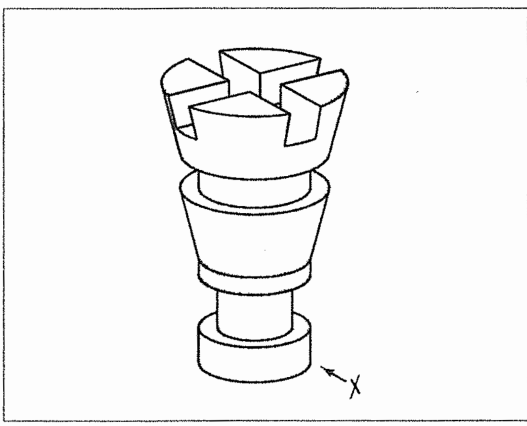
[Ex. 3. Fig. 17-127(6)]
Fig. 17-185



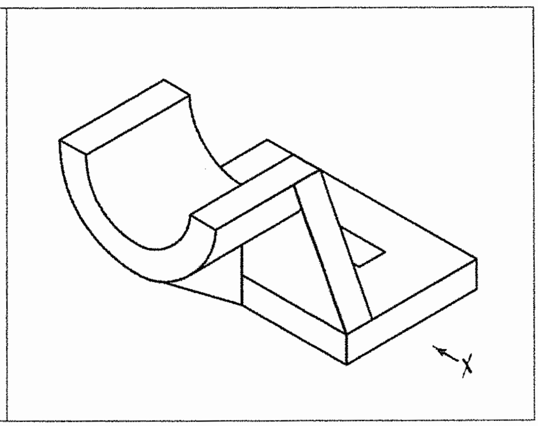
[Ex. 3. Fig. 17-127(7)]
Fig. 17-186



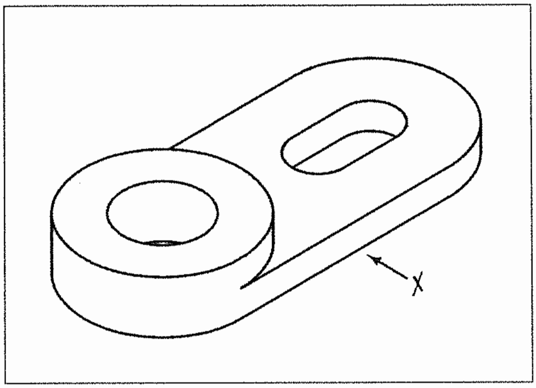
[Ex. 3. Fig. 17-127(8)]
Fig. 17-187



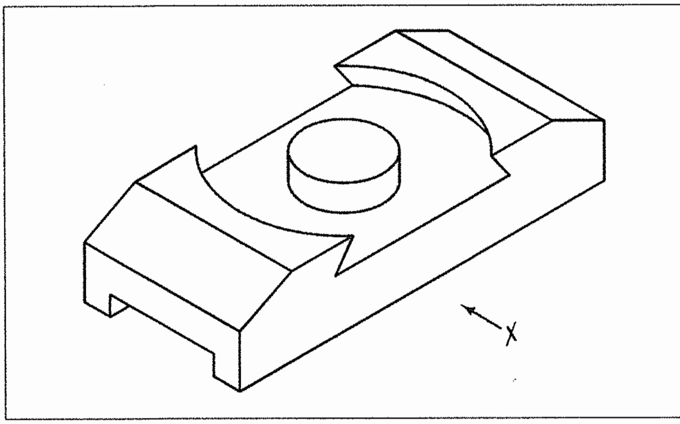
[Ex. 3. Fig. 17-127(9)]
Fig. 17-188



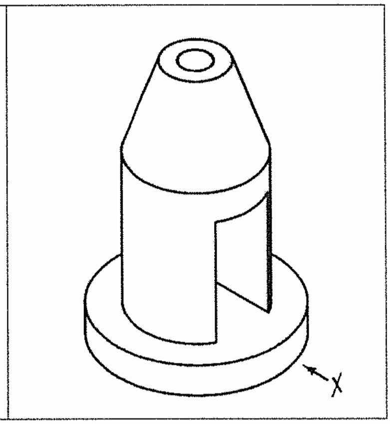
[Ex. 3. Fig. 17-127(10)]
Fig. 17-189



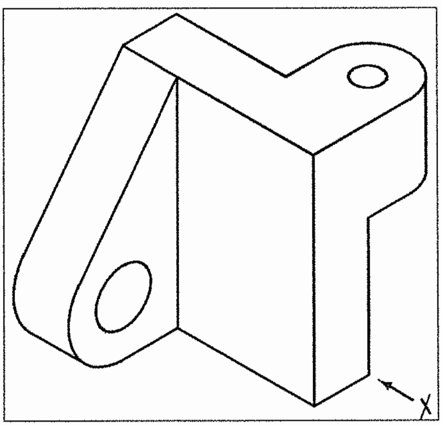
[Ex. 3. Fig. 17-127(11)]
Fig. 17-190



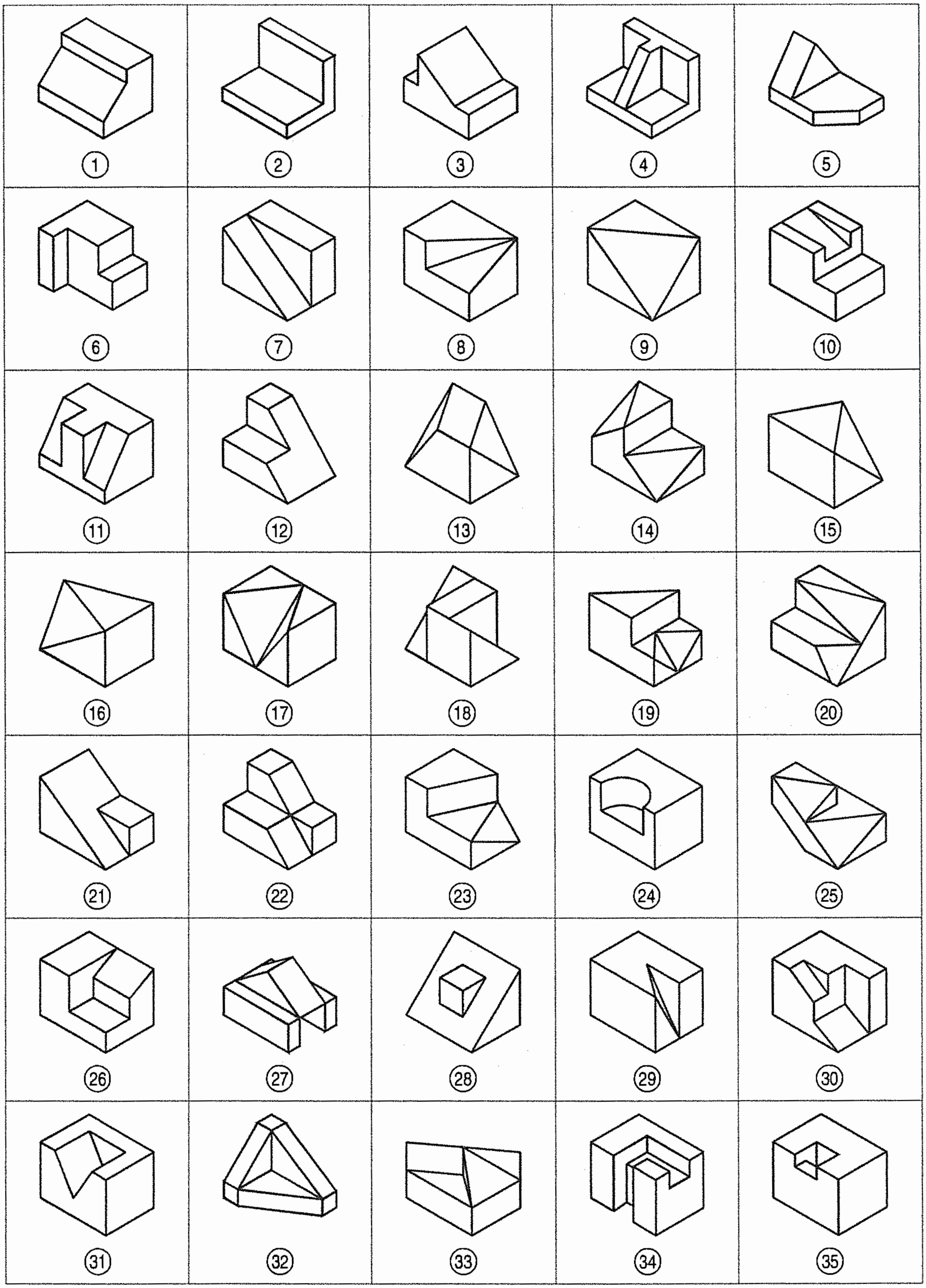
[Ex. 3. Fig. 17-127(12)]
Fig. 17-191



[Ex. 3. Fig. 17-127(13)]
Fig. 17-192



[Ex. 3. Fig. 17-127(14)]
Fig. 17-193



(Ex. 4. Fig. 17-128)
Fig. 17-194